Evaluation of osmotic energy extraction via FEM modeling and exploration of PRO operational parameter space

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A B S T R A C T

Power generation via pressure retarded osmosis (PRO) was explored based on a detailed two dimensional finite element (2-D-FEM) PRO model. Using the numerical model, an approach is presented for determining the draw and feed crossflow velocities for maximizing peak power generation. The dependence of PRO power generation on channel dimensions, membrane transport parameters were then evaluated, followed by assessing the impact of frictional pressure losses and pumping and energy recovery device (ERD) efficiencies. Illustrative test cases are presented for three different draw/feed streams representing seawater/brackish water (SW/BW), seawater RO-brine/brackish water (SWB/BW), and Dead Sea water/Seawater RO-brine (DSW/SWB). The maximum peak power density attainable via PRO was for DSW/SWB (35.3 W/m²), followed by SWB/BW (7.29 W/m²) and SW/BW (3.53 W/m²) for the case of ideal pumps and ERD. For the optimistic Power generation from DSW/SWB PRO, high efficiency pumps (98%) and ERD (96%) would be required for peak power density to approach ~12 W/m² and 1.6 W/m² for the cases of DSW/SWB and SWB/BW, respectively, while net positive power generation is not expected for SW/BW. Higher permeability membranes could provide somewhat increased PRO performance; however, frictional pressure losses and less than ideal pumps and ERDs present a barrier for PRO as a viable approach for energy generation.

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1. Introduction

In recent years there has been a renewed interest in energy production through the use of salinity (or osmotic pressure) gradients via the process of pressure retarded osmosis (PRO). The approach relies on having high and low salinity solutions flowing in two channels separated by a semi-permeable membrane that allows water permeation (from the low to high salinity solution) while rejecting the passage of salt ions [1,2]. It has been proposed that the extraction of osmotic energy may be practically (or commercially) feasible when streams of high salinity difference are readily available, as for example at the location where a river flows into the sea [2]. Another example of a location often touted as promising for PRO energy production is the meeting zone between the Baltic and the North Seas known as the halocline [3,4]. Along the halocline surface water salinity on the North Sea side of the halocline at Kattegat is about 34–35 g/L [4] and the salinity at surface on the Baltic side is about 8 g/L [3]. Energy production by PRO has also been proposed via harnessing the salinity difference between the Dead-Sea and either the Red Sea or the Mediterranean Sea [5].

Energy production via PRO requires adequate water permeation across the PRO membrane from low (feed) to high (draw) salinity streams and higher hydraulic pressure in the draw than in the feed channels. The generated power (expressed as power density in units of W/m²), under ideal conditions (neglecting all losses and assuming ideal pumps and energy recovery devices), is approximated [6] as

\[ W = J_w \Delta P \]

where \( J_w \) and \( \Delta P \) are the average water flux and transmembrane pressure. For the above ideal conditions, the maximum peak power generation is

\[ W_{max} = A \Delta \pi^2 / 4 \]

where \( A \) is the membrane water permeability and \( \Delta \pi \) is the transmembrane osmotic pressure difference. Peak power generation [7–9], for a given membrane and channel geometries and stream salinities, is dictated by the PRO operating conditions (i.e., imposed hydraulic pressure, cross flow velocities). The latter govern the concentration field development and salt back diffusion and associated reduction in the water permeation driving force [10,11]. Bench-scale experimental PRO studies, which have generally been carried out with short PRO channels (typically ~8–15 cm, [9,12–14]), have shown that PRO power generation increases with membrane permeability and draw solute concentration but decreases with feed concentration and the membrane length. It is important to recognize that PRO power generation may be more significantly impacted by concentration polarization (CP), salt back diffusion and...
frictional pressure losses with increased channel length. For example, modeling studies, utilizing the 1-D film model, along with various approximations [13,15], concluded that due to CP one should expect up to about 10% flux decline [15] in long membrane channels (⋯m) and 40% lower power generation than attained in short laboratory channels [13].

The majority of PRO studies have relied on process analysis based on the one-dimensional film model (assuming fully developed axial flow and concentration fields) [2,16–18]. In these studies the PRO membrane has been characterized by the water permeability and salt transport coefficient, denoted by $\bar{A}$ and $\bar{B}$, respectively, and the PRO membrane porous support structural parameter $S$ (i.e., $S = \frac{t}{\varepsilon}$, in which $t$, $\varepsilon$ and $\tau$ are the support layer thickness, tortuosity and porosity, respectively). Various studies have argued [13,19–21] that the PRO membrane support layer represents the major resistance to water permeation due to internal CP (i.e., ICP). For example, it was reported [2] that by tuning the support layer thickness to be in the range of 70–100 μm and with $S \leq 0.5$ mm it may be possible to achieve power density higher than 5 W/m² which is often cited as an acceptable level for PRO power generation [9]. It is noted that various studies on osmotically-driven FO [10,20] have also concluded that the support layer offers greater resistance to water permeation compared to both the active membrane layer and external CP (in the feed and draw channels). In contrast, recent forward osmosis (FO) studies [20,22,23] have presented detailed analysis, relying on CFD models in conjunction with reported literature data, demonstrating that the membrane active layer and external CP layers represent a far greater resistance to water permeation relative to the support layer.

Previous PRO studies have also documented that power generation, for a given specific feed and draw streams, PRO membrane type, and height and lengths of the draw and feed channels, is highly dependent on the crossflow velocities of the PRO draw and feed streams [7,24]. The above should be expected since the crossflow velocities in the feed and draw channels will impact both external concentration polarization (ECP) and internal concentration polarization (ICP), as well as frictional pressure losses. Accordingly, in the present work a systematic approach is presented, making use of a detailed 2-D FEM model of the coupled hydrodynamics and mass transfer equations, to explore the impact of PRO operating conditions on the attained maximum level of power generation. The present analysis considers feed and draw channel frictional pressure losses, feed pumping energy and efficiency of PRO hydraulic to energy conversion, as well as the influence of membrane channel height and length, in addition to potential improvements that may be afforded by increasing membrane permeability. Specific examples of the attainable PRO power production are then presented for the draw/feed combinations of seawater/brackish-water, RO-concentrate/brackish-water and Dead-Seawater/RO-concentrate.

2. Modeling of pressure-retarded osmosis

2.1. Work flow for maximizing osmotic power generation

The maximum osmotic power that can be generated via PRO, for a given feed and draw streams, specific PRO membrane type, and RO channel height and length, was first evaluated consisted of the major steps as per the workflow described in Fig. 1. In the first step membrane water permeability and salt transport parameters should be extracted from suitable experimental PRO membrane performance data. In the present illustration of the approach, membrane transport parameters were extracted from available literature bench-scale data using a 2-dimensional (2-D) CFD model (Section 2.2) following the method described in [23,25]. Subsequently, for specific feed and draw streams and prescribed PRO feed and draw channels (i.e., of a prescribed height and length), an iterative process is followed to find the inlet draw stream velocity that will maximize the peak power generation; here, one must recognize that utilization of the feed stream should be sufficiently high in order to maximize the energy that can be extracted for a given draw stream volumetric inflow rates. In other words, for each selected inlet draw stream cross flow velocity there should be a corresponding inlet feed stream velocity that will result in high level of feed stream utilization. Accordingly, for a selected draw stream velocity the CFD model is solved iteratively varying the feed inlet cross flow velocity, $V_{f_in}$ until a solution is reached whereby the feed channel outlet feed crossflow velocity, $V_{f_out}$, vanishes to within a tolerance level, i.e., $|V_{f_out}/V_{f_in}| < 0.5$, which signifies near complete utilization of the feed stream. The above process is repeated for different values of inlet draw stream velocity in order to determine the stream velocities that maximize the peak power generation. It is emphasized that the CFD model solution for the above considers both ECP and ICP as well as frictional pressure losses. Following the above analysis, the impacts of channel length and height on the attained maximum energy production is evaluated first considering PRO operation with ideal (i.e., 100% efficient) feed pumps and hydraulic energy recovery devices (ERD). Subsequently, we estimate the reduction in net PRO power generation due to the use of non-ideal pumps and ERDs (i.e., efficiencies < 100%) in addition to the effect of frictional pressure losses.

2.2. PRO fluid flow and mass transport

In the PRO process high salinity (draw) and low salinity (feed) salt solutions, which flow through a draw and feed channels, respectively, are separated by a semi-permeable membrane (Fig. 2). The feed solution is fed on one side of the membrane and water then permeates through the membrane to the draw channel side. The draw solution is maintained under pressure which is set below the draw osmotic pressure. The energy gained is the product of the added water flow rate on the draw side (via permeation from the lower salinity feed channel) and the draw stream exit pressure. The available hydraulic energy is then converted (or recovered) using a suitable energy recovery device (ERD; e.g., turbine or pressure recovery device). In determining the net energy gain, however, one must account for any energy expenditures due to pumping of the draw and feed streams and any frictional energy losses in the membrane channels.

Water permeation across the PRO membrane (from feed to draw side) is governed by the osmotic pressure and hydraulic pressure differences. Countercurrent flow is typically utilized where the hydraulic pressure is higher in the draw channel relative to the feed channel. In the present analysis, the development of the flow and salt concentration fields along the channels, permeate flux and power generation were determined from numerical solution of the 2-dimensional (2-D) CFD model consisting of the coupled hydrodynamic and mass transfer governing equations.

The hydrodynamics in the feed and draw channels, for the case of 2-D steady-state flow, is described by the Navier-Stokes equations of motion and the continuity equation,

$$\frac{\partial u_i}{\partial t} - \nabla p_i + \nabla \cdot \left[ \nu_i \left( \nabla u_i + \left( \nabla u_i \right)^T \right) \right] - \nabla \cdot u_i = 0 \quad (1)$$

in which the subscript $i$ denotes either the feed $(i = f)$ or draw $(i = d)$ channels, and the solutions density and kinematic viscosity are denoted by $\rho_i$, $\nu_i$, respectively. The pressure term $p_i = P_i - P_0$ represents the difference between the actual applied pressure $P_i$ and the pressure $P_0$, at the outlet of the channel. The 2-D differential operator, $\nabla = (\partial/\partial x, \partial/\partial y)$ is along $x$ and $y$ being the coordinates normal to the membrane surface and in the cross flow direction, respectively, and $u_i$ is the velocity vector in the $x, y$ domain. The axial velocity in the support layer at $y = 0.1$ is set to zero and the velocity profile at the channel entrance (i.e., $y = 0$) was taken as parabolic. Within the porous support layer the flow field is described by the Brinkman’s equation [26],

$$\frac{1}{\kappa} \nabla p = \frac{1}{\nu_f} \nabla \cdot \left[ \kappa \left( \nabla u + \left( \nabla u \right)^T \right) \right] - \nabla \cdot u = 0 \quad (2)$$
where $\kappa$ and $\varepsilon_p$ are hydraulic permeability of the support layer and its porosity, respectively.

Salt transport in the feed and draw channels and the support layer is described by the convection and diffusion equations [26],

$$\nabla \cdot (D_k \nabla c_k) = u_{ki} \cdot \nabla c_k$$

(3)

in which index $i$ refers to either the feed channel ($i = f$), porous support ($i = p$) or draw channel ($i = d$), and where subscript $k$ represents the solute. The mass diffusivity of solute $k$ is denoted by $D_k$, and where in the porous support $D_{kp} = D_k \cdot \varepsilon_p / \tau$, in which $\tau$ is the porous layer and $D_k$ is the solute mass diffusivity in the solution.

The draw and feed solutions enter the PRO channels at salt concentrations of $C_{df}$ and $C_{fi}$, respectively, and the velocity profiles at the entrance to the two channels are taken to be laminar. Water permeates from the feed to the draw channel and the local flux is given as [2],

$$J_w = A \cdot \left[ (\pi_d - \pi_f) - (P_d - P_f) \right]$$

(4)

in which $A$ is the membrane intrinsic water permeability, $\pi_d$ and $\pi_f$ are the solution osmotic pressures at the membrane active layer surfaces at the draw and feed-sides, respectively, The hydraulic pressures $P_f$ and $P_d$ are in the feed and draw channels, respectively, where the corresponding hydraulic pressures in the draw and the feed channels are given as $P_{do} = p_d + P_{do}$ and $P_f = p_f + P_{fo}$. The outlet pressures in the draw and feed channels are denoted by $P_{do}$ and $P_{fo}$, respectively, and the pressures difference variables $p_d$ and $p_f$ are those used in Eqs. (1) and (2). Solute flux (across the membrane) from the draw to the feed channel is described by,

$$J_s = B \cdot (C_d - C_p)$$

(5)

in which $B$ is the intrinsic salt transport coefficient, $c_d$ and $c_p$ are draw and support concentrations on both sides of the membrane skin surfaces. Additional boundary conditions listed in Table 1 and referenced in Fig. 3, which are necessary for the solution of the model Eqs. (1)-(3), include symmetry, no-slip ($u_i = 0$) and solute no-flux boundary conditions.

**Fig. 1.** Work flow for evaluating the impact of various factors on PRO energy production.
conditions (insulation or impermeable wall, \( J_s = 0 \)), and viscous stress and pressure conditions at the channels [26].

Eqs. (1)–(7), PRO channel dimensions and associated boundary conditions (Table 1) constitute the PRO 2-D CFD model (Fig. 3). Model input parameters include the following concentration-dependent parameters: dynamic viscosity (\( \eta \)), density (\( \rho \)), osmotic pressure (\( \pi \)), and solute mass diffusivity (\( D \)). Additional parameters included solution permeability (\( k \)) through the porous support, porous support layer porosity (\( \eta_p \)) and tortuosity (\( \tau \)), and membrane water permeability (\( A \)) and salt transport parameter (\( B \)). In the present study, the parameters \( A \) and \( B \) were extracted from experimental membrane performance data (Section 3.1 and Appendix A, Section A.1), using the CFD transport model, as described elsewhere [23,25].

The net power generation, \( W_n \) (expressed as power generation per membrane area, i.e., power density, W/m²), derived from the PRO process, accounting for internal pressure losses in the draw and feed channels and efficiencies of feed pumps and energy recovery device (ERD) [9] is calculated from:

\[
W_n = J_w P_d \rho \eta_d - \left[ \frac{Q_d}{S_m} \left( \frac{P_d}{\eta_d} - P_d \rho \eta_d \right) + \frac{Q_f}{S_m} \frac{F_j}{\eta_f} \right]
\]

where \( Q \), and \( P \) are the volumetric flowrate and pressure, average flux \( J_w \) along the membrane channel, indices \( d, f \) designate the draw and feed streams respectively and \( i, o \) denote the channels inlet and outlet, respectively, and where feed outlet is taken to be at atmospheric pressure.

The efficiencies of the draw (\( \eta_d \)) and feed (\( \eta_f \)) high and low pressure pumps, respectively, are expected to be in the range of 0.6–0.8 [27,28], and efficiency of energy recovery devices (e.g., turbines), \( \eta_{ERD} \), at the high end is in the range of 0.90–0.96 [29]. Here it is noted that the condition of \( \eta_d = 1 \) and \( \eta_f = \eta_{ERD} = 1 \) would represent an ideal scenario which is unlikely to be realized in practice. Moreover, given the added pressure added losses due to channel spacers, the pressures \( P_{j,o} \) at the feed and draw channel outlets is reduced as expressed below,

\[
P_{j,o} = P_{j,i} - \Delta P_{j,loss}
\]

in which \( j = f \) for feed and \( j = d \) for draw and where \( \Delta P_{j,loss} \) represents the pressure loss in the channels (draw or feed) calculated (Appendix A, Section A.2) following the method of [30] which accounts for channel spacers. This approach was taken since CFD analysis of PRO with channel spacers is computationally prohibitive given the need for iterative simulations to arrive at the optimal operating conditions. Nonetheless, the present approach provides a reasonable estimate of the expected pressure losses.

Once the permeate flux is determined for a given PRO channels geometry and operating conditions, the net power generation, \( W_n \), can be calculated via Eq. (6) accounting for pressure losses as per [30]. The cross flow velocities of the feed and draw streams, \( V_f \) and \( V_d \), respectively, affect the net achievable power density, \( W_n \), given that pumping of both the feed and draw solutions is likely to require energy input. Therefore, the minimal inlet applied feed pressure required for maintaining

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**Table 1**

Boundary conditions\(^{*}\) for the PRO 2-D system with SFD/counter-current configuration shown in Fig. 3.

<table>
<thead>
<tr>
<th>#</th>
<th>Boundary-conditions(^{*})</th>
<th>Navier-Stokes equations</th>
<th>Convection-diffusion equation</th>
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<td></td>
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<td>Feed</td>
<td>Porous</td>
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<td>1</td>
<td>Symmetry(^{b})</td>
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<tr>
<td>2</td>
<td>Outlet(^{c})</td>
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<td>Inlet(^{a})</td>
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<td>5</td>
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<td>Wall(^{d})</td>
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\(^{a}\) Boundary numbers are defined in Fig. 3. Details of the mathematical expressions for the boundary conditions are provided in [26].

\(^{b}\) Zero velocity normal to the plane of symmetry.

\(^{c}\) Zero viscous stresses and zero applied pressures \( p_f \) and \( p_d \).

\(^{d}\) No slip condition or zero velocity.

\(^{e}\) Zero convective flux normal to the symmetry plane.

\(^{f}\) Vanishing (within a prescribed tolerance, See Section 2.1) axial convective fluxes at the feed channel outlet.
the feed flow was set as \( P_{\text{fin}} \approx P_{\text{floss}} \). Furthermore, in order to assess the beneficial impact of developing membranes of higher permeability than the current generation of PRO membranes, simulations were also conducted over a wider range of membrane permeability \((A)\) and salt transport coefficient \((B)\). Also, since pump and ERD efficiencies reduce the obtained net power, in the present approach simulations are first conducted searching for the PRO operating conditions that maximize the net power production for ideal conditions \((i.e., 100\%\) efficiencies of feed pumps and ERD). Subsequently, the impact of feed pumps and ERD efficiencies on the net power generation is determined to assess the practically achievable osmotic power production.

### 2.3. Simulations

In order to evaluate the influence of various PRO operational factors on the feasible level of osmotic energy extraction it is necessary to first establish the membrane transport parameters \((A\) and \(B)\). In the present study, as per the approach summarized in Fig. 1 and outlined in Section 2, simulations were conducted based on PRO membrane of water permeability \((A)\) and salt transport coefficient \((B)\) as determined from experimental bench-scale PRO data (Appendix A). Accordingly, the PRO model equations (Section 2.2) were solved via the finite-element approach [26] seeking the membrane parameters that lead to best matching of the experimental and calculated salt fluxes with deviations of \(<3.3\%\) and \(9.8\%\) for \(J_{\text{w}}\) and \(J_{\text{s}}\), respectively. The extracted \(A\) and \(B\) values (Appendix A, Section A.1), were then utilized in simulations to arrive at the inlet feed and draw streams crossflow velocities that would maximize PRO power generation for a 1 m long channel (Table 2) which is about the length typically expected in commercial deployment [6]. Using the PRO model (Section 2.2), PRO analysis was subsequently carried out for feed and draw solutions of salt concentrations that reflect the salinity levels for brackish water and brine from seawater desalination \(i.e., RO\) concentrate, as well as Dead Sea water as draw for PRO; the latter has been proposed for commercial deployment of PRO power generation [31].

Simulations were carried out with mesh density that increased from the bulk toward the membrane surfaces. For short PRO channels the element density was set at \(-560\) elements/mm\(^2\) in proximity of the membrane surface and close to the channels ends and decreased to \(-110\) elements/mm\(^2\) toward the channel center and near the walls. The mesh density was doubled for the long channel simulations. Grid size independence of the numerical solution was evaluated for all simulations as per the convergence criterion described in [23], whereby global convergence was ascertained when the calculated water flux did not significantly change \((<0.001\%)\) upon doubling the mesh density.

### 3. Results and discussion

#### 3.1. Determining the draw and feed stream cross flow velocities that maximize the net power production for the case of ideal pumps and energy recovery device (ECD)

The approach to assessing the operating conditions that will maximize PRO net power generation was demonstrated for a membrane having the specific membrane transport parameters extracted from the experimental data in [7]. This dataset was selected given the relatively high power density \((W = 12.8 \text{ W/m}^2)\) reported, along with corresponding water and salt flux data. Extraction of the membrane \(A\) and \(B\) parameters from the dataset revealed those parameters to be pressure dependent (Appendix A, Section A.1). Subsequently, assessment of the maximum attainable power production for a long \((1\) m) channel (Table 2) proceeded with the net power density \((W_n)\) calculated considering pressure losses (Eqs. (6) and (7), and Appendix A, Section A.2) while also assessing the impact of pumps and ERD efficiencies.

The generated power production has a maximum with respect to the applied pressure difference (Fig. 4) as expected for the PRO process.
Frictional losses (expressed as power density) for a long membrane (1 m) under the ideal case (i.e., \( W = f_w \Delta P \), where \( f_w = A(\Delta \tau - \Delta P) \) and constant \( A \) set as the average value from Fig. A1) was higher by a factor of 4.8 and 4.2 than predicted by the CFD model with and without pressure losses, respectively, assuming pumps and ERD of 100% efficiency. The above behavior is not surprising considering that the CFD model accounts for the decline in osmotic pressure driving force (due to concentration polarization and salt back diffusion) and channel frictional pressure losses. As the draw stream crossflow velocity increases, the impact of CP decreases which tends to elevate the level of energy production; however, frictional pressure losses increase with increased crossflow velocity (Appendix A, Section A.2). As a consequence, a maximum achievable peak power density increases by 1.7% (from 6.89 to 7.01 W/m²) and a sthe minor power density gain of ~4% (i.e., from 7.3 W/m² to ~7.6 W/m²). For a given crossflow velocity, thinner channels will develop thinner ECP (given the higher fluid shear rate at the channel fluid/membrane interface) and thus higher water flux and correspondingly higher \( W_p \). However, pressure losses can increase significantly for thin channels once one accounts for the impact of spacers. For example, relative to a PRO channel of 1 m in length and 0.71 mm height, decreasing the channel height to ~0.5 mm (i.e., by ~29%), for a given crossflow velocity, would result in a vanishingly low level of power generation, when compared at the same crossflow velocity, given the low flow rate which cannot sustain the required flux. Frictional pressure losses increase with channel length (i.e., lower pressure loss for shorter channels; Appendix Section A.2). For example, decreasing the membrane length from 1 m to 0.5 m or 0.25 m would increase the energy density (relative to power density of ~7.3 W/m² for 1 m long channel) by ~13% and 33%, respectively. Here it is noted that the use of shorter membrane elements could increase capital cost (due to increased number of membranes, fittings, valves, etc.); therefore, the selection of membrane element length is clearly a decision that would involve economics and engineering considerations.

As the channel height of ~0.4 mm is reached, a maximum peak power density is attained which decreases rapidly with further decrease in channel height. Simulations for lower channel height down to 0.35 mm would result in a vanishingly low level of power generation, when compared at the same crossflow velocity, given the low flow rate which cannot sustain the required flux. Frictional pressure losses increase with channel length (i.e., lower pressure loss for shorter channels; Appendix Section A.2). For example, decreasing the membrane length from 1 m to 0.5 m or 0.25 m would increase the energy density (relative to power density of ~7.3 W/m² for 1 m long channel) by ~13% and 33%, respectively. Here it is noted that the use of shorter membrane elements could increase capital cost (due to increased number of membranes, fittings, valves, etc.); therefore, the selection of membrane element length is clearly a decision that would involve economics and engineering considerations.

### 3.2. Impact of PRO channel length and height on the attainable net peak power production

Frictional losses, concentration polarization and salt back diffusion, which are all impacted by PRO channel length and height, reduce the attainable PRO energy generation. As an illustration, the impacts of channel length and height are shown in Fig. 6, for the case of ideal pumps and hydraulic energy recovery device (i.e., 100% efficiency) and draw and feed solutions as in Fig. 5 at inlet draw and feed velocities of 14 cm/s and 6 cm/s, respectively. As seen in Fig. 6, the net PRO power density is marginally impacted by the channel dimensions. For a given crossflow velocity, thinner channels will develop thinner ECP (given the higher fluid shear rate at the channel fluid/membrane interface) and thus higher water flux and correspondingly higher \( W_p \). However, pressure losses can increase significantly for thin channels once one accounts for the impact of spacers. For example, relative to a PRO channel of 1 m in length and 0.71 mm height, decreasing the channel height to ~0.5 mm (i.e., by ~29%), for a given crossflow velocity, would result in minor power density gain of ~4% (i.e., from 7.3 W/m² to ~7.6 W/m²).

### 3.3. Effects of membrane permeability (A) and salt transport coefficient (B)

An assessment of the impact of increasing membrane permeability (A) and decreasing salt transport (B) on the level of PRO energy generation is illustrated in Fig. 7 for the case of draw/feed streams as in Fig. A1 (Table A3 in Appendix A), representing the case mimicking Seawater RO concentrate/low salinity brackish water as draw/feed streams), for the channel dimensions given in Table 2, considering frictional losses but with ideal pumps and ERD and crossflow velocities as in Figs. 4–6. For the purpose of the present illustrative analysis, the achievable peak PRO energy density was estimated considering constant A and B values relative to the corresponding nominal values of 3.8 · 10⁻¹² m/s/Pa and 5.1 · 10⁻⁷ m/s (i.e., the intrinsic A and B values derived from [7] (Fig. A1, Appendix A). Such analysis is somewhat speculative since information on the variation of salt transport coefficient relative to water permeability is unavailable for membranes of characteristics removed from the nominal values of the presented analysis. Notwithstanding the above, as expected (Fig. 7a) higher A would lead to higher water flux and thus higher attained power peak density. As demonstrated in Fig. 7a, there is a sharp increase in energy production, by up to a factor of ~2.75, as the ratio A/A_{in} increased from 1 to 10.
draw/feed streams for constant water is about 0.79 kWh/m³ [36]; thus, a reversible process of osmotic (SWB). It is interesting to note that the osmotic energy available in sea-

energy density (based on membrane area), \( W_{n} \), and feed water volume utilized per unit energy produced (m³/kWh) denoted as \( V_{e} \). Simulation are shown for the case of SWB/BW draw/feed streams for constant \( A \) and \( B \) values with \( B = B_{\text{in}} \) in 7(a) and \( A = A_{\text{in}} \) for 7(b) considering ideal pumps and ERD and crossflow velocities that maximize peak power generation for the channel dimensions as in the simulation scenario of Fig. 4. (Note: \( A_{\text{in}} \) and \( B_{\text{in}} \) are 3.8 - 10^{-7} m/s and 5.1 - 10^{-7} m/s, respectively, based on analysis of the data in [7]).

The peak power density that can be generated via PRO from the above SW/BW, SWB/BW and DSW/SWB draw/feed water streams were determined, from CFD model simulations and using Eq. (6), to be 3.53 W/m², 7.29 W/m², and 35.3 W/m², respectively, for the case of ideal pumps (i.e., \( \eta_{p} = 1 \)) and ERD (i.e., \( \eta_{k} = 1 \)). The peak power density decreased linearly with decreasing pump and ERD efficiencies as illustrated in Figs. 8–9. It is noted, that for high end ERD efficiency \( \eta_{k} = 0.96 \) [29] and ideal pumping efficiency \( \eta_{p} = 1 \) the peak power density expected from DSW/SWB PRO is 19 W/m². The results of Fig. 9 illustrate that ERD efficiency would have to be above 0.91, 0.92 and 0.95, for the DSW/SWB, SWB/SW and SW/BW draw/feed streams, in order for the PRO process to provide net positive peak energy production even with the use of ideal pumps (\( \eta_{p} = 1 \)). Similar analysis reveals that, for the case of an ideal ERD \( \eta_{k} = 1 \), the minimum pump efficiency required to achieve positive net peak power production for the DSW/SWB, SWB/BW and SW/BW draw/feed streams is 0.91, 0.92 and 0.96, respectively. As shown in Fig. 9, for the case of high efficiency ERD \( \eta_{k} = 0.96 \), even the most optimistic scenario of high PRO energy productivity for DSW/SWB with commercially available high efficiency pumps (i.e., \( \eta_{p} = 0.98 \), [27,28]), the maximum peak power density is not likely to exceed -12 W/m² and 2 W/m² for the DSW/SWB and SWB/BW cases, respectively, while essentially no power generation is expected for the SW/BW case.

It is instructive to quantify the sensitivity of PRO peak power generation (at the optimal conditions that maximize power production) with respect to pump and ERD efficiencies for the three different feed/draw stream evaluated. Accordingly, one can express the change in the maximum peak power generation as \( \Delta W_{i} = \alpha_{i}(\Delta \eta_{i}) \) where \( \alpha_{i} \) is a sensitivity coefficient and the subscript \( i \) denotes either the system pumps or ERD. These sensitivity coefficients are essentially the slopes of the net energy production versus efficiency in Figs. 8 and 9. Accordingly, 1% change in pumping or ERD efficiencies would result in the peak power generation as listed in Table 3 demonstrating that improvements in power generation by pump or ERD efficiencies will be more significant for the DSW/ SW draw/feed combination. The power generation gain by increasing pump efficiency will be marginal for the SWB/BW and SW/BW given that relatively high pump and ERD efficiency are required (Fig. 9) for net power generation. Likewise, increasing ERD efficiency beyond 92%
and 95% for SWB/BW and SW/BW are required with essentially ideal pumps for reasonable net energy production. The above analysis suggests that only the case of DSW/SWB could possibly provide reasonable net energy production (e.g., ≥ 5 W/m² [37]) but this provided that pumps of high efficiency (e.g., well above 90%) would be available and with ERD efficiency of ~98% and preferably higher.

### 3.5. Implications of parameter estimation with respect to PRO performance assessment

The present PRO process analysis was based on fundamental treatment that considers the coupling of fluid flow and mass transfer whereby membrane transport parameters and other relevant geometrical channel parameters are extracted from experimental data reported in the literature. It is noted, however, that fundamental membrane transport parameters and accompanying PRO performance data are generally reported based on experiments performed in small PRO cells of short channel length and small membrane coupons. In such studies, the crossflow velocities of the draw and feed streams in the channels are often higher than the optimal range that would maximize the produced energy while not revealing the excessive frictional pressure losses that would be otherwise experienced in longer channels. Also, the channel heights in bench scale systems are usually high to the extent that channel Reynolds numbers are higher (often ≥ 1000) [10,20] than one would expect for a PRO process operating under optimal conditions (e.g., those required for high utilization of the feed volume). It is noted that bench-scale laboratory PRO data were readily available in the literature for seawater/freshwater PRO evaluation of specific membrane transport properties. In contrast, literature PRO data were insufficient or unavailable for higher concentration feed and draw streams. Therefore, in the present analysis, the values of A and B utilized for the high concentration streams (e.g., seawater RO brine and Dead Sea water) were taken to be those extracted for the lower concentrations. In this regard, it is expected that membrane water permeability would be expected to be lower and salt passage higher for the above higher concentration feed and draw streams. As a consequence, the present estimation of PRO performance with respect to energy extraction from seawater, seawater RO brine and Dead Sea water is expected to be overly optimistic with regard to the level of attained power production.

It is known that increased crossflow velocity in the feed and draw channels will reduce concentration polarization which in turn reduces water permeation across the membrane. However, increased velocity...
within the flow channels also increases frictional pressure losses (i.e., greater the pressure drop) which reduces the net extracted energy. As in optimized RO membrane elements, the channel height and stream velocity along the membranes are established so as to minimize pressure losses, and maximize element product water recovery, while also striving to minimize fouling which increases with increased permeate flux. In PRO, the feed stream velocity will decrease along the channel (from inlet to outlet), thereby increasing the importance of CP in this channel (Appendix B) to an extent which is more significant in longer channels than in short laboratory test cells. In contrast, the draw stream velocity increases along the channel (from inlet to exit), given the gain of water from the feed side; thus, frictional pressure losses are expected to increase along the draw channel (Appendix B), while CP will only slightly reduced for the expected Reynolds number increase. The above intricate coupling of hydrodynamics and CP development along the channels calls for optimization of the velocities in both channels to reduce both CP and pressure losses. It is estimated that in 1 m long membrane, the possible tolerance in the extracted energy as per the present analysis is <10%. Since the overall energy gain in the calculated cases presented above (see Section 3.4) is very low and considering a realistic range of pumps and ERD efficiencies, even 20% improvement in the energy balance will not significantly alter the conclusions as portrayed in the analysis leading to the results provided in Figs. 8 and 9.

4. Summary and conclusions

An approach to quantifying the feasible level of osmotic power generation via pressure retarded osmosis (PRO) was developed based on detailed two-dimensional finite element (2-D-FEM) model consisting of the fully-coupled hydrodynamics and salt mass transfer equations. The analysis was carried out for a channel length of 1 m which is the typical size expected for commercial size PRO elements. Pressure losses along the feed and draw channels and the efficiencies of pumping and energy recovery were also considered in the analysis. The approach to arriving at the operating conditions that maximize peak power generation was illustrated for the case of PRO membrane of a relatively high permeability. PRO simulations, using experimental membrane transport parameters extracted from experimental data via a 2-D-FEM PRO model, were carried out to determine the required inlet draw and feed inlet velocities that maximize the generated power with illustrative test cases discussed for three different draw/ feed stream salinities representing seawater/brackish water (SW/BW), seawater RO-brine/brackish water (SWB/BW), and Dead Sea water/Seawater (DSW/SWB). The analysis revealed that the maximum generated peak power density was highest for DSW/SW (35.3 W/m²), followed by SWB/BW (7.29 W/m²) and SW/BW (3.53 W/m²). Positive net power generation, even for the case of high efficiency (96%), would only be reached if pump efficiencies would be in excess of 99.5%, 96.5% and 94.5%, for PRO with the draw/feed stream pairs of SW/BW, SWB/BW and DSW/SWB, respectively. In assessing the expected performance of PRO for the above cases, it is emphasized that with the selection of optimal draw and feed stream crossflow velocities, the loss (or reduction) in net peak power production is in the ranges of 10–36% caused by reduction in osmotic pressure driving force due to CP, 0.63–1.1% due to frictional pressure losses in the feed, and draw channels and 6–46% due to less than ideal efficiencies (i.e., set as 98%) ERD and pumps. It is noted that with ideal pumps a loss of ERD efficiency of 2%, relative to an ideal ERD, is manifested by 3–23% reduction in the attainable peak power production, as well as about 3–23% loss of net peak power production that would occur for the case of pumps of 98% efficiency but ideal pumps ERD.

Overall, it appears that even for the most optimistic scenario of high PRO energy productivity, from DSW/SW with energy recovery devices and pumps of extremely high efficiencies (96% and 98%, respectively), the maximum peak power density is not likely to exceed ~2 W/m² and 12 W/m² for the DSW/SWB and SWB/BW PRO cases, while no net positive power generation should be expected for the SW/BW case. It is conceivable that higher permeability PRO membranes could provide somewhat higher performance; however, pressure losses and the need for extremely high efficiency pumps and ERDs would present a significant challenge for PRO as a viable energy generation approach.

Acknowledgements

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Appendix A

A.1. Extraction of intrinsic water A and salt B permeability coefficients from PRO data

The intrinsic membrane water permeability (A) and salt transport coefficient (B) were extracted from experimental data channel with spacers as provided in [7]. The average feed and draw stream velocities in the above dataset were calculated for the data in Fig. 4, PEI 2# membrane of [7], as per the equations of [30] resulting in the velocities presented in Table A1. The CFD PRO model was then utilized to determine the values of A and B by matching model predictions of the water (jw) and salt (js) fluxes with the reported experimental data for these variables. The resulting A and B values in Fig. A1, which were found to vary with the applied transmembrane pressure, were used in the CFD simulations of PRO performance in a long (1 m) channels as described in Section 3.1.

Table A1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Draw (cm/s)</th>
<th>Feed (cm/s)</th>
<th>Equations of [30]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ε</td>
<td>0.765</td>
<td>0.543</td>
<td>Spacer’s porosity, Eqs. (20)–(22)</td>
</tr>
<tr>
<td>dₒ [mm]</td>
<td>1.31</td>
<td>0.2</td>
<td>Hydraulic diameter, Eqs. (17), (23)</td>
</tr>
<tr>
<td>v [cm/s]</td>
<td>11.8</td>
<td>10.4</td>
<td>Cross flow velocity, Eqs. (11), (12)</td>
</tr>
</tbody>
</table>

* Spacers SHW2 and SHW3 used in [7] for the draw and feed channels, respectively.
* Crossflow velocities and pressure losses in feed and draw channels with spacers were calculated as per the equations in [30].

Fig. A1. Dependence of intrinsic membrane permeability coefficients A and B on the hydraulic pressure difference Δp. Dots are of the coefficients derived by the PRO model based on the data in Fig. 4, PEI 2# membrane of [7]. The dashed lines are curves fitted to the extracted A and B values.
Table A2
Information sources utilized for estimation of the pressure loss in the feed and draw channels\(^a\).

<table>
<thead>
<tr>
<th>Channel</th>
<th>Spacers type</th>
<th>Spacers porosity</th>
<th>Spacers thickness [mm]</th>
<th>Hydraulic diameter(^b)</th>
<th>Friction coefficients(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed (Table A1)</td>
<td>Toray PEC-1000</td>
<td>0.59</td>
<td>0.72</td>
<td>Eqs. (17), (23)</td>
<td>Fig. 7</td>
</tr>
<tr>
<td>Draw (Table A1)</td>
<td>Naltex 1228</td>
<td>0.90</td>
<td>0.75</td>
<td>Eqs. (17), (23)</td>
<td>Fig. 6</td>
</tr>
</tbody>
</table>

\(^a\) Pressure losses were calculated using the data and approach provided in [30].
\(^b\) The hydraulic diameter, \(d_h\), was calculated via Eqs. (17), (23) of [30].
\(^c\) Figures of [30] from which friction coefficients were calculated.

A.2. Estimation of pressure losses in the draw and feed channels

Assessment of the net power generation, \(W_{n}\) (Eq. (6)), requires determination of the losses in both the feed (\(P_{fe}\)) and draw (\(P_{dr}\)) channels taken to be of 1 m length in the present analysis. Pressure loss in the draw channel (\(P_{dr}\)) was calculated via Eq. (24) of [30] with the parameters reported for the spacer Naltex 1228, Table 11 of [30]. Pressure loss in the feed channel (\(P_{fe}\)) was determined for parameters reported for the Toray PEC-1000, Table A2 and Fig. 7 of [30]. It is noted that the characteristics of the above spacers are similar to those used in [7] which were adopted for the present RO simulations. It is noted that the calculations were accomplished for the equivalent set of spacers that could fit in the channel spacing and that the use of different spacers in the feed and draw channels is common in spiral wound membrane elements [30,38].

Pressure losses were calculated following the approach in [30] whereby the pressure loss is calculated from \(P_{loss} = 0.5 \rho V^2 L/d_b\), in which \(\rho\) is the solution density, \(V\) is the average crossflow velocity and \(d_b\) is the hydraulic diameter, i.e., \(d_b = 4\epsilon/(2(b + h)bh + (1 - \epsilon)S_{sp}/bh)\) where \(\epsilon\) is the porosity of the channel containing the spacers, \(b\) and \(h\) are feed channel width and height, respectively and \(S_{sp}\) is the specific spacer surface area. The friction coefficient, \(\lambda\), for the draw channel is given by \(\lambda = 13 Re^{-0.8}\) and as \(\lambda = 6.23 Re^{-0.3}\) for the feed channel corresponding to the spacer arrangements as per Table A2. Finally, the Reynolds number is defined as \(Re = V/d_b/\nu\) in which \(\nu\) is the kinematic viscosity of the flowing fluid stream.

Table A3
Illustration of power production losses due to frictional losses and pumps and ERD efficiencies for the scenario as in Fig. 9.

<table>
<thead>
<tr>
<th>PRO operational scenario</th>
<th>PRO power production or loss, W/m² (also indicated as % of (W_{n}))</th>
<th>DSW/SWB</th>
<th>SWB/BW</th>
<th>SW/BW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum theoretical power generation, (W_{ax})</td>
<td>1039</td>
<td>22.2</td>
<td>9.37</td>
<td></td>
</tr>
<tr>
<td>Max power generation considering CP losses, (W_{1} = J_{m} \Delta p)</td>
<td>36.0</td>
<td>7.75</td>
<td>3.98</td>
<td></td>
</tr>
<tr>
<td>Draw side frictional pressure losses, (W_{fdr})</td>
<td>0.632</td>
<td>0.42</td>
<td>0.407</td>
<td></td>
</tr>
<tr>
<td>Feed side frictional pressure losses, (W_{ffe})</td>
<td>0.043</td>
<td>0.037</td>
<td>0.038</td>
<td></td>
</tr>
<tr>
<td>Power production after accounting for frictional losses, (W_{n} = (1 - \eta_{pmp}) W_{ax})</td>
<td>22.9</td>
<td>5.33</td>
<td>2.39</td>
<td></td>
</tr>
<tr>
<td>((\eta_{pmp} = 0.98)) and frictional losses for the case of ideal pumps</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Power production accounting for loss of pump efficiency, (\eta_{pmp} = 0.98) and frictional losses for the case of ideal ER pumps</td>
<td>23.4</td>
<td>5.44</td>
<td>2.44</td>
<td></td>
</tr>
<tr>
<td>Net power generation considering frictional losses and 98% and 96% pumping and ERD efficiencies, respectively</td>
<td>10.9</td>
<td>1.96</td>
<td>−0.0</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) The power production loss are expressed as percentage of the maximum theoretically achievable power density; \(^b\) \(A\) and \(B\) coefficients were derived via CFD model and experimental data. The coefficients of other two cases were estimated since no data available.
\(^c\) Peak power production for the PRO systems of SW/BW, SWB/BW and DSW/SWB was achieved for draw channel inlet pressures of 11.7 bars, 18.1 bars and 117.6 bars, respectively.

Appendix B. Illustration of profiles of solute concentration, osmotic pressure, velocity profiles, water flux and PRO power density

In PRO operation (in the preferred counter-current mode) the osmotic pressure in the draw channel, at the membrane surface, decreases rapidly from inlet toward the exit as the draw stream gains water from the feed side (Fig. B1(a)). At the same time, the osmotic pressure on the feed side, at the support layer skin-side, increases from inlet (i.e., \(x = 1\) m in Fig. B1(b)) toward the feed channel outlet (i.e., \(x = 0\)). In the counter-current configuration there is a severe decline in osmotic pressure in the draw stream inlet region and feed stream outlet regions.
(a) Velocity profile for SW/BW

(b) Concentration profile for SW/BW

(c) Concentration profile from feed channel centerline to draw side channel centerline

(d) Local power density along the membrane

**Fig. B2.** Illustration of (a) velocity, (b) 2-D salt concentration profile, (c) traverse salt concentration profile (from feed to draw channel across the membrane and support layer, and (d) axial power density profiles. Simulations are for SW (0.6 M)/BW (0.01 M) PRO showing the profiles for channels 1 m long and each (feed and draw) being 0.71 mm in height, with inlet draw and feed crossflow velocities being 14 cm/s and 6 cm/s, respectively and where the peak power density was attained at draw side inlet pressure of 13.7 bars. Note that the support layer domain in the figures is expanded (i.e., it is not to scale) in order to show the profile in this region. In Fig. B2(c) the concentration profile from left to the right is denoted as: feed bulk \( (x = 0) = 0.0103 \text{ M} \), feed end of the support layer \( (x = 0.3555 \text{ mm}) = 0.0279 \text{ M} \); here we note that \( ECP_f = 0.0176 \text{ M} \), concentration at membrane end of the support layer \( (x = 0.4315 \text{ mm}) = 0.0292 \text{ M} \), i.e. \( ICP = 0.001 \text{ M} \), membrane surface concentration on the draw side \( (x = 0.4315 \text{ mm}) = 0.418 \text{ M} \), and concentration difference across the skin \( (CDS) \), i.e. \( CDS = 0.389 \text{ M} \) and \( ECP_d = 0.182 \text{ M} \). Concentration drop ratio \( ECP_d/ICP = 182 \). (Note: \( ECP_f \) = concentration difference across the feed-side concentration polarization layer, \( ICP \) = concentration difference across the support layer).
Fig. B3. Illustration of (a) velocity, (b) 2-D salt concentration profile, (c) traverse salt concentration profile (from feed to draw channel across the membrane and support layer, and (d) axial power density profiles. Simulations are for SWB (1 M)/BW (0.01 M) PRO showing the profiles for channels 1 m long and each (feed and draw) being 0.71 mm in height, with inlet draw and feed crossflow velocities being 14 cm/s and 6 cm/s, respectively and where the peak power density was attained at draw side inlet pressure of 13.7 bars. Note that the support layer domain in the figures is expanded (i.e., it is not to scale) in order to show the profile in this region. In Fig. B3(c) the concentration profile from left to the right is denoted as: feed bulk concentration (at x = 0) = 0.0108 M, feed end of the support layer (x = 0.3555 mm) = 0.048 M; here we note that $ECP_f = 0.0372$ M, membrane end of the support layer (x = 0.4315 mm) = 0.054 M, i.e. $ICP = 0.006$ M, membrane surface concentration on the draw side (x = 0.4315 mm) = 0.631 M, and concentration difference across the skin (CDS) i.e. $CDS = 0.577$ M and $ECP_d = 0.423$ M. Concentration drop ratio $ECP_d/ICP = 72$. 

(a) Velocity field of the PRO unit for SWB/BWB.

(b) Concentration field of the PRO unit for SWB/BW

(c) Concentration profile from feed channel centerline to draw side channel centerline

(d) Local power density along the membrane
References


Table B3

<table>
<thead>
<tr>
<th></th>
<th>Feed</th>
<th>Draw</th>
<th>Feed</th>
<th>Draw</th>
<th>Feed</th>
<th>Draw</th>
<th>Feed</th>
<th>Draw</th>
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<tbody>
<tr>
<td>( \rho ) [kg/m(^3)]</td>
<td>997</td>
<td>1022</td>
<td>8.91</td>
<td>9.41</td>
<td>2.13</td>
<td>4.98</td>
<td>8.94</td>
<td>141</td>
</tr>
<tr>
<td>( \eta ) [10(^{-4}) Pa s]</td>
<td>997</td>
<td>1039</td>
<td>8.91</td>
<td>9.71</td>
<td>2.13</td>
<td>4.98</td>
<td>8.94</td>
<td>139</td>
</tr>
<tr>
<td>( Q ) [10(^{-5}) m(^3)/s]</td>
<td>139</td>
<td>1238</td>
<td>9.71</td>
<td>27.2</td>
<td>2.13</td>
<td>4.98</td>
<td>8.55</td>
<td>58.2</td>
</tr>
</tbody>
</table>

(a) Simulations were carried out for 1 m long channels each being 0.71 mm in height.

Fig. B4. Illustration of (a) velocity, (b) 2-D salt concentration profile, (c) traverse salt concentration profile (from feed to draw channel across the membrane and support layer, and (d) axial power density profiles. Simulations are for DSW (5.8 M)/SWB (1 M) PRO showing the profiles for channels 1 m long and each (feed and draw) being 0.71 mm in height, with inlet draw and feed crossflow velocities being 14 cm/s and 6 cm/s, respectively and where the peak power density was attained at draw side inlet pressure of 13.7 bars. Note that the support layer domain in the figures is expanded (i.e., it is not to scale) in order to show the profile in this region. In Fig. B4(c) the concentration profile from left to the right is denoted as: feed bulk \( x = 0 \) = 1.011 M, feed end of the support layer \( x = 0.3555 \) mm) = 1.527 M, here we note that \( ECP_f \) = 0.516 M, membrane end of the support layer \( x = 0.4315 \) mm) = 1.624 M, i.e., \( ICP = 0.097 \) M, membrane surface concentration on the draw side = 3.689 M, and concentration difference across the skin (\( CBD, x = x = 0.4315 \) mm), i.e., \( CBD = 2.065 \) M and \( ECP_d = 3.735 \) M. Concentration drop ratio \( ECP_d/ICP = 47.3 \).
[4] Anon, Management Unit of the North sea Mathematical Model (MUMM), (Department of the Royal Belgian Institute of Natural Sciences, (2002–2015)).