Effect of Stream Mixing on RO Energy Cost Minimization

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Abstract

Various mixing operations between the feed, retentate and permeate streams are studied in this work to determine their effectiveness in decreasing the specific energy consumption (SEC) of single-stage (single-pass), two-pass and two-stage reverse osmosis (RO) processes operated at the limit of the thermodynamic restriction. The results show that in a single-stage RO process, partial retentate recycling to the feed stream does not change the SEC, while partial permeate recycling to the feed stream increases the SEC if targeting the same overall water recovery. Energy optimization of two-pass membrane desalination, with second-pass retentate recycling to the first-pass feed stream and operated at the limit imposed by the thermodynamic restriction, revealed the existence of a critical water recovery. When desalting is accomplished at recoveries above the critical water recovery, two-pass desalination with recycling is always less efficient than single-pass desalination. When desalting is accomplished at recoveries below the critical water recovery, an operational sub-domain exists in which the SEC for a two-pass process with recycling can be lower than for a single-pass counterpart, when the latter is not operated at its globally optimal state. For the two-stage RO process, diverting part of the raw feed to the second stage, in order to dilute the feed to the second-stage RO, does not decrease the minimal achievable SEC of a two-stage RO process. The various mixing approaches, while may provide certain operational or system design advantages (e.g., with respect to achieving target salt rejection for certain solutes or flux balancing), do not provide an advantage from an energy usage perspective.

1. Introduction

Energy cost remains one of the most important factors contributing to the cost of water desalination via reverse osmosis (RO) processes [1–3]. The introduction of highly permeable RO membranes has led to a significant reduction in the energy consumption in RO desalination [4,5]. As a result, the feasible operating pressure for the new generation of high permeability membranes is approaching the limit imposed by the thermodynamic restriction [6,7]. This constraint specifies that the feed-side pressure cannot be lower than the sum of the osmotic pressure of the exit brine stream and pressure losses (in the membrane channel) in order to ensure that permeate product water is produced along the entire membrane surface area [9]. As argued in a previous study [8], significant reduction in the cost of RO water desalination is less likely to arise from the development of significantly more permeable membranes, but it is more likely to arise from: (a) optimization of process configuration [9,11], (b) implementation of advanced control schemes (e.g., to account for feed salinity fluctuation [10] and even temporal fluctuation of electrical energy purchasing price), (c) utilization of low cost renewable energy sources, and (d) more effective and lower cost feed pretreatment and brine management strategies [12].

Recent studies have demonstrated that when a membrane desalting process can be operated up to the limit imposed by the thermodynamic restriction, there is an optimal product water recovery at which the specific energy consumption (i.e., energy consumption per volume of permeate produced) is minimized [9]. For example, the optimization model was successfully demonstrated in a recent study showing significant energy savings (up to 22%) under fluctuating feed salinity (up to 43%) [10]. It has been shown, via a formal optimization procedure, that the optimal operating condition shifts to higher recovery with increased membrane and brine management costs [9]. It has also been suggested that the energy consumption for membrane desalting would decrease with increased number of desalting stages where inter-stage pumps are utilized.

More recently, a two-pass membrane desalination process was evaluated and compared to a single-pass process when both processes operate at the limit of the thermodynamic restriction [11]. Considerations of energy recovery, pump efficiency and the limitations imposed by membrane rejection level have led to the conclusion that a single-pass process is more energy efficient relative to a two-pass process.
However, in our previous works [8,9,11], the impact of various stream mixing and recycling configurations on the SEC of an RO plant was not systematically studied.

Extending previous studies on RO optimization for operation at the thermodynamic limit, this work evaluates the effect of possible mixing/blending of various streams (feed, retentate, permeate) on the specific energy consumption (SEC) of RO desalination. To address this problem, the analysis begins with the simplest configuration: single-stage RO desalination, in which two possible recycling (partial retentate recycling and partial permeate recycling) operations are examined. Based on the results from the single-stage RO configuration, two-pass and two-stage desalting with recycling are then studied to determine the effect of various mixing/blending operations on the resulting SEC.

2. Effect of partial recycling operation on the SEC of single-stage RO desalting at the thermodynamic limit

For single-stage RO desalting, full recycling of either the retentate or permeate streams is not possible for a continuous process operation. Therefore, in this section only partial recycling is studied. In the partial retentate recycling operation, part of the retentate stream is diverted to the feed stream immediately before the RO module (Fig. 1(a)), while in the partial permeate recycling operation, part of the permeate stream is diverted to the raw feed (Fig. 1(b)).

2.1. Partial retentate recycling in single-stage RO desalting

For single-stage RO desalting with partial retentate recycling as shown in Fig. 1(a), one can show, via a salt mass balance, that the brine–permeate osmotic pressure difference is \( \Delta \pi_{\text{brine}} = \frac{\eta_f}{\eta_p} \pi_f \) (\( \pi_f \) is feed osmotic pressure, \( \eta_f \) salt rejection), assuming linear relationship between the osmotic pressure and salt concentration [13]. When desalting at the limit of the thermodynamic restriction and neglecting the pressure drop in the system [9], the feed pressure is given by:

\[
\Delta P = P_f - P_0 = \Delta \pi_{\text{brine}} = \frac{\eta_f}{\eta_p} \pi_f \tag{1}
\]

Since the recycled retentate stream of a pressure \( P_f \) is fed directly into the inlet of the RO unit, there is no additional pump work involved to pressurize it to \( P_f \); thus, the rate of pump work for the RO system in Fig. 1(a) is given by:

\[
W = \Delta P \times Q_{\text{raw}} = \frac{\eta_f}{\eta_p} \pi_f Q_{\text{raw}}\tag{2}
\]

Therefore, the specific energy consumption (SEC) is given by:

\[
\text{SEC} = \frac{W}{Q_{\text{raw}}} = \frac{\eta_f}{\eta_p} \pi_f \frac{1}{1 - \eta_f} \tag{3}
\]

which is consistent with the SEC for a single-stage RO system (without recycling) that operates at the limit of the thermodynamic restriction [9]. This means that partial retentate recycling will not change the SEC of a single-stage RO desalting. The inclusion of an energy recovery device (ERD) will not alter this conclusion since the brine stream flow rate \( (Q_b = Q_{\text{raw}} - Q_p) \) and pressure \( (P_f) \), which determine the amount of energy that can be recovered [9], are the same for operation with and without partial retentate recycling.

2.2. Partial permeate recycling in single-stage desalting

For single-stage RO desalting with partial permeate recycling as shown in Fig. 1(b), the brine–permeate stream osmotic pressure difference is also given by \( \Delta \pi_{\text{brine}} = \frac{\eta_f}{\eta_p} \pi_f \) assuming linear relationship between osmotic pressure and salt concentration [13]. When desalting at the limit of the thermodynamic restriction, the feed pressure is also as in Eq. (1). Given a recycled stream flow rate of \( Q_{\text{rec}} = \alpha Q_p \), where \( \alpha \) is the recycle-to-product ratio (\( \alpha < 0 \)), the rate of pump work for a feed flow rate \( Q_f \) is given as

\[
W = \Delta P \times Q_f = \frac{\eta_f}{\eta_p} \pi_f \times (\alpha Q_p + Q_{\text{raw}})\tag{4}
\]

where

\[
Q_f = Q_{\text{rec}} + Q_{\text{raw}} = \alpha Q_p + Q_{\text{raw}}\tag{5}
\]

Therefore, the SEC for this system is given by

\[
\text{SEC} = \frac{W}{Q_f} = \frac{\eta_f}{\eta_p} \pi_f \frac{1}{1 - \eta_f} \times \frac{\alpha Q_p + Q_{\text{raw}}}{Q_f}\tag{6}
\]

In Eq. (6), the first term, \( \frac{\eta_f}{\eta_p} \pi_f \frac{1}{1 - \eta_f} \), is the SEC for a single-stage RO desalting at a water recovery of \( Y \) (Section 2.1, if one replaces the configuration inside the dashed region of Fig. 1(b) by a single-stage RO system without recycling). Thus, the SEC of single-stage RO desalting with partial permeate recycling is less energy favorable than single-stage RO desalting without partial permeate recycling. It is noted that if the pressure drop is taken into account, the SEC of partial permeate recycling operation will increase further. Likewise, the effect of an ERD will not change the above conclusion since the brine stream flow rate \( (Q_b = Q_{\text{raw}} - Q_p) \) and feed pressure \( (P_f) \) are the same for operation with and without partial permeate recycling [9].

The conclusion from the above simple analysis is that in a single-stage RO operation, permeate recycling increases the SEC, while retentate recycling does not change the SEC.

3. Effect of second-pass retentate recycling to the first-pass feed in a two-pass membrane desalting process

A two-pass RO/NF desalting has been proposed in the literature as a potential approach to lower energy consumption [14] or to achieve target salt rejection which is not feasible with a single-pass [15]. Recent analysis has shown that the two-pass process without recycling has no...
advantage with respect to energy savings and its optimized configuration defaults to a single-pass process [11]. The two-pass with retentate recycling from the second-pass to the first-pass feed could provide the means for reducing the energy consumption in the two-pass system. This approach is therefore investigated and compared with a single-pass process at the same overall water recovery and salt rejection.

3.1. General governing equations

Specifically, the case of full retentate recycling from the second-pass to the first-pass is studied in this section (Fig. 2). For this case, the SEC for permeate water production, normalized with respect to the feed water pressure (i.e., \( \eta_0 \)) at a target water recovery of \( Y_t \) and target salt rejection \( R_t \) for operation at the thermodynamic limit is given as [11]:

\[
S_{\text{EC}}^{\text{norm,2passes}} = \frac{[1 - Y_t(1 - Y_{t2})]}{1 - Y_t(1 - R_t)(1 - Y_{t2}(1 - R_{t2}))} \times \left( \frac{R_t}{1 - Y_t(1 - R_t)} \right) \left( \frac{R_t(1 - R_t)Y_t}{1 - Y_t(1 - R_t)} \right) \frac{Q}{Y_{t2}}
\]

subject to the following constraints:

\[
Y_t = \frac{Y_tY_2}{1 - Y_t(1 - Y_{t2})}
\]

\[
R_t = 1 - \frac{(1 - R_t)(1 - R_{t2})[1 - Y_t(1 - Y_{t2})]}{1 - Y_t(1 - R_t)(1 - Y_{t2}(1 - R_{t2}))}
\]

\[
0 \leq R_t < 1.0 \leq R_{t2} < 1.0 \leq Y_t \leq 1.0 - Y_{t2} \leq 1
\]

where \( \eta_0, \eta_r, \) and \( \eta_p \) are the pump and ERD efficiencies for the first and second-passes, respectively, \( R_t \) and \( R_{t2} \) are salt rejections in the first and second-passes, respectively, \( C_{t1} \) and \( C_{t2} \) are the feed and permeate concentrations of the first-pass, and \( Q_{t1} \) and \( Q_{t2} \) are the permeate concentrations of the second-pass, \( Y_t \) and \( Y_{t2} \) are the water recoveries in the first and second-passes, respectively ( \( Y_t = \frac{Q_t}{Q_{t2}} \) where \( Q_{t2} \) is the raw feed water rate).

3.2. Critical water recovery

In studying the impact of recycling the second-pass retentate stream to the first-pass feed stream, the efficiencies of the feed pumps are taken to be independent of water recovery and feed pressure. The feed flow rate to the second pass will be lower than the feed to the first pass. Therefore, the second-pass feed pump is expected to operate at a lower efficiency relative to the first-pass feed pump—a well-known characteristic pump behavior. However, a conservative analysis can be carried out by considering the efficiency of the first and second-pass feed pumps to be identical. As a consequence, energy optimization is only affected within a pump efficiency factor which will drop out of the comparative analysis when considering the ratio of energy consumption for the two-pass and single-pass processes.

Extensive numerical optimizations have been carried out with respect to different water recoveries, salt rejections and ERD efficiencies in the range [0 1]. For the special case of the two-pass process with retentate recycling and ideal pumps (i.e., \( \eta_p = 1 \)), it is possible to arrive at an analytical solution for the minimum SEC of a two-pass process, since the optimal solutions fall on the boundaries of \( R_t = 0 \) or \( R_{t2} = 0 \) as shown previously [11]. When \( R_t = 0 \), \( R_{t2} \) is computed from Eq. (9) as follows:

\[
R_{t2} = \frac{R_t(1 - Y_t) + Y_tY_2}{(1 - Y_t + Y_tY_2R_t)}
\]

Upon substituting Eqs. (9) and (11) into Eq. (7), the normalized SEC of this two-pass process with retentate recycling, is given by:

\[
S_{\text{EC}}^{\text{norm,2passes}} | R_t=0 = R_t \left( \frac{A(Y_t + B)}{(1 - Y_t)(1 - Y_{t2}) - Y_t} + C \right)
\]

where \( A = \frac{1 - Y_{t2}}{1 - Y_t(Y_t - Y_{t2})} \), \( B = \frac{Y_t(Y_t - 1)}{(1 - Y_{t2})} \) and \( C = \frac{1 - Y_t}{1 - Y_{t2}} \). It is noted that \( A, B \) and \( C \) are constants for each given target water recovery and second-pass ERD efficiency. Determination of the minimum normalized SEC is equivalent to finding the minimum of \( \frac{A(Y_t + B)}{(1 - Y_t)(1 - Y_{t2}) - Y_t} \), since the optimal \( Y_t \) in Eq. (7) is always greater than zero. It is also equivalent to finding the maximum of \( \frac{A(Y_t + B)}{(1 - Y_t)(1 - Y_{t2}) - Y_t} \) since \( \frac{A(Y_t + B)}{(1 - Y_t)(1 - Y_{t2}) - Y_t} \) is always greater than zero. Thus \( Y_{1,\text{opt}} \) and corresponding minimum SEC are found to be:

\[
Y_{1,\text{opt}} = \sqrt{Y_t + B(1 + Y_t + B)} - B
\]

\[
S_{\text{EC}}^{\text{norm,2passes}} | R_t=0 = \frac{R_t}{Y_t(1 - Y_t)} \left[ \frac{(1 - Y_{t2})Y_t^2}{(1 - Y_t - Y_{t2})} \right] + \frac{(1 - Y_t)Y_t - 1}{(1 - Y_t)Y_tY_{t2} + 1}
\]

For Eqs. (13) and (14) to be valid, \( Y_{1,\text{opt}} = \sqrt{Y_t + B(1 + Y_t + B)} - B \) has to be in the range \( [Y_t, 1] \). From Eq. (12), \( (B + Y_{1,\text{opt}})^2 = Y_t + B \)

\[
(1 + Y_t + B) = Y_t + B + BY_{1,\text{opt}} + B^2, \text{ which is less than } B^2 + 2B + 1 = (B + 1)^2 \text{ and larger than } Y_t^2 + BY_{1,\text{opt}} + B^2 = (Y_t + B)^2, \text{ thus } Y_{1,\text{opt}} \text{ is in the range } [Y_t, 1].
\]

Similarly, when \( R_{t2} = 0 \), \( R_t \) is computed from Eq. (9) as follows:

\[
R_t = 1 - \frac{(1 - R_{t2})}{1 - Y_t(1 - Y_{t2}) + Y_{t2}Y_t(1 - R_{t2})}
\]

\[
R_t = 1 - \frac{(1 - R_{t2})}{1 - R_{t2}Y_t(1 - Y_{t2})}
\]
The normalized SEC of this two-pass process with retentate recycling, is obtained by substituting Eqs. (9) and (15) into Eq. (7):

$$\text{SEC}_{\text{norm.2passes}} | R_0 = 0 = \frac{R_t}{Y_1} \left( \frac{1}{1-Y_1} - \eta_{k1} \right)$$

(16)

The optimum $Y_1$ value is obtained from $\left( \frac{\partial \text{SEC}_{\text{norm.2passes}} | R_0 = 0}{\partial Y_1} \right)_t = 0$, leading to

$$Y_{1,\text{opt}} = Y_t$$

(17)

$$\left. \left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right) \right|_{min} = \frac{[1-\eta_{k1}(1-Y_1)]R_t}{Y_t(1-Y_t)}$$

(18)

It is noted that, the global minimum SEC is the minimum of the above two minima Eqs. (14) and (18). The SEC of the single-pass (or single-stage) counterpart is given by Eq. 15 and it is the same as Eq. (18). Therefore, if $\left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right)_{min} > \left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right)_{critical}$, a single-pass process will always be more energy efficient than its two-pass counterpart. However, if $\left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right)_{min} < \left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right)_{critical}$, there will be a sub-domain where a two-pass process can be of greater energy efficiency relative to a single-pass process. Finally, if $\left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right)_{min} = \left( \text{SEC}_{\text{norm.2passes}} | R_0 = 0 \right)_{critical}$, the optimized two-pass process is as efficient as its single-pass counterpart, but it will be less efficient if not optimized.

The critical total recovery, $Y_t$, critical, at which the transition occurs is determined by equating Eqs. (14) and (18) to give:

$$Y_{t,\text{critical}} = \frac{1-\eta_{k1}}{2\sqrt{1-\eta_{k1} - (\eta_{k2} - 1)}}$$

(19)

If $\eta_{k1} = \eta_{k2} = \eta_c$, the critical overall water recovery is given by:

$$Y_{1,\text{critical}} = \frac{\sqrt{1-\eta_c}}{2(1 + \sqrt{1-\eta_c})}$$

(20)

Furthermore, if $\eta_{k1} = \eta_{k2} = 0$, $Y_{1,\text{critical}} = 0.25$, while if $\eta_{k1} = \eta_{k2} = 1$, $Y_{1,\text{critical}} = 0$. Eq. 20 (Fig. 3) indicates that in the absence of energy recovery (i.e., $\eta = 0$) $Y_{1,\text{critical}}$ reduces to half the optimal recovery for a single-pass process [11] (i.e., $Y_{1,\text{critical}} = 0.5Y_{\text{opt}} = 0.25$, Eq. (19)). On the other hand, for an ideal ERD ($\eta_t = 1$),$Y_{1,\text{critical}} = 0$, indicating that a single-pass process is always more energy efficient than a two-pass process. For $Y_1 \geq Y_{1,\text{critical}}$, a single-pass is always equally or more energy efficient than a two-pass process, but for $Y_1 < Y_{1,\text{critical}}$, there can be a sub-domain in which a two-pass process will be more energy efficient; this would be the case only when the single-pass process is not operating at the optimal recovery at which the global minimum SEC is achieved. It should be recognized, however, that the globally optimized two-pass process, for the configuration shown in Fig. 2, will always reduce to a single-pass process. Specific examples, that illustrate the process with second-pass retentate recycling are presented in Sections 3.3–3.5 for desalting with energy recovery at 100% and 80% efficiency and without energy recovery to demonstrate the impact of ERD efficiency on the SEC optimization of a two-pass process with retentate recycling.

3.3. Two-pass desalting with complete retentate recycling and ideal energy recovery

For the case of desalting with ideal energy recovery (i.e., 100%), the normalized two-pass SEC is obtained from Eq. (7) by setting $\eta_{k1}$ and $\eta_{k2}$ to unity. The critical water recovery as computed from Eq. (20) is zero and thus a single-pass process without recycling is always more energy efficient than a two-pass process with second-pass retentate recycling. As an example, the normalized SEC, with the feed pumps taken to be ideal (i.e., $\eta_{p1} = \eta_{p2} = 1$) is plotted in Fig. 4, for desalting operation up to the limit of the thermodynamic restriction, for a target overall water recovery ($Y_t$) and salt rejection ($R_t$) of 48% (typical water recovery in ADC pilot study [5]) and 99%, respectively. The bottom plane in Fig. 4 is the normalized SEC for a single-pass process without recycling, also operating up to the limit of the thermodynamic restriction, with the same target recovery and salt rejection as above. The results depicted in Fig. 4 show that a single-pass process (without recycling the second-pass retentate stream to the first-pass feed stream) is more energy efficient than a two-pass process with retentate recycling, provided that both cases target the same overall water recovery and salt rejection. It is only when the two-pass process reduces to a single-pass process (the plane in Fig. 4) that it can be as efficient as the single-pass process.

![Fig. 3. Critical water recovery vs. ERD efficiency for the process depicted in Fig. 2.](image-url)

![Fig. 4. Variation of the normalized SEC of a two-pass membrane desalting process (with ERDs of 100% efficiency in each pass, therefore the critical target water recovery is zero according to Eq. (20)), with respect to salt rejection and water recovery in the first-pass, operated up to the limit of the thermodynamic restriction, for operation with full recycling of the second-pass brine stream to the first-pass feed stream. The target water recovery and salt rejection are 48% and 99%, respectively. The plot is truncated at a normalized SEC of 4 in order to zoom in on the lower SEC region.](image-url)
3.4. Two-pass desalting with complete retentate recycling and non-ideal energy recovery

Illustration of the effect of non-ideal energy recovery on the normalized two-pass SEC is shown in Fig. 5, for the case of 80% energy recovery (i.e., $\eta_E^1 = \eta_E^2 = 0.8$ in Eq. 7) and ideal pumps (i.e., $\eta_P^1 = \eta_P^2 = 1$). According to Eq. (20), the critical overall water recovery ($Y_t^{(\text{critical})}$) is 15.45%. Fig. 5a and b shows the normalized SEC of a two-pass membrane desalting process operated up to the limit of the thermodynamic restriction, with recycling of the second-pass brine stream to the first-pass feed stream, with respect to salt rejection and water recovery in the first-pass. The target salt rejection is 99% in both Fig. 5a and b. The target water recovery in Fig. 5a is 15% (i.e., $<Y_t^{(\text{critical})}$), while in Fig. 5b it is 16% (i.e., $>Y_t^{(\text{critical})}$). Both plots are truncated at a normalized SEC of 2.55 in order to zoom in on the lower SEC region. Fig. 5a shows that at this specific condition ($Y_t < Y_t^{(\text{critical})}$), there is a sub-domain in which the two-pass process has a lower SEC than a single-pass process operated at the same water recovery (the higher plane in Fig. 5a). However, the optimized two-pass process has the same SEC as a single-pass process when operated at the critical water recovery. On the other hand as shown in Fig. 5b, when the target water recovery (16%) is higher than $Y_t^{(\text{critical})}$, the two-pass process would always be of a higher SEC than its single-pass counterpart operated at the same water recovery (16%, the lower plane in Fig. 5b). It is only when the two-pass process reduces to a single-pass (the lower plane in Fig. 5b) that it can be as efficient as the single-pass process.

3.5. Two-pass desalting with complete retentate recycling without energy recovery

The normalized SEC for two-pass desalting with ideal pumps, but without energy recovery devices (i.e., $\eta_E^1 = \eta_E^2 = 0$ and $\eta_P^1 = \eta_P^2 = 1$ in Eq. 7), is illustrated in Fig. 6a and b for recoveries above and below the critical recovery of 20% (Eq. 20). The two-pass membrane desalting process operated up to the limit of the thermodynamic restriction, with the recycling of the second-pass brine stream to the first-pass feed stream, has a total target salt rejection of 99%. The target water recovery in Fig. 6a is 24% (i.e., $<Y_t^{(\text{critical})}$), while in Fig. 6b it is 26% (i.e., $>Y_t^{(\text{critical})}$). Both plots are truncated at a normalized SEC of 5.6 in order to zoom in on the lower SEC region. When $Y_t < Y_t^{(\text{critical})}$ (Fig. 6a), there is a sub-domain in which the two-pass process has a lower SEC than a single-pass process when operated at the same water recovery (the higher plane in Fig. 6a). However, the optimized two-pass process has the same SEC as a single-pass process when operated at the critical water recovery. When $Y_t > Y_t^{(\text{critical})}$ (Fig. 6b), the two-pass process always has a higher SEC than its single-pass process counterpart when operated at the same water recovery (26%, the plane in Fig. 6b). At the optimal energy consumption state,
the two-pass process reduces to a single-pass (or single-stage) process (the plane in Fig. 6b).

Although membrane desalting via a two-pass process with or without recycling is less energy efficient than a single-pass (or single-stage) process, there can be situations where a two-pass process is preferred, particularly in situations of difficult to achieve rejection of certain species (boron removal [15]).

4. SEC optimization of two-stage RO desalting with feed diversion to the second-stage

In considering the operation of a two-stage process, it is interesting to evaluate the potential impact of diverting part of the feed stream of the first-stage to the second-stage (in order to reduce the salinity of the feed to the second-stage RO, Fig. 7) on the SEC optimization. Following recent analysis of the process [9], the rates of work done by the first-stage pump, $W_{1st}^w$, and second-stage pump, $W_{2nd}^w$, at the limit of the thermodynamic restriction, are given by:

$$W_{1st}^w = \frac{\pi_0}{1-Y_1} \times (Q_{p,1} + Q_{d,1})$$

$$W_{2nd}^w = \frac{\pi_0}{1-Y_2} Q_{d,2} [1-\eta_{E2} (1-Y_2)] - \frac{\pi_0}{1-Y_1} Q_{d,1}$$

where $\eta_{E2}$ is the efficiency of the ERD in the second stage; $Y_1$ and $Y_2$ are the water recoveries in the first and second stages, respectively ($Y_1 = Q_{p,1}/Q_{f,1}$, $Y_2 = Q_{p,2}/Q_{f,2}$); $Q_{d,1}$, $Q_{p,1}$, $Q_{d,2}$ and $Q_{p,2}$ are the feed and permeate flow rates to the first and second stages, respectively ($Q_{d,2} = Q_{d,1} + (1-Y_1)Q_{f,1}$, where $Q_{f,1}$ is the raw water flow rate to the second-stage); and $\pi_0$ and $\pi_2$ are the osmotic pressures of the feed to the first and second stages, respectively, and are related by the following (assuming 100% of salt rejection in each stage) expression:

$$\pi_0 = \frac{Q_{d,1}}{Q_{d,1} + (1-Y_1)Q_{f,1}} \pi_2$$

Therefore, the average SEC of this two-stage process, normalized with respect to the feed osmotic pressure, $\pi_0$, is given by

$$SEC_{two-stage} = \frac{W_{1st}^w + W_{2nd}^w}{(Q_{p,1} + Q_{p,2})\pi_0}$$

where

$$Q_{p,1} + Q_{p,2} = Y_1 (Q_{f,1} + Q_{d,1})$$

Fig. 6. Variation of the normalized SEC of a two-pass membrane desalting process (without ERDs, therefore the critical target water recovery is 25% according to Eq. (20)) operated up to the limit of the thermodynamic restriction, with full recycling of the second-pass brine stream to the first-pass feed stream, with respect to salt rejection and water recovery in the first-pass. The target salt rejection is 99% with the target water recovery of (a) 24% (i.e., less than the critical target water recovery), and (b) 26% (i.e., greater than the critical water recovery). Both plots are truncated at a normalized SEC of 5.6 in order to zoom in on the lower SEC region.
Combining Eqs. (21)−(25), the average SEC targeting a desired water recovery, $Y_t$, is dependent on the fractional water recovery in each stage and the diverted raw feed fraction, $f_d$, as follows:

$$\text{SEC}^{2\text{stgs}}_{\text{tr,norm}} = \frac{Y_t(1-f_d)}{Y_t-1} + \frac{1-Y_t + Y_t f_d - \eta_b}{Y_t}$$ (26)

where the diverted raw feed fraction is $f_d = \frac{Q_{d1}}{Q_{f1} + Q_{d1}}$. The objective is to minimize the function $\text{SEC}^{2\text{stgs}}_{\text{tr,norm}}$ in Eq. (26) in order to minimize the SEC, with respect to the following constraints:

$$0 \leq f_d \leq 1$$ (27)

$$0 < Y_t < 1$$ (28)

$$0 < Y_2 < 1$$ (29)

The constraint $0 < Y_2 < 1$ requires $Y_t < \frac{V_t}{1-f_d}$ or $f_d > 1 - \frac{Y_t}{1-Y_t}$ based on the overall mass balance in Eq. (25).

The average SEC of a two-stage RO process without diverting the raw feed to the second stage, but targeting the same overall water recovery, $Y_t$, is determined by setting $f_d = 0$ in Eq. (26) leading to:

$$\text{SEC}^{2\text{stgs,nd}}_{\text{tr,norm}} = \frac{Y_t}{Y_t-1} + \frac{1-Y_t - \eta_b}{Y_t}$$ (30)

where the superscript $\text{nd}$ denotes “no diversion”. In Eq. (30), $\text{SEC}^{2\text{stgs,nd}}_{\text{tr,norm}}$ is only a function of the water recovery in the first stage. Consistent with the optimization result reported previously [9], the optimum water recovery and minimum $\text{SEC}^{2\text{stgs,nd}}_{\text{tr,norm}}$ are given by:

$$Y_{t,\text{opt}} = 1 - \sqrt{1-Y_t}$$ (31)

$$\left(\text{SEC}^{2\text{stgs,nd}}_{\text{tr,norm}}\right)_{\text{min}} = \frac{2}{\sqrt{1-Y_t}} - \frac{1}{Y_t}$$ (32)

The optimum $(f_d, Y_t)$ set is obtained via a similar search algorithm used previously [11]. A typical result is shown in Fig. 8, in which the bottom plane represents the minimum SEC of a two-stage RO process without diversion of the raw feed (Eq. 32). Fig. 8 shows that the minimum SEC of a two-stage process with raw feed diversion occurs when $f_d = 0$, which is simply a two-stage process without feed diversion [9]. To help understand this point, one can take the diverting operation to its extreme situation, where all the feed to the first stage is diverted to the second stage: in this case, the two-stage RO process with diversion of the feed evolves into a single-stage RO process. As shown in our previous work [9], a single-stage RO process is less energy efficient than a two-stage RO process.

5. Conclusions

Various mixing operations between the feed, retentate and permeate streams were explored to assess their potential effectiveness for decreasing the specific energy consumption of single-stage, two-pass and two-stage RO desalination processes operated at the limit of the thermodynamic restriction. The analysis clarifies that in a single-stage RO process, partial retentate recycling to the feed stream does not change the SEC, while partial permeate recycling to the feed stream increases the SEC when targeting the same overall water recovery. For a two-stage RO process, diverting part of the raw water feed from the first-stage to the second-stage RO does not decrease the minimum achievable SEC in the two-stage RO process. For a two-pass membrane desalination, second-pass retentate recycling to the first-pass feed stream reduces the energy consumption relative to the case of no recycling. However, the optimal two-pass process always reduces to a single-pass (single-stage) process. In closure, the various mixing approaches considered in the present study, while may be...

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**Fig. 7.** Schematic of a two-stage RO process with part of the raw feed diverted to the second stage.

**Fig. 8.** Variation of SEC for a two-stage RO (targeting 50% of water recovery, ERD efficiency 100%) with respect to the diverted fraction and the first-stage water recovery. $f_d$ is the fraction of the raw feed diverted from the first to the second-stage RO.
useful for various operational reasons, do not provide an advantage from the viewpoint of energy use reduction.

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