PERGAMON

Automatica 37 (2001) 61–69

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www.elsevier.com/locate/automatica

Brief Paper

Robust control of parabolic PDE systems with time-dependent spatial domains☆

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Received 24 April 1999; revised 3 December 1999; received in final form 12 May 2000

Abstract

We synthesize robust nonlinear static output feedback controllers for systems of quasi-linear parabolic partial differential equations with time-dependent spatial domains and uncertain variables. The controllers are successfully applied to a typical diffusion–reaction process with moving boundary and uncertainty. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Galerkin's method; Robust stabilization; Static output feedback control; Diffusion–reaction processes with moving boundaries

1. Introduction

Quasi-linear parabolic partial differential equation (PDE) systems with time-dependent spatial domains arise very frequently in the modeling of diffusion–reaction processes with moving boundaries (e.g., crystal growth, metal casting, gas–solid reaction systems and coatings). In addition to being nonlinear and time-varying, such systems are usually characterized by the presence of uncertain variables owing to unknown or partially known process parameters and disturbances. Uncertain variables, if they are not appropriately accounted for in the controller design, can significantly deteriorate the nominal closed-loop performance, and even lead to closed-loop instability.

In the past, most of the research on control of linear/quasi-linear parabolic PDEs with uncertainty has focused on systems with fixed spatial domains. Initially, H^{∞} control algorithms were developed for linear parabolic PDE systems in the frequency domain (e.g., Palazoglu & Owens, 1987; Curtain & Glover, 1986; Gauthier & Xu, 1989). The development of concrete relations between frequency-domain and state-space control theoretic concepts (e.g., controllability and observability) in Jacobson and Nett (1988) allowed the derivation of the state-space counterparts of the frequency domain H^{∞} results (see, for example, the book by Keulen (1993)). More recently, Lyapunov-based robust controller design methods were proposed (Ydstie & Alonso, 1997; Christofides, 1998; Christofides & Baker, 1999) for quasi-linear parabolic PDEs with uncertainty. Other results on control of PDE systems with uncertainty include disturbance decoupling for linear parabolic PDEs (Curtain, 1986), Lyapunov-based (Christofides & Daoutidis, 1998) and sliding-mode control (Hanczyc & Palazoglu, 1995) of hyperbolic PDE systems, Lyapunov-based (Kang & Ito, 1992) and H^{∞} (e.g., Burns & Ou, 1994; King & Qu, 1995) control of Navier–Stokes equations, as well as adaptive control (e.g., Byrnes, 1987; Wen & Balas, 1989; Demetriou, 1994).

Even though the subject of feedback control of parabolic PDE systems with fixed spatial domains has received great attention, few results are available on control and estimation of parabolic PDE systems with time-dependent spatial domains. Such systems exhibit properties which are not encountered in parabolic PDEs with fixed spatial domains including variation of controllability and observability properties with time and space, and they are characterized by the presence of time-dependent terms that describe convective transport owing to variation of the length of the domain with time. These characteristics, if not appropriately accounted for in the

☆This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Frank Allgöwer under the direction of Editor Sigurd Skogestad.

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PII: S0005-1098(00)00123-0

synthesis of the controller, may lead to poor performance and closed-loop instability (see, for example, Armaou & Christofides, 1999a,b). In the area of control of parabolic PDEs with time-dependent spatial domains, important contributions include Wang's work on the synthesis of linear optimal controllers (see, for example, Wang, 1990) and their application to temperature and thermal gradient regulation in crystal growth processes (Wang, 1995), as well as the synthesis of nonlinear distributed state estimators using stochastic methods in Ray and Seinfeld (1975). More recently, we proposed in Armaou and Christofides (1999a,b) a method for the synthesis of nonlinear dynamic output feedback controllers for a broad class of quasi-linear parabolic PDEs with time-dependent spatial domains that enforce stability and output tracking in the closed-loop infinite-dimensional system. The controllers are synthesized on the basis of low-order approximations of the PDE system obtained through combination of Galerkin's method with approximate inertial manifolds. Despite this progress, at this stage, there is no general method for robust nonlinear control of highly uncertain parabolic PDE systems with time-dependent spatial domains.

This paper focuses on a broad class of uncertain quasi-linear parabolic PDE systems with time-dependent spatial domains whose dynamics can be partitioned into slow and fast ones. Such systems arise naturally in the modeling of diffusion-reaction processes with moving boundaries. The objective is to develop a general method for the synthesis of robust time-varying static output feedback controllers. Initially, Galerkin's method is used to derive an approximate ODE model that describes the slow dynamics of the parabolic PDE for all times. Then, under the assumption that the number of available measurements is equal to the number of slow modes for all times, the approximate ODE model is used to synthesize robust static output feedback controllers through combination of geometric techniques and Lyapunov's direct method. We show that the proposed controllers guarantee boundedness of the state and output tracking with arbitrary degree of asymptotic attenuation of the effect of the uncertain variables on the controlled output of the closed-loop system, provided that the separation between the slow and fast dynamical phenomena is sufficiently large. The proposed controllers are successfully applied to a typical diffusion–reaction process with moving boundary and uncertainty, and are shown to outperform controllers that do not account for the presence of uncertainty and the variation of the spatial domain with time.

2. Preliminaries

We consider quasi-linear uncertain parabolic PDE systems with uncertain variables of the form

$$\frac{\partial \bar{x}}{\partial t} = A\frac{\partial \bar{x}}{\partial z} + B\frac{\partial^2 \bar{x}}{\partial z^2} + wb(z,t)u + f(t,\bar{x}) + W(t,\bar{x},r(z)\theta(t)),$$

$$y_{c_i} = \int_0^{l(t)} c_i(z,t)k\bar{x}\,\mathrm{d}z, \quad i = 1,\dots,l,$$

$$y_{m_\kappa} = \int_0^{l(t)} s_\kappa(z,t)\omega\bar{x}\,\mathrm{d}z, \quad \kappa = 1,\dots,q \tag{1}$$

subject to the boundary conditions

$$C_1\bar{x}(0,t) + D_1\frac{\partial \bar{x}}{\partial z}(0,t) = R_1,$$

$$C_2\bar{x}(l(t),t) + D_2\frac{\partial \bar{x}}{\partial z}(l(t),t) = R_2 \tag{2}$$

and the initial condition

$$\bar{x}(z,0) = \bar{x}_0(z), \tag{3}$$

where $\bar{x}(z,t) = [\bar{x}_1(z,t) \cdots \bar{x}_n(z,t)]^{\mathrm{T}}$ denotes the vector of state variables, $[0,l(t)] \subset \mathbb{R}$ is the domain of definition of the process, $z \in [0,l(t)]$ is the spatial coordinate, $t \in [0,\infty)$ is the time, $u = [u_1\ u_2\ \cdots\ u_l]^{\mathrm{T}} \in \mathbb{R}^l$ denotes the vector of manipulated inputs, $y_{c_i} \in \mathbb{R}$ denotes the ith controlled output, $\theta(t) = [\theta_1(t) \cdots \theta_q(t)] \in \mathbb{R}^q$ denotes the vector of uncertain variables, which may include unknown/partially known process model parameters or exogenous disturbances, and $y_{m_\kappa} \in \mathbb{R}$ denotes the κth measured output. $\partial\bar{x}/\partial z$, $\partial^2\bar{x}/\partial z^2$ denote the first- and second-order spatial derivatives of $\bar{x}$, $f(t,\bar{x})$, $W(t,\bar{x},r(z)\theta(t))$ are nonlinear vector functions, $l(t)$ is a known and smooth function of time which describes the change of the domain size, k,w,ω are constant vectors, A,B,C_1,D_1,C_2,D_2 are constant matrices, R_1,R_2 are column vectors, and $\bar{x}_0(z)$ is the initial condition.

The smooth vector function $b(z,t)$ is of the form $b(z,t) = [b_1(z,t)\,b_2(z,t)\ \cdots\ b_l(z,t)]$, where $b_i(z,t)$ describes how the control action $u_i(t)$ is distributed in the interval $[0,l(t)]$ (e.g., point/distributed actuation), $r(z) = [r_1(z)\ \cdots\ r_q(z)]$, where $r_k(z)$ is a known smooth function of z which specifies the position of action of the uncertain variable $\theta_k(t)$ on $[0,l(t)]$, $c_i(z,t)$ is a known smooth function of (z,t) which is determined by the desired performance specifications in the interval $[0,l(t)]$ (e.g., regulation of the entire temperature profile of a crystal or regulation of the temperature at a specific point), and $s_\kappa(z,t)$ is a known smooth function of (z,t) which is determined by the location and type of the κth measurement sensor (e.g., point/distributed sensing). In the case of point actuation (i.e., the control action enters the system at a single point z_0, with $z_0 \in [0,l(t)]$), the function $b_i(z,t)$ is taken to be nonzero in a finite spatial interval of the form $[z_0 - \varepsilon, z_0 + \varepsilon]$, where ε is a small positive real number, and zero elsewhere in $[0,l(t)]$. A schematic of a typical process with moving boundaries is shown in

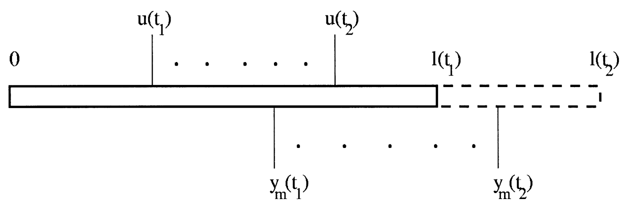

Fig. 1. Schematic of a prototype system with time-dependent spatial domain, moving control actuators and moving measurement sensors.

Fig. 1, in the case of point control actuators and sensors. Note that as the length of the process changes with time, the position of the control actuators and sensors may also change with time. The latter can be described mathematically by allowing the control actuator and sensor "shape" functions $b_i(z,t)$ and $s_k(z,t)$, respectively, to depend explicitly on time.

Throughout the paper, we will use the order of magnitude notation $O(\varepsilon)$ (i.e., $\delta(\varepsilon) = O(\varepsilon)$ if there exist positive real numbers k_1 and k_2 such that $|\delta(\varepsilon)| \le k_1|\varepsilon|, \forall |\varepsilon| < k_2$) and $\|\theta\|$ denotes the $ess\,sup\,|\theta|$, $t \ge 0$ for any measurable function $\theta : \mathbb{R}_{\ge 0} \to \mathbb{R}^q$.

Our assumptions on the properties of $l(t)$ are precisely stated below:

Assumption 1. *$l(t)$ is a known smooth (i.e., $\dot{l}$ exists and is bounded, $\forall t \in [0,\infty)$) function of time which satisfies $l(t) \in (0, l_{\max}]$, $\forall t \in [0,\infty)$, where $l_{\max}$ denotes the maximum length of the spatial domain.*

We consider the Hilbert space $\mathscr{H}(t)$ of n-dimensional vector functions defined on $[0, l(t)]$ that satisfy the boundary conditions of Eq. (2), with inner product and norm

$$(\omega_1, \omega_2) = \int_0^{l(t)} (\omega_1(z), \omega_2(z))_{\mathbb{R}^n}\,\mathrm{d}z,$$

$$\|\omega_1\|_2 = (\omega_1, \omega_1)^{1/2}, \tag{4}$$

where ω_1, ω_2 are two elements of $\mathscr{H}(t)$ and the notation $(\cdot,\cdot)_{\mathbb{R}^n}$ denotes the standard inner product in $\mathbb{R}^n$. In $\mathscr{H}(t)$, we define the state x as:

$$x(t) = \bar{x}(z,t), \quad t > 0,\ z \in [0, l(t)], \tag{5}$$

the time-varying operator

$$\mathscr{A}(t)x = A\frac{\partial \bar{x}}{\partial z} + B\frac{\partial^2 \bar{x}}{\partial z^2} + \dot{l}(t)\frac{z}{l(t)}\frac{\partial \bar{x}}{\partial z},$$

$$x \in D(\mathscr{A}(t)) = \Big\{ x \in \mathscr{H}(t)\colon C_1\bar{x}(0,t) + D_1\frac{\partial \bar{x}}{\partial z}(0,t) = R_1,$$

$$C_2\bar{x}(l(t),t) + D_2\frac{\partial \bar{x}}{\partial z}(l(t),t) = R_2 \Big\} \tag{6}$$

and the input, controlled output and measurement operators as

$$\mathscr{B}(t)u = wb(t)u, \quad \mathscr{C}(t)x = (c(t), kx), \quad \mathscr{S}(t)x = (s(t), \omega x) \tag{7}$$

where $c(t) = [c_1(t)\,c_2(t)\,\cdots\,c_l(t)]^{\mathrm{T}}$ and $s(t) = [s_1(t)\,s_2(t)\,\cdots\,s_q(t)]^{\mathrm{T}}$, and $c_i(t) \in \mathscr{H}(t)$, $s_\kappa(t) \in \mathscr{H}(t)$. Then, in $\mathscr{H}(t)$, the system of Eqs. (1)–(3) takes the form

$$\dot{x} = \mathscr{A}(t)x + \mathscr{B}(t)u + f(t,x) + \mathscr{W}(t,x,\theta), \quad x(0) = x_0,$$

$$y_c = \mathscr{C}(t)x, \quad y_m = \mathscr{S}(t)x, \tag{8}$$

where $f(t,x(t)) = f(t,\bar{x}(z,t))$, $\mathscr{W}(t,x(t),\theta(t)) = W(t,\bar{x}(z,t), r(z)\theta(t))$ and $x_0 = \bar{x}_0(z)$.

Finally, we precisely state our assumption that the dynamics of the infinite-dimensional system of Eq. (8) can be partitioned into slow (which are finite-dimensional) and fast (which are infinite-dimensional) ones. We note that this assumption is satisfied by most diffusion–reaction processes with moving boundary (e.g., Armaou & Christofides, 1999a,b).

Assumption 2. *Let $\{\phi_j(t)\}$, $j = 1,\ldots,\infty$, be an orthogonal and countable, $\forall t \in [0,\infty)$, basis of $\mathscr{H}(t)$. Let also $\mathscr{H}_s(t) = span\{\phi_1(t), \phi_2(t), \ldots, \phi_m(t)\}$ and $\mathscr{H}_f(t) = span\{\phi_{m+1}(t), \phi_{m+2}(t), \ldots\}$ be two subspaces of $\mathscr{H}(t)$ such that $\mathscr{H}_s(t) \oplus \mathscr{H}_f(t) = \mathscr{H}(t)$, and define the orthogonal (pointwise in time) projection operators $P_s : \mathscr{H}(t) \to \mathscr{H}_s(t)$ and $P_f : \mathscr{H}(t) \to \mathscr{H}_f(t)$ so that the state x of the system of Eq. (8) can be written as $x = x_s + x_f = P_s x + P_f x$. Using this decomposition for x, we assume that the system of Eq. (8) can be written in the following singularly perturbed form:*

$$\frac{\mathrm{d}x_s}{\mathrm{d}t} = \mathscr{A}_s(t)x_s + \mathscr{B}_s(t)u + f_s(t,x_s,x_f) + \mathscr{W}_s(t,x_s,x_f,\theta),$$

$$x_s(0) = P_s x_0,$$

$$\varepsilon\frac{\partial x_f}{\partial t} = \mathscr{A}_{f\varepsilon}(t)x_f + \varepsilon\mathscr{B}_f(t)u + \varepsilon f_f(t,x_s,x_f)$$

$$+ \mathscr{W}_f(t,x_s,x_f,\theta), \quad x_f(0) = P_f x_0,$$

$$y_c = \mathscr{C}(t)(x_s + x_f), \quad y_m = \mathscr{S}(t)(x_s + x_f), \tag{9}$$

where $\mathscr{A}_s(t) = P_s\mathscr{A}(t)$, $\mathscr{B}_s(t) = P_s\mathscr{B}(t)$, $f_s(t,x_s,x_f) = P_s f(t,x) + P_s\mathscr{A}(t)x_f$, $\mathscr{W}_s(t,x_s,x_f,\theta) = P_s\mathscr{W}(t,x,\theta)$ $\mathscr{A}_{f\varepsilon}(t) = \varepsilon P_f\mathscr{A}(t)$ is an unbounded differential operator, $\mathscr{B}_f(t) = P_f\mathscr{B}(t)$, $f_f(t,x_s,x_f) = P_f f(t,x) + P_f\mathscr{A}(t)x_s$, $\mathscr{W}_f(t,x_s,x_f,\theta) = P_f\mathscr{W}(t,x,\theta)$, $\varepsilon \ll 1$ and the operators $\mathscr{A}_s(t)$, $\mathscr{A}_{f\varepsilon}(t)$ generate semigroups with growth rates which are of the same order of magnitude.

Note that the partial derivative in the term $\partial x_f/\partial t$ is used to denote that x_f belongs to an infinite dimensional Hilbert space.

Remark 1. Referring to Assumption 1, we note that the requirement that the length of the domain is a smooth function of time is needed for the system of Eq. (8) to be well posed in $\mathscr{H}(t)$, while the requirement that the length of the domain is always finite is necessary for the dominant dynamics of the parabolic PDE system to be described by a finite number of degrees of freedom (Assumption 2). We note that both requirements are usually satisfied in practice: a typical example is the Czochralski crystal growth process where a seed crystal is pulled smoothly from the melt within a well-regulated thermal environment with the final crystal size being finite.

Remark 2. In the formulation of the PDE system of Eqs. (1)–(3) in $\mathscr{H}(t)$, the time-varying term $\dot{l}(t)[z/l(t)]\partial\bar{x}/\partial z$ in the expression of $\mathscr{A}(t)$ (Eq. (6)) accounts for convective transport owing to the motion of the domain. This term makes $\mathscr{A}(t)$ an explicit function of time. We also note that in contrast to the case of parabolic PDE systems defined on a fixed spatial domain (Christofides, 1998), we allow the actuator, performance specification and measurement sensor functions to depend explicitly on time (i.e., moving control actuators and objectives, and measurement sensors).

Remark 3. We note that the representation of Eq. (9) can be obtained by using basis functions, $\{\phi_j(t)\}$, $j=1,\ldots,\infty$, of $\mathscr{H}(t)$ taken from standard basis functions sets, or computed by solving an eigenvalue problem of the form $\mathscr{A}(t)\phi_j(t)=\lambda_j(t)\phi_j(t)$, or computed by applying Karhunen–Loève expansion on an appropriately chosen ensemble of solutions of the system of Eq. (8) (see Holmes, Lumley and Berkooz (1996) and Armaou and Christofides (1999b) for details on Karhunen–Loève expansion). The terms $P_s\mathscr{A}(t)x_f$ in $f_s(t,x_s,x_f)$ and $P_f\mathscr{A}(t)x_s$ in $f_f(t,x_s,x_f)$ account for the use of basis function sets which are not solutions of the problem $\mathscr{A}(t)\phi_j(t)=\lambda_j(t)\phi_j(t)$. We finally note that when $\phi_j(t)$ solve $\mathscr{A}(t)\phi_j(t)=\lambda_j(t)\phi_j(t)$, then $\varepsilon=\sup_{t\in[0,\infty)}|Re\,\lambda_1(t)|/|Re\,\lambda_{m+1}(t)|$, where $\lambda_1(t)$ is the largest eigenvalue of the matrix $\mathscr{A}_s(t)$ and $\lambda_{m+1}(t)$ is the largest eigenvalue of the operator (infinite range matrix) $P_f\mathscr{A}(t)$ (see example in Section 4).

3. Robust nonlinear output feedback control

We consider the synthesis of robust time-varying output feedback control laws of the form

$$u_0=p_0(t,y_m)+Q_0(t,y_m)\bar{v}+r_0(t,y_m), \tag{10}$$

where $p_0(t,y_m)$, $r_0(t,y_m)$ are vector functions, $Q_0(t,y_m)$ is a matrix, and $\bar{v}$ is a vector of the form $\bar{v}=\mathscr{V}(v_i,v_i^{(1)},\ldots,v_i^{(r_i)})$ where $\mathscr{V}(v_i,v_i^{(1)},\ldots,v_i^{(r_i)})$ is a smooth vector function, $v_i^{(k)}$ is the kth time derivative of the external reference input v_i (which is assumed to be a smooth function of time) and r_i is a positive integer.

We initially state our stability requirements on the fast dynamics of Eq. (9). To this end, we write the system of Eq. (9) in the fast time-scale $\tau=t/\varepsilon$ and set $\varepsilon=0$, to derive the following infinite-dimensional fast subsystem:

$$\frac{\partial x_f}{\partial\tau}=\mathscr{A}_{f\varepsilon}(t)x_f. \tag{11}$$

Assumption 3. *The fast subsystem of Eq.* (11) *is exponentially stable, uniformly in* $t\in[0,\infty)$.

Setting $\varepsilon=0$ in the system of Eq. (9), we end up with the following m-dimensional slow system:

$$\begin{aligned}\frac{dx_s}{dt}&=\mathscr{A}_s(t)x_s+f_s(t,x_s,0)+\mathscr{B}_s(t)u+\mathscr{W}_s(t,x_s,0,\theta)\\&=:F_0(t,x_s)+\sum_{i=1}^{l}\mathscr{B}_0^i(t)u_0^i+\mathscr{W}_0(t,x_s,0,\theta),\\y_{c_i}&=\mathscr{C}_i(t)x_s=:h_{i_0}(t,x_s),\ y_m=\mathscr{S}(t)x_s,\end{aligned} \tag{12}$$

where the subscript 0 in $(F_0,\mathscr{B}_0^i,u_0^i,\mathscr{W}_0,h_{i_0})$ denotes that they are elements of the $O(\varepsilon)$ approximation of the x_s-subsystem of Eq. (9) (see Theorem 1 below for a precise characterization of the accuracy of the system of Eq. (12)).

We now state three assumptions on the system of Eq. (12) which are needed to synthesize a robust state feedback controller of the form of Eq. (10). To simplify the statement of these assumptions, we use the Lie derivative of the scalar function $h_{i_0}(t,x_s)$ with respect to the vector function $f_0(t,x_s)$, defined as $L_{f_0}h_{i_0}(t,x_s)=(\partial h_{i_0}/\partial x_s)f_0(t,x_s)+\partial h_{i_0}/\partial t$ (this definition of Lie derivative was introduced in Palanki and Kravaris (1997) and is different than the standard one used in Isidori (1989) for the case of time-invariant h_{i_0}, f_0). Furthermore, $L_{f_0}^k h_{i_0}(t,x_s)$ denotes the kth-order Lie derivative and $L_{g_{0_i}}L_{f_0}^k h_{i_0}(t,x_s)$ denotes the mixed Lie derivative, where $g_{0_i}(t,x_s)$ is a vector function.

Assumption 4. *Referring to the system of Eq.* (12), *there exist a set of integers* $(r_1,r_2,\ldots,r_l)$ *and a coordinate transformation* $(\zeta,\eta)=T(x_s,\theta)$ *such that the representation of the system, in the coordinates* (ζ,η), *takes the form*:

$$\begin{aligned}\dot{\zeta}_1^{(1)}&=\zeta_2^{(1)},\\&\ \vdots\\\dot{\zeta}_{r_1-1}^{(1)}&=\zeta_{r_1}^{(1)},\\\dot{\zeta}_{r_1}^{(1)}&=L_{F_0}^{r_1}h_{1_0}(t,x_s)+\sum_{i=1}^{l}L_{\mathscr{B}_0^i}L_{F_0}^{r_1-1}h_{1_0}(t,x_s)u_0^i\\&\quad+L_{\mathscr{W}_0}L_{F_0}^{r_1-1}h_{1_0}(t,x_s),\\&\ \vdots\\\dot{\zeta}_1^{(l)}&=\zeta_2^{(l)},\\&\ \vdots\\\dot{\zeta}_{r_l-1}^{(l)}&=\zeta_{r_l}^{(l)},\\\dot{\zeta}_{r_l}^{(l)}&=L_{F_0}^{r_l}h_{l_0}(t,x_s)+\sum_{i=1}^{l}L_{\mathscr{B}_0^i}L_{F_0}^{r_l-1}h_{l_0}(t,x_s)u_0^i\\&\quad+L_{\mathscr{W}_0}L_{F_0}^{r_l-1}h_{l_0}(t,x_s),\end{aligned} \tag{13}$$

$$\dot{\eta}_1 = \Psi_1(t,\zeta,\eta,\theta,\dot{\theta}),$$
$$\vdots$$
$$\dot{\eta}_{m-\sum_i r_i} = \Psi_{m-\sum_i r_i}(t,\zeta,\eta,\theta,\dot{\theta}),$$
$$y_{c_i} = \zeta_1^{(i)}, \quad i = 1,\dots,l,$$

where $x_s = T^{-1}(\zeta,\eta,\theta)$, $\zeta = [\zeta^{(1)}\cdots\zeta^{(l)}]^{\mathrm{T}} \in \mathbb{R}^{r_1+r_2\cdots+r_l}$, $\eta = [\eta_1 \;\cdots\; \eta_{m-\sum_i r_i}]^{\mathrm{T}} \in \mathbb{R}^{m-(r_1+r_2\cdots+r_l)}$.

The above assumption is always satisfied for systems for which $r_i = 1$, for all $i = 1,\dots,l$; this requirement can be easily achieved by selecting $b_i(z,t) \neq \phi_j(z,t)$, for $i = 1,\dots,l$, $j = 2,\dots,\infty$.

Referring to the system of Eq. (13), we assume that the matrix:

$$C_0(t,x_s) = \begin{bmatrix} L_{\mathscr{B}_0^1} L_{f_0}^{r_1-1} h_{1_0}(t,x_s) & \cdots & L_{\mathscr{B}_0^l} L_{f_0}^{r_1-1} h_{1_0}(t,x_s) \\ L_{\mathscr{B}_0^1} L_{f_0}^{r_2-1} h_{2_0}(t,x_s) & \cdots & L_{\mathscr{B}_0^l} L_{f_0}^{r_2-1} h_{2_0}(t,x_s) \\ \vdots & \cdots & \vdots \\ L_{\mathscr{B}_0^1} L_{f_0}^{r_l-1} h_{l_0}(t,x_s) & \cdots & L_{\mathscr{B}_0^l} L_{f_0}^{r_l-1} h_{l_0}(t,x_s) \end{bmatrix} \tag{14}$$

is nonsingular $\forall t \in [0,\infty)$, $\forall\; x_s \in \mathscr{H}_s(t)$. This assumption is made to simplify the presentation of the controller synthesis results and can be relaxed (see Isidori, 1989 for details). The following assumption is a standard one in geometric nonlinear control and states that the zero dynamics of the finite-dimensional slow subsystem is exponentially stable.

Assumption 5. *The dynamical system*

$$\begin{aligned} \dot{\eta}_1 &= \Psi_1(t,\zeta,0,0,0), \\ &\vdots \\ \dot{\eta}_{m-\sum_i r_i} &= \Psi_{m-\sum_i r_i}(t,\zeta,0,0,0) \end{aligned} \tag{15}$$

is locally exponentially stable.

To design a nonlinear controller that explicitly compensates for the effect of uncertain variables, we need to assume the existence of a nonlinear time-varying bounding function which captures the size of the uncertain terms in the ζ-subsystem of Eq. (13). Such a bounding function may be computed from physical considerations, preliminary simulations and experimental data.

Assumption 6. *There exists a known function* $c_0(t,x_s)$ *such that the following condition holds*:

$$|[L_{\mathscr{W}_0} L_{F_0}^{r_1-1} h_{1_0}(t,x_s) \cdots L_{\mathscr{W}_0} L_{F_0}^{r_l-1} h_{l_0}(t,x_s)]^{\mathrm{T}}| \leq c_0(t,x_s) \tag{16}$$

for all $t \in [0,\infty)$, $x_s \in \mathscr{H}_s(t)$, $\theta \in \mathbb{R}^q$, $t \geq 0$.

The following assumption is needed in order to obtain estimates of the states x_s of the system of Eq. (12) from the measurements y_{m_κ}, $\kappa = 1,\dots,p$.

Assumption 7. $p = m$ (*i.e., the number of measurements is equal to the number of slow modes*), *and the inverse of the operator* $S(t)$ *for all* $t \in [0,\infty)$ *exists so that* $\hat{x}_s = S^{-1}(t)y_m$, *where* $\hat{x}_s$ *is an estimate of* x_s.

Theorem 1 that follows provides an explicit formula for the robust controller, conditions that ensure boundedness of the state, and a precise characterization of the ultimate uncertainty attenuation level. To simplify the statement of the theorem, we set $\bar{v}_i = [v_i\, v_i^{(1)} \cdots v_i^{(r_i)}]^{\mathrm{T}}$ and $\tilde{v} = [\bar{v}_1^{\mathrm{T}}\, \bar{v}_2^{\mathrm{T}} \cdots \bar{v}_m^{\mathrm{T}}]^{\mathrm{T}}$.

Theorem 1. *Consider the parabolic PDE system of Eq.* (8) *for which Assumptions* 1–3 *hold, and the finite-dimensional system of Eq.* (12), *for which Assumptions* 4–7 *hold, under the robust output feedback controller*:

$$\begin{aligned} u_0 &= a_0(x_s,x_f,\bar{v},t) \\ &:= [C_0(t,\hat{x}_s)]^{-1}\Bigg\{\sum_{i=1}^{l}\sum_{k=1}^{r_i}\frac{\beta_{ik}}{\beta_{ir_i}}(v_i^{(k)} - L_{F_0}^{k} h_{i_0}(t,\hat{x}_s)) \\ &\quad + \sum_{i=1}^{l}\sum_{k=1}^{r_i}\frac{\beta_{ik}}{\beta_{ir_i}}(v_i^{(k-1)} - L_{F_0}^{k-1} h_{i_0}(t,\hat{x}_s)) \\ &\quad - \chi[c_0(t,\hat{x}_s)] \\ &\quad \frac{\sum_{i=1}^{l}\sum_{k=1}^{r_i}(\beta_{ik}/\beta_{ir_i})(L_{F_0}^{k-1} h_{i_0}(t,\hat{x}_s) - v_i^{(k-1)})}{|\sum_{i=1}^{l}\sum_{k=1}^{r_i}(\beta_{ik}/\beta_{ir_i})(L_{F_0}^{k-1} h_{i_0}(t,\hat{x}_s) - v_i^{(k-1)})| + \phi}\Bigg\}, \end{aligned} \tag{17}$$

where $\hat{x}_s = S^{-1}y_m, \beta_{ik}/\beta_{ir_i} = [\beta_{ik}^1/\beta_{ir_i}^1 \cdots \beta_{ik}^l/\beta_{ir_i}^l]^{\mathrm{T}}$ *are column vectors of parameters chosen so that the roots of the equation* $det(B(s)) = 0$, *where* $B(s)$ *is an* $l \times l$ *matrix, whose* (i,j)*th element is of the form* $\sum_{k=1}^{r_i}(\beta_{jk}^i/\beta_{jr_i}^i)s^{k-1}$, *lie in the open left-half of the complex plane, and* χ,ϕ *are adjustable parameters with* $\chi > 1$ *and* $\phi > 0$. *Then, there exist positive real numbers* (δ,ϕ^*) *such that for each* $\phi \leq \phi^*$, *there exists* $\varepsilon^*(\phi)$, *such that if* $\phi \leq \phi^*$, $\varepsilon \leq \varepsilon^*(\phi)$ *and* $\max\{|x_s(0)|, \|x_f(0)\|_2, \|\theta\|, \|\dot{\theta}\|, \|\tilde{v}\|\} \leq \delta$,

(a) *the state of the infinite-dimensional closed-loop system is bounded, and*
(b) *the outputs of the infinite-dimensional closed-loop system satisfy*:

$$\limsup_{t\to\infty} |y_{c_i} - v_i| \leq d_0, \quad i = 1,\dots,l \tag{18}$$

where $d_0 = O(\phi + \varepsilon)$ *is a positive real number.*

Note that we use the notation $|x_s|$ to denote the norm of x_s in the finite-dimensional Hilbert space $\mathscr{H}_s(t)$.

Remark 4. The nonlinear controller of Eq. (17) was derived through combination of geometric and Lyapunov techniques (the details of the derivation of this controller are omitted for brevity and the reader may refer to Christofides (1998) for details on the synthesis of a similar controller for parabolic PDE systems with fixed spatial domains). The controller tuning parameters include β^i_{jk}, which can be chosen to influence the speed of the output response, and ϕ, χ which can be chosen to achieve the desired degree of attenuation (d_0) of the effect of the uncertain variables (unknown/partially known process model parameters or exogenous disturbances), $\theta_k(t)$, on the controlled output.

Remark 5. Regarding the practical implementation of the result of Theorem 1, one has to initially construct an approximate finite-dimensional system which captures the dominant dynamics of the PDE system (making certain that Assumption 3 is satisfied), then verify that such a system satisfies Assumptions 4–6, and the number and location of the measurement sensors satisfies Assumption 7, and finally, use the formula of Eq. (17) to synthesize the controller on the basis of the finite-dimensional system and tune the controller (i.e., pick the values for the parameters β^i_{jk}, ϕ, χ) so that the desired degree of uncertainty attenuation is achieved in the closed-loop system.

Remark 6. We note that even though the controller of Eq. (17) uses static feedback of the measured outputs y_{m_κ}, $\kappa = 1, \dots, p$, and thus, it feeds back both x_s and x_f, the use of x_f feedback cannot lead to destabilization of the stable fast subsystem. This is due to the large separation of the slow and fast modes of the spatial differential operator (i.e., assumption that ε is sufficiently small) and the fact that the controller does not include terms of the form $O(1/\varepsilon)$.

Proof of Theorem 1. Substituting the controller of Eq. (17) into the parabolic PDE system of Eq. (8), we obtain:

$$\dot{x} = \mathscr{A}(t)x + \mathscr{B}(t)a_0(t, x_s, x_f, \bar{v}) + f(t, x) + \mathscr{W}(t, x, \theta),$$
$$y_c = \mathscr{C}(t)x, \quad y_m = \mathscr{S}(t)x. \tag{19}$$

One can easily verify that Assumption 2 *holds for the above system*, and thus, it can be written in the following form:

$$\frac{dx_s}{dt} = \mathscr{A}_s(t)x_s + \mathscr{B}_s(t)a_0(t, x_s, x_f, \bar{v}) + f_s(t, x_s, x_f) + \mathscr{W}_s(t, x_s, x_f, \theta),$$

$$\varepsilon\frac{\partial x_f}{\partial t} = \mathscr{A}_{f\varepsilon}(t)x_f + \varepsilon\mathscr{B}_f(t)a_0(t, x_s, x_f, \bar{v}) + \varepsilon f_f(t, x_s, x_f) + \varepsilon\mathscr{W}_f(t, x_s, x_f, \theta). \tag{20}$$

Rewriting the above system in the fast time-scale $\tau = t/\varepsilon$ and setting $\varepsilon = 0$, we obtain the following infinite-dimensional fast subsystem from the system of Eq. (20):

$$\frac{\partial x_f}{\partial \tau} = \mathscr{A}_{f\varepsilon}(t)x_f \tag{21}$$

which is globally exponentially stable. Setting $\varepsilon = 0$ in the system of Eq. (20), we have that $x_f = 0$ and thus, the finite-dimensional slow system takes the form

$$\frac{dx_s}{dt} = F_0(t, x_s) + \sum_{i=1}^{l} \mathscr{B}^i_0(t)a^i_0(t, x_s, 0, \bar{v}) + \mathscr{W}_0(t, x_s, 0, \theta),$$
$$y_{c_i} = \mathscr{C}_i(t)x_s =: h_{i_0}(t, x_s). \tag{22}$$

For the above system, there exists a $\phi \in (0, \phi^*]$ such that if $\max\{|x_s(0)|, \|\theta\|, \|\dot{\theta}\|, \|\bar{v}\|\} \leq \delta$, then its state is bounded and its outputs satisfy $\limsup_{t\to\infty}|y_{c_i} - v_i| \leq O(\phi)$, $i = 1, \dots, l$ (this can be shown using similar arguments as in the proof of Theorem 1 in Christofides (1998)). Finally, since the infinite-dimensional fast subsystem of Eq. (11) is exponentially stable, we can use standard singular perturbation arguments to obtain that there exists an $\varepsilon^*(\phi)$, such that if $\varepsilon \in (0, \varepsilon^*(\phi)]$, $\max\{|x_s(0)|, \|x_f(0)\|_2, \|\theta\|, \|\dot{\theta}\|, \|\bar{v}\|\} \leq \delta$, then the state of the closed-loop parabolic PDE system of Eq. (19) is bounded and that its outputs satisfy the relation of Eq. (18). □

4. Control of a diffusion–reaction process with moving boundary

We consider a diffusion–reaction process with moving boundary which is described by the following parabolic PDE:

$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial^2 \bar{x}}{\partial z^2} + \beta_T e^{-\gamma/(1+\bar{x})} + \beta_U(b(z,t)u(t) - \bar{x}) - \beta_{T,n}e^{-\gamma} \tag{23}$$

subject to the Dirichlet boundary conditions

$$\bar{x}(0, t) = 0, \quad \bar{x}(l(t), t) = 0 \tag{24}$$

and the initial condition:

$$\bar{x}(z, 0) = 0.5, \tag{25}$$

where $\bar{x}$ is the state, γ, β_u are dimensionless process parameters, β_T denotes a dimensionless heat of reaction (which is assumed to be unknown and time-varying; uncertain variable), $\beta_{T,n}$ denotes a *nominal* dimensionless heat of reaction, $b(z,t) = [b_1(z,t) \;\; b_2(z,t)]$ are the actuator distribution functions and $u = [u_1 \;\; u_2]^T$ is the manipulated input vector. The spatial domain is assumed to change according to the relation

$$l(t) = \pi(1.4 - 0.4e^{(-0.02t^{2.7})}) \tag{26}$$

(it can be easily seen that the above function satisfies the requirements of Assumption 1) and the following typical values were given to the process parameters:

$$\beta_{T,n} = 60.0, \quad \beta_U = 2.0, \quad \gamma = 4.0. \tag{27}$$

The following countable, $\forall t \in [0, \infty)$, orthogonal basis of $\mathscr{H}(t)$ was used in our calculations:

$$\phi_j(z,t) = \sqrt{\frac{2}{l(t)}} \sin\left(j\pi \frac{z}{l(t)}\right), \quad j = 1, \dots, \infty. \tag{28}$$

Note that the above set of basis functions satisfies the conditions of Assumption 2.

A 20th-order Galerkin truncation of the system of Eq. (23), with $\phi_j(z,t)$ of Eq. (28) as the basis functions, was used in our simulations (it was verified that further increase of the order of the Galerkin model provided no substantial improvement on the accuracy of the simulation results). It was found that the operating steady state $\bar{x}(z,t) = 0$ is an unstable one (the open-loop state starting from initial conditions close to the steady state $\bar{x}(z,t) = 0$ moves to another stable steady state characterized by a maximum at $z = l(t)/2$). Moreover, the linearization of the system of Eq. (23) around the steady-state $\bar{x}(z,t) = 0$ possesses one positive eigenvalue $\forall l(t) \in [\pi, 1.4\pi]$ and a second eigenvalue which becomes periodically positive as the domain size $l(t)$ and the value of β_T change.

The control objective is to stabilize the system at the unstable steady-state $\bar{x}(z,t) = 0$ in the presence of time-varying uncertainty in the dimensionless heat of the reaction β_T, (i.e., $\beta_T = \beta_{T,n} + \theta(t)$ where $\theta(t) = 0.35\beta_{T,n}\sin(0.524t)$) by employing a nonlinear output feedback controller which uses two point measurements of the state at $z = 0.25l(t)$ and $z = 0.625l(t)$ (i.e., moving sensors with $s(z,t) = \delta(z - 0.25l(t))$ and $s(z,t) = \delta(z - 0.625l(t))$, respectively, where $\delta(\cdot)$ is the standard Dirac function). We note that this selection for $\theta(t)$ satisfies the requirements of Theorem 1 that $\theta(t)$, $\dot{\theta}(t)$ should be sufficiently small (see closed-loop simulations below), while it leads to a very poor open-loop behavior for $\bar{x}(z,t)$ (see Fig. 2). Since the maximum open-loop value of $\bar{x}(z,t)$ occurs for $z = l(t)/2$ and the first two modes of the process become unstable for some $l(t) \in [\pi, 1.4\pi]$, the controlled outputs were defined as

$$y_{c_1}(t) = \int_0^{l(t)} \phi_1(z,t)\bar{x}(z,t)\,dz,$$

$$y_{c_2}(t) = \int_0^{l(t)} \phi_2(z,t)\bar{x}(z,t)\,dz. \tag{29}$$

The actuator distribution functions were taken to be $b_1(z,t) = 1$ (uniform in space, distributed control action) and $b_2(z,t) = \delta(z - \frac{2}{3}l(t))$ (moving point control actuation).

For the system of Eq. (23), we considered the first two eigenvalues as the dominant ones ($\varepsilon = 0.11$) and used Galerkin's method to derive a two dimensional ODE that was used for the synthesis of a nonlinear robust output feedback controller through an application of Eq. (17). The controller was implemented with the following values of the parameters $\theta_{max} = 21$ (where θ_{max} is an upper bound of the disturbance $\theta_{max} \geq \sup_{t\in[0,\infty)} \{|\theta(t)|\}$), $\chi = 1.2$, $\phi = 0.005$, and achieved an uncertainty attenuation level $d = 0.05$ (note that $0.05 = O(\varepsilon + \phi) = O(0.11 + 0.005)$).

Fig. 3 shows the evolution of the closed-loop rod temperature profile under the nonlinear robust output feedback controller, while Fig. 4 shows the corresponding manipulated input profile. Clearly, the proposed controller regulates the temperature profile at $\bar{x}(z,t) = 0$, attenuating the effect of the uncertain variable (note that the requirement $\limsup_{t\to\infty} |y_c| \leq 0.05$ is enforced in the closed-loop system; Fig. 5). For the sake of comparison, we also implemented on the process the same controller as before without the term which compensates for the effect of the uncertainty (i.e., $\chi = 0$). Fig. 6 shows the

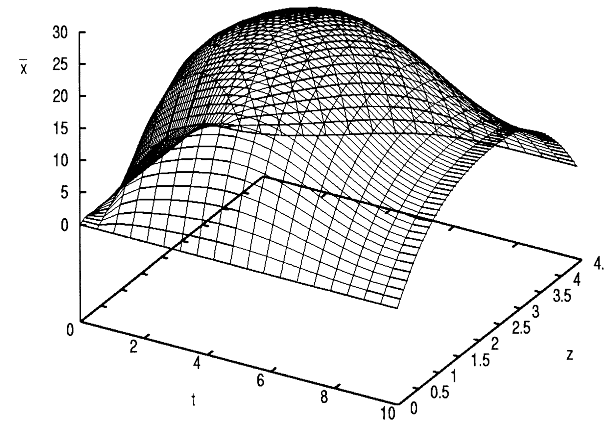

Fig. 2. Open-loop profile of $\bar{x}$ in the presence of uncertainty.

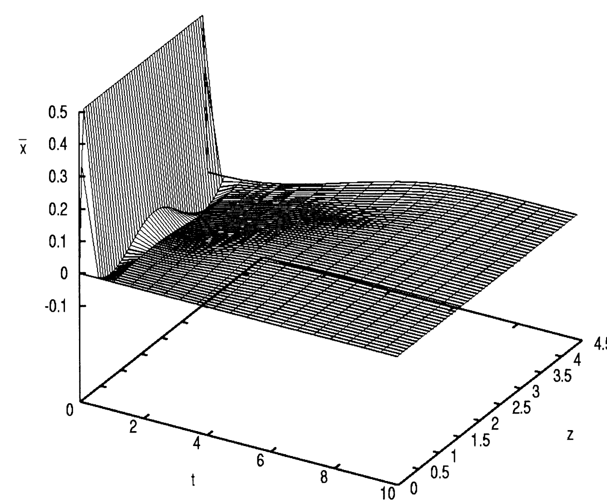

Fig. 3. Closed-loop profile of $\bar{x}$ under nonlinear robust output feedback control.

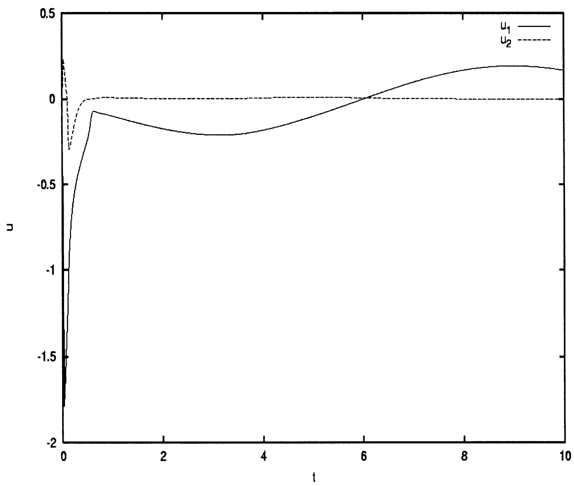

Fig. 4. Manipulated input profiles for nonlinear robust output feedback controller.

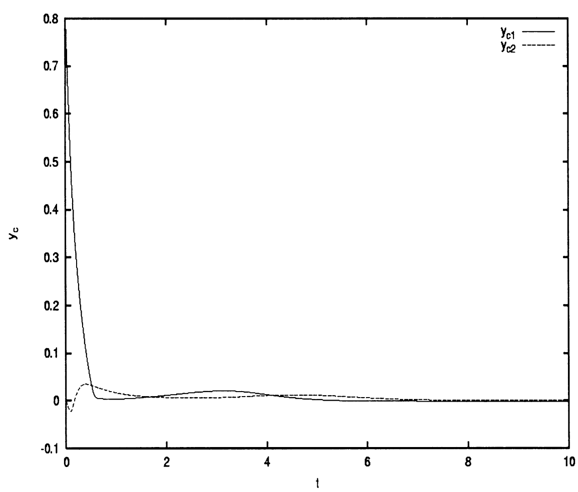

Fig. 5. Closed-loop output profile under nonlinear robust output feedback control.

evolution of the closed-loop rod temperature profile. It is clear that this controller cannot regulate the temperature profile at the desired steady state, leading to close-loop instability.

We finally note that we also tested through simulations nonlinear robust static output feedback controllers which were synthesized under the assumption of fixed spatial domain (i.e., $l(t) = \pi$ and $l(t) = 1.4\pi$, for all times) which led to an unstable closed-loop system; the detail simulations are omitted for brevity.

5. Conclusions

In this work, we considered a broad class of quasi-linear parabolic PDE systems with time-dependent spatial domains and time-varying uncertain variables whose

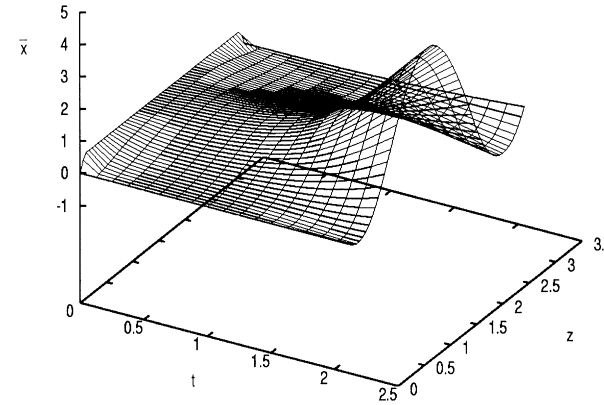

Fig. 6. Closed-loop profile of $\bar{x}$ under nonlinear output feedback control (no uncertainty compensation).

dynamics can be partitioned into slow and fast ones. For these systems, under the assumption that the number of available measurements is equal to the number of slow modes for all times, we synthesized robust static output feedback controllers through combination of geometric techniques and Lyapunov's direct method. The developed controllers guarantee boundedness of the state and output tracking with arbitrary degree of asymptotic attenuation of the effect of the uncertain parameters on the controlled output of the closed-loop system, provided that the separation between the slow and fast dynamical phenomena is sufficiently large. The controllers were successfully applied to a typical diffusion–reaction process with moving boundary and time-varying uncertainty and were shown to outperform controllers that do not account for the presence of the uncertainty.

Acknowledgements

Financial support from an NSF CAREER award, CTS-9733509, is gratefully acknowledged.

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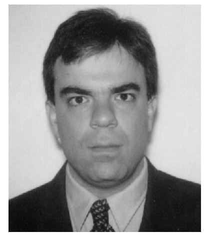

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