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Handling Input and State Constraints in Networked Predictive Control

David Muñoz de la Peña and Panagiotis D. Christofides*

Department of Chemical and Biomolecular Engineering
and Department of Electrical Engineering
University of California, Los Angeles
Los Angeles, CA 90095-1592
Email: davidmps@ucla.edu, pdc@seas.ucla.edu

Abstract. In this work, we focus on the problem of stabilization of constrained nonlinear systems subject to data losses. We extend previous results on Lyapunov-based model predictive control (LMPC) of nonlinear systems subject to data losses to explicitly handle the presence of input and state constraints. In addition, a hybrid LMPC controller for systems subject to soft state constraints with the objective of minimizing the time for which the state constraints are violated is proposed. The theoretical results are demonstrated through a chemical process example.

Keywords. Networked control systems; Predictive control for nonlinear systems; Constrained control; Process control applications; Uncertain systems.

1 Introduction

Nowadays there is an increasing interest on studying networked control systems; that is, control systems in which the control loop is closed using a shared communication network, see for example [27, 26, 20]. Networked control systems, in addition to dealing with nonlinearities, constraints and performance issues, have to account for the dynamics introduced in the closed-loop system by the network. These dynamics are often modeled as time varying delays, data quantization or data losses. Systems subject to data losses are of particular interest for wireless networked control systems, see [21, 22, 7] and the references therein. Wireless technology is used in new control applications like sensor networks [1, 4], multi-agent systems [3, 25] and in distributed process control systems [11, 12] to increase the flexibility and fault-tolerance of the closed-loop system by adding redundancy in the control loops.

There are several works in the control systems literature that deal with nonlinear systems subject to data losses. A common approach is to design a stabilizing feedback law for the system under continuous measurements, and then study the robustness properties of the closed-loop system subject to data losses. In [23, 24, 19] it was proved that if the maximum time in which the system operates in open-loop is sufficiently small, the closed-loop system is practically stable. A similar result has been obtained for output feedback schemes based on high gain observers in [17]. In [12], the stability properties of nonlinear systems under Lyapunov-based control subject to data losses and input constraints were studied. In this work, however, closed-loop system performance issues were not taken into account.

Most process control systems need to explicitly deal with input and state constraints. These constraints typically represent hard limits on the capacity of control actuators (input constraints) or operating constraints imposed by performance objectives or safety considerations (state constraints). Controllers that are able to deal with nonlinearities, performance issues and state and inputs constraints are of interest for the control community. Model predictive control is one control method that is able to deal with state and input constraints in an explicit manner. Lyapunov-based model predictive control (LMPC) was first proposed in [9] for nonlinear switched systems and further developed for constrained systems [10] and applied to fault-tolerant control schemes [11]. More recently in [16], a Lyapunov-based model predictive controller that takes data losses into account in the optimization problem has been proposed. However, the control problem formulation in [16] does not deal with input and state constraints.

In the present work, we extend the results of [16] to constrained nonlinear systems and propose a Lyapunov-based model predictive controller that takes explicitly into account performance issues, constraints, and data losses. Furthermore, a hybrid LMPC controller for systems subject to soft state constraints with the objective of minimizing the time on which the state constraints are violated is presented. The proposed controllers allow for an explicit characterization of the stability region and guarantee that this region is an invariant set for the closed-loop system under data losses if the maximum time in which the loop is open is shorter than a given constant. This constant depends on the parameters of the system and the Lyapunov-based controller that is used to formulate the optimization problem. The theoretical results are demonstrated through a chemical process example.

2 Preliminaries

In this work we consider the problem of stabilization of continuous-time nonlinear systems subject to data losses with state and input constraints, with the following state-

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space description

$$\dot{x}(t) = f(x(t), u(t), w(t)) \quad (1)$$

where $x(t) \in X \subseteq R^{n_x}$ denotes the vector of state variables, $u(t) \in U \subseteq R^{n_u}$ denotes the vector of input variables, $w(t) \in W \subseteq R^{n_w}$ denotes the vector of disturbance variables, and f is locally Lipschitz on $R^{n_x} \times R^{n_u} \times R^{n_w}$. The sets X and U denote the constraints in the state variables and in the manipulated inputs, respectively. The disturbance vector is bounded in

$$W := \{w \in R^{n_w} \text{ s.t. } |w| \leq \theta, \theta > 0\}^1.$$

The system is subject to data losses; that is, the actual state of the system is not always available for the controller to decide the input. This concept is made clear below in this section when signal $s(\cdot)$ is introduced.

We assume that the nominal closed-loop system (system (1) with $w(t) \equiv 0$ for all t) has an asymptotically stable equilibrium at the origin $x = 0$ for a given feedback control $h : R^{n_x} \rightarrow R^{n_u}$ which satisfies $h(0) = 0$ (this assumption is equivalent to the existence of a control Lyapunov function (CLF) for the system $\dot{x} = f(x, u, 0)$). Using converse Lyapunov theorems (see [8]), this assumption implies that there exists a Lyapunov-function $V(x)$ for the nominal closed-loop system such that

$$\begin{array}{c} \dot{V}(x(t)) \leq 0 \\ h(x) \in U \end{array} \quad (2)$$

for all $x \in D$, where D is an open neighborhood of the origin, see [8].

Lyapunov-based model predictive control is based on using an existing Lyapunov-based feedback law in the design of the model predictive control optimization problem. Using an appropriate set of constraints, it is guaranteed that the LMPC controller inherits the stability properties of the Lyapunov-based feedback controller used in the design. The feedback law $h(x)$ will be used in the design of the LMPC controller in the following section.

In the present work, we take into account both the cases in which the state constraints have to be fulfilled for all times (hard constraints) and in which the constraints can be relaxed for a short period of time (soft constraints).

For this reason, we are going to define two different regions of attraction defined as level sets of $V(x)$. First we define Ω_u as

$$\Omega_u = \{x \in R^{n_x} : V(x) \leq c_u^{max}\}$$

where c_u^{max} is the largest number for which $\Omega_u \in D$. Starting from any initial state in Ω_u, asymptotic stability and satisfaction of the input constraints of the nominal closed-loop system are guaranteed. To take into account state constraints, we define Ω_{ux} as

$$\Omega_{ux} = \{x \in R^{n_x} : V(x) \leq c_{ux}^{max}\}$$

where c_{ux}^{max} is the largest number for which $\Omega_{ux} \in X \bigcap \Omega_u$. Starting from any initial state in Ω_{ux}, asymptotic stability and satisfaction of the state and input constraints of the nominal closed-loop system are guaranteed.

[1] $|\cdot|$ denotes Euclidean norm of a vector.

Although system (1) is defined as a continuous time system, the Lyapunov-based controller is implemented in a sample and hold fashion with sample time Δ. For this reason, in order to model data losses, we define a signal $s(t_k)$ with $t_k = k\Delta,\ k = 0, 1, \ldots$ such that at sampling time t_k, $s(t_k) = 1$ implies that the state of the system is available and $s(t_k) = 0$ implies that at sampling time t_k the system must operate in open-loop.

We assume that the data losses are random, but satisfy a given constraint on the maximum number of sampling times in which the system operates in open-loop. Following the notation introduced in [23], the maximum time n_o is refereed as the maximum allowable transmission interval (MATI). Another parameter that characterizes the data losses of a given system is the rate of data losses; that is, the fraction of samples that are lost along a large period of time.

The controller must take into account $s(t_k)$ to decide the input at a given sampling time. There are several different strategies for the controller when feedback is lost. In general, the controller makes the decision based on the last received state. Following this idea, we characterize a control law in the presence of data losses as a function that depends on the last available state and the sampling time in which that state was received; that is, $u = k(x, i)$ where x is the last available state, and i is the number of sampling times that have passed since the last measurement was received. Note that $i = 0$ implies that the loop is closed. Taking into account sampling and data losses, the controller takes the following form:

$$\begin{cases} s(t_k) = 1 \rightarrow u(t) = K(x(t_k), 0),\ t \in [t_k, t_{k+1}] \\ s(t_k) = 0 \rightarrow u(t) = K(x(t_i), k - i),\ t \in [t_k, t_{k+1}] \end{cases}$$

where t_i is the last sampling time in which a measurement was received.

A general approach is to design a feedback control law for the system under continuous measurements, for example the Lyapunov-based controller $h(x)$, and then set the input to zero or to the last available input when feedback is lost. For the zero control strategy, $K(x, i) = h(x)$ for $i = 0$, and zero for $i > 0$. For the last available input trajectory, $K(x, i) = h(x)$ for all i. In a recent line of work, Antsaklis and co-workers [13, 14] have proposed a strategy based on using an estimate of the state computed via the nominal model of the plant to decide the control input over the period of time in which feedback is lost between consecutively received measurements. Using the model to predict the state when data losses occur yields the following controller:

$$\begin{array}{l} K(x, i) = h(\hat{x}(t_i)) \\ \dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_k)), 0),\ t \in [t_k, t_{k+1}] \end{array} \quad (3)$$

where $t_k = t_0 + k\Delta,\ k = 0, 1, \ldots$ and $\hat{x}(t_0) = x$. The trajectory $\hat{x}(t)$ is denoted as the nominal sampled trajectory of system (1) associated with a feedback law $h(x)$ with sampling time Δ starting at x. The main idea is that the controller uses the nominal model of the system to predict the current state from the last available state and applies the corresponding manipulated input trajectory. Note that the above strategy is applied in a sample

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and-hold technique and takes data losses explicitly into account.

This is the strategy used in the LMPC controller presented in [16] and in the present work. This strategy is appropriate to apply in a model predictive control scheme, because the solution of the optimization problem is the optimal future trajectory of inputs computed via the nominal model of the plant, so it can be used to update the input when feedback is lost. The main idea behind Lyapunov-based model predictive control is that of using the nominal sampled trajectory of system (1) associated with a feedback law $h(x)$ to design the contractive constraints in the optimization problem. It can be proved that the closed-loop system under the LMPC controller inherits the same stability properties of the Lyapunov-based controller when implemented following (3). The Lyapunov-based controller $h(x)$ has robustness properties that preserve practical stability of the closed-loop system when no data losses occur for sufficiently small sampling time and uncertainties, and guarantee that the stability region is invariant if the data losses are small enough. These properties were proved in [16] where unconstrained LMPC of nonlinear systems subject to data losses was studied.

In the following section, an LMPC controller that takes into account input and state constraints, as well as a hybrid LMPC controller that deals with soft state constraints are introduced. Both strategies satisfy the same set of contractive constraints and inherit the properties of the Lyapunov-based controller when implemented following (3).

3 Lyapunov-based Model Predictive Control

3.1 Handling state and input constraints

In this section we present a Lyapunov-based model predictive controller based on the results of [16]. At each sampling time, the manipulated input trajectory is obtained solving the following optimal control problem with prediction horizon N

$$\min_{u \in S(\Delta)} \quad J(x(\tau), u(\tau)) \tag{4a}$$
$$\text{s.t.} \quad \dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), u(\tau), 0) \tag{4b}$$
$$\tilde{x}(t_0) = x \tag{4c}$$
$$u(\tau) \in U \tag{4d}$$
$$V(\tilde{x}(\tau)) \leq V(\hat{x}(\tau)),\ \forall \tau \in [t_0, t_{N_R}] \tag{4e}$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ, $\tilde{x}(\tau)$ with $\tau \in [t_0, t_N]$ is the predicted sampled trajectory of the nominal system for the input trajectory computed by the LMPC (4), $\hat{x}(\tau)$ with $\tau \in [t_0, t_N]$ is the nominal sampled trajectory under the Lyapunov-based controller with initial state $\hat{x}(t_0) = x$. This trajectory is obtained solving (3). Depending on the cost function and on the value of the horizon N_R of the contractive constraint, different LMPC schemes are defined. Standard LMPC schemes [9, 10, 11] minimize a quadratic function of the state and the input, i.e.,

$$J(x(\tau), u(\tau)) = \int_{t_0}^{t_N} [x(\tau)^T Q_c x(\tau) + u(\tau)^T R_c u(\tau)] d\tau \tag{5}$$

in order to improve a given performance index. The LMPC controller is defined as follows

$$K_L(x, i) = u^*(t_i) \tag{6}$$

where $u^*(t)$ is the optimizer of problem (4) with $x(t_0) = x$. The main idea is that when data losses occur the controller implements the last evaluated optimal trajectory.

When no data losses are taken into account, the contractive constraint has to be satisfied only for the first time step, that is, $N_R = 1$. In [16] it was proven that in order to inherit the stability properties of the Lyapunov-based controller when implemented following (3), N_R must be greater or equal than n_o. This is the approach taken in the present work while introducing state and input constraints. The closed-loop system properties under the LMPC controller (6) are formalized in Theorem 1 below.

Theorem 1 *Consider system (1) in closed-loop with the LPMC scheme (6) with $N_R \geq n_o$ based on a Lyapunov-based controller $h(x)$. Then, given any positive number d, there exist positive real numbers Δ^*, θ^* and N_{ux} such that if $\Delta \leq \Delta^*$ and $\theta \leq \theta^*$, the following statements hold:*

- *If there are no data losses and $x(0) \in \Omega_{ux}$, then $x(t) \in \Omega_{ux}$ for all times and $\lim V(x(t)) \leq d$.*
- *If $n_o \leq N_{ux}$ and $x(0) \in \Omega_{ux}$, then $x(t) \in \Omega_{ux}$ for all times.*

Proof: The proof of this theorem is similar to the one presented in [16] for the unconstrained case. The main difference is that to take constraints into account, X and U have to be used to obtain the stability region Ω_{ux}. In what follows a sketch of the full proof is provided.

Part 1. In this part we prove that the model predictive optimization problem (4) is feasible for all $x \in \Omega_{ux}$. Feasibility follows from the fact that $u(t) = h(\hat{x}(t_k))$ for $t \in [t_k, t_{k+1}]$, where $\hat{x}$ is the nominal (assuming zero disturbance) sampled trajectory of the closed-loop system under the Lyapunov-based controller $h(x)$ with initial state $x(t_0) = x$, satisfies all the constraints of problem (4) because for a sufficiently small Δ (see [6, 18]), $\hat{x}(t) \in \Omega_{ux}$ for all t, so by definition $u(t) \in U$.

Part 2. Assuming that no data losses are taken into account ($n_o = 0$), at each time step the optimization problem is solved and a new optimal manipulated input trajectory is obtained. Taking into account that the constraints guarantee that

$$V(\tilde{x}(\tau)) \leq V(\hat{x}(\tau)),\ \forall t \in [t_0, t_0 + \Delta]$$

it can be proved that $V(x(t)) \leq V(\check{x}(t))$ for all times where $\check{x}(t)$ is the state trajectory of (1) in closed-loop with the Lyapunov-based controller $h(x)$ implemented following (3) (taking into account the disturbance trajectory

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$w(t)$). In this case, using standard sampled-data results, see for example [6, 18], it can be proved that given any positive number d there exist real numbers Δ^* and θ^* such that if $\Delta \leq \Delta^*$ and $\theta \leq \theta^*$ then $V(\check{x}(t)) \leq c_{ux}^{max}$ and $\lim V(\check{x}(t)) \leq d$.

Part 3. In the presence of data losses, in order to prove that $V(x(t)) \leq V(\check{x}(t))$ for all times, the contractive constraint must hold at least for the maximum time in which the system can operate in open-loop, i.e., the n_o. Following sampled-data results, given Δ and θ, there exist n_o^* such that if $n_o \leq N_R \leq n_o^*$ and $x(0) \in \Omega_{ux}$ then $V(\check{x}(t)) \leq c_{ux}^{max}$ for all times. A detailed proof is provided in [16]. **QED**

Remark 1 *The stability and robustness properties with respect to data losses stated in Theorem 1 stem from the contractive constraints and the properties of the Lyapunov-based controller $h(x)$ implemented following (3). These properties are independent of the objective function.*

Remark 2 *It is also important to remark that the invariance property of the set Ω_{ux} guarantees that the state and input constraints are satisfied for all times.*

Remark 3 *Complexity of the optimization problem is an important issue in model predictive control. This issue is particularly important for MPC schemes for nonlinear uncertain systems. Standard MPC schemes like min-max controllers have a very high computational burden, in general, a computational burden that grows exponentially with the prediction horizon, see for example [2, 15] and the references therein. Lyapunov-based MPC is based on a reduced complexity deterministic optimization problem, and moreover, an initial feasible solution guess is available (the input trajectory of the nominal sampled closed-loop system). This makes possible to apply LMPC to a broad family of control applications.*

3.2 Handling soft constraints via controller switching

In many control applications, state constraints represent desired bounds on the state variables that provide improved closed-loop performance. In this case, the state constraints can be violated, although it is desirable that the time in which the state constraints are not satisfied is minimized. In [11], a hybrid controller was proposed to deal with soft constraints in an LMPC scheme. Following the same ideas, we propose a hybrid controller that switches between two different LMPC controllers: one that guarantees state and input constraints satisfaction and minimizes a given performance objective, and another that guarantees only input constraints satisfaction while minimizing the time needed to regulate the system to the stability region Ω_{ux}. The only difference between this two LMPC controllers is the objective function. Both are based on the same stability and input constraints of problem (4). The hybrid controller is defined as follows

$$\begin{array}{l}\text{If } x \in \Omega_{ux} \text{then } K_L(x,i) = u_H^*(t_i) \\ \text{Else } K_L(x,i) = u_S^*(t_i)\end{array} \tag{7}$$

where $u_L^*(t)$ is the optimizer of problem (4) with

$$J(x(\tau), u(\tau)) = \int_{t_0}^{t_N} [x(\tau)^T Q_c x(\tau) + u(\tau)^T R_c u(\tau)] d\tau$$

and $x(t_0) = x$, while $u_S^*(t)$ is the optimizer of problem (4) with

$$J(x(\tau), u(\tau)) = V(x(t_N))$$

and $x(t_0) = x$. Both problems are defined using the same prediction horizon N and horizon N_R for the contractive constraint. In this way, when the state is outside the stability region Ω_{ux}, but inside the region Ω_u, the controller switches to minimize directly the Lyapunov function, reducing, in general, the time in which the state constraints are violated.

Theorem 2 *Consider system (1) in closed-loop with the LPMC scheme (7) with $N_R \geq n_o$ based on Lyapunov-based controller $h(x)$. Then, given any positive real number d, there exist positive real numbers Δ^*, θ^*, N_u and N_{ux} such that if $\Delta \leq \Delta^*$ and $\theta \leq \theta^*$, the following hold:*

- *If there are no data losses and $x(0) \in \Omega_u$, then $x(t) \in \Omega_u$ for all times and $\lim V(x(t)) \leq d$.*
- *If $n_o \leq N_u$ and $x(0) \in \Omega_u$, then $x(t) \in \Omega_u$ for all times.*
- *If $n_o \leq N_{ux}$ and $x(0) \in \Omega_{ux}$, then $x(t) \in \Omega_{ux}$ for all times.*

Proof: Because both u_H^* and u_S^* are computed solving an optimization problem subject to the same contractive constraints of problem (4), Theorem 1 can be applied to characterize any closed-loop trajectories under the hybrid LMPC controller. In order to obtain the positive real numbers that guarantee that the three statements hold, first, we assume that $X = R^{nx}$, that is, there are no constraints on the state. In this case, $\Omega_{ux} = \Omega_u$ so Theorem 1 guarantees that for any $d > 0$, there exists Δ^*, θ^*, $N_u > 0$ such that the first two claims hold. To prove the third claim, state constraints are taken into account. Theorem 1 is applied for d, Δ^* and θ^* to obtain N_{ux} that guarantees that Ω_{ux} is invariant for data losses which satisfy $n_o \leq N_{ux}$. **QED**

Remark 4 *The main idea is that when the state constraints can be violated, a higher amount of data losses can be tolerated while guaranteeing that the system remains in Ω_u, which is the set of states that can be regulated to the origin while satisfying the input constraints. The values of Δ^*, θ^*, N_u and N_{ux} are obtained from the properties of the Lyapunov-based controller $h(x)$ implemented following (3).*

4 Application to a Chemical Reactor Example

Consider a well-mixed, non-isothermal continuous stirred tank reactor where an irreversible elementary exothermic reaction takes place of the form $A \rightarrow B$. The feed to the

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Table 1: Process parameters

F	0.1 $[m^3/min]$	k_0	72*10^9 $[min^{-1}]$
V_r	0.1$[m^3]$	R	8.314 $[KJ/kmol \cdot K]$
T_{A0}	310 $[K]$	E	8.314*10^4 $[KJ/kmol]$
C_{A0}	1 $[kmol/m^3]$	ΔH	-4.78*10^4 $[KJ/kmol]$
σ	1000 $[kg/m^3]$	c_p	0.239 $[KJ/kg \cdot K]$

reactor consists of pure A at flow rate F, temperature T_{A0} and molar concentration $C_{A0} + \Delta C_{A0}$ where ΔC_{A0} is one of the manipulated inputs. Due to the non-isothermal nature of the reactor, a jacket is used to remove/provide heat to the reactor. The heat removed/provided to the reactor is another manipulated input. Using first principles and standard modeling assumptions, the following mathematical model of the process is obtained

$$\frac{dC_A}{dt} = \frac{F}{V_r}(C_{A0} + \Delta C_{A0} - C_A) + k_0 e^{\frac{-E}{RT}} C_A + w_1$$

$$\frac{dT}{dt} = \frac{F}{V_r}(T_{A0} - T) - \frac{\Delta H}{\sigma c_p} k_0 e^{\frac{-E}{RT}} C_A + \frac{Q}{\sigma c_p V_r} + w_2 \tag{8}$$

where C_A denotes the concentration of the reactant A, T denotes the temperature of the reactor, Q denotes the rate of heat input/removal, V_r denotes the volume of the reactor, $\Delta H, k_0, E$ denote the enthalpy, pre-exponential constant and activation energy of the reaction and c_p, σ denote the heat capacity and the density of the fluid in the reactor. Variables w_1 and w_2 are time-varying disturbances which correspond to unmodelled process dynamics. The values of the process parameters are shown in Table 1.

System (8) has three steady-states for $w_1 = w_2 = 0$ (two locally asymptotically stable and one unstable). The control objective is to stabilize the system at the open-loop unstable steady state $C_{As} = 0.57 kmol/m^3$, $T_s = 395.3K$ while keeping C_A between $0.41 kmol/m^3$ and $0.73 kmol/m^3$, and T between $392.3K$ and $398.3K$. These constraints are soft constraints. The manipulated inputs are the deviation from the nominal concentration of the inflow ΔC_{A0} and the rate of heat input Q with constraints $|\Delta C_{A0}| \leq 1 kmol/m^3$ and $|Q| \leq 60 KJ/min$. We consider time-varying uncertainties that satisfy $|w_1| \leq 0.5 kmol/m^3 min$ and $|w_2| \leq 1.2552 K/min$. The control system is subject to data losses in the communication links.

To demonstrate the theoretical results, we first design a Lyapunov based feedback law using the method presented in [5]. System (8) belongs to the following class of nonlinear systems

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + w(x(t))\theta(t)$$

where $x^T = [(T - T_s)\ (C_A - C_{As})]$ is the state vector, $u^T = [\Delta C_A\ Q]$ is the input vector and $\theta^T = [w_1\ w_2]$ is a time-varying bounded disturbance vector. Consider the control Lyapunov function $V(x) = x^T P x$ with

$$P = \begin{bmatrix} 9.3548 & 0.4068 \\ 0.4068 & 0.0202 \end{bmatrix}.$$

The values of the weights have been chosen to account for the different range of numerical values for each state. The following feedback law [5] asymptotically stabilizes the open-loop unstable steady-state of the nominal system (i.e., $\theta(t) = 0$) and is of the form:

$$h(x) = \begin{cases} -L_G V^T c(x) & \text{if } |L_G V| \neq 0 \\ 0 & \text{if } |L_G V| = 0 \end{cases} \tag{9}$$

with

$$c(x) = \frac{L_f^* V + \sqrt{(L_f^* V)^2 + (u_{max}|L_G V|)^4}}{|L_G V|^2[1 + \sqrt{1 + (u_{max}|L_G V|)^2}]}$$

where $L_f V$ denotes the Lie derivative of the scalar function V with respect to the vector field f, $L_G V = [L_{g^1} V \cdots L_{g^m} V]$, where g^i is the ith column of G, $L_f^* V = L_f V + \beta V$ where $\beta > 0$, and u_{max} is a real number such that $|u| \leq u_{max}$ implies that $u \in U$. In the following simulations we have chosen $\beta = 0.5$.

Following Lyapunov arguments, it can be proved that whenever the closed-loop state evolves within the set

$$D = \{x \in R^n : L_f^* V \leq u_{max}|L_G V|\}$$

then the controller satisfies the input constraints and the time-derivative of the Lyapunov function is negative. The set D is a complex nonconvex set. In the simulation, we use a grid to estimate D and evaluate c_u^{max} and c_{ux}^{max}. Figure 1 shows the constrained stability regions Ω_u, Ω_{ux} together with sets D and X. In this figure $c_u^{max} = 0.2$ and $c_{ux}^{max} = 0.023$.

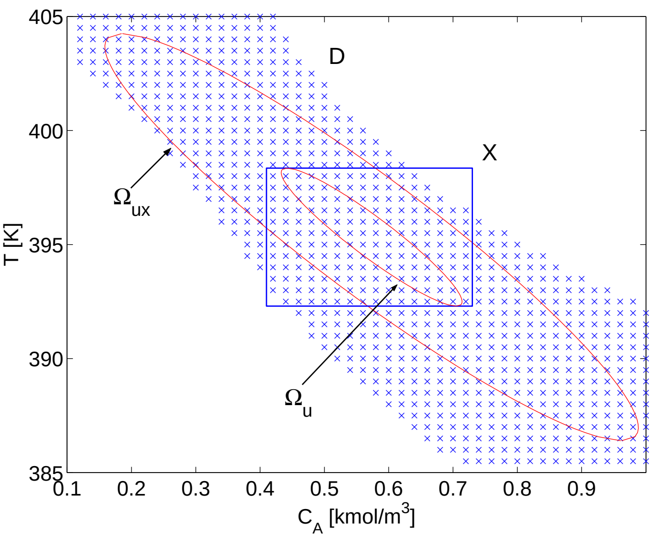

Figure 1: Representation of the constrained stability regions Ω_u, Ω_{ux} together with sets D and X.

Data losses are modeled with a random sequence $s(t_k)$. To obtain this sequence two parameters are given, first a probability p of data loss and a maximum allowable time between two successful transmissions n_o. At each time step t_k, if $k - i > n_o$ where t_i is the last sampling time with no data losses, then $s(t_k) = 1$. This guarantees that the maximum time without measurements is less than or

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equal to n_o. If $k - i \le n_o$, a random number $\xi \in [0, 1]$ is obtained from a uniform distribution. If $\xi > p$ then $s(t_k) = 1$, and if $\xi \le p$ then $s(t_k) = 0$. This constructive method provides a random sequence that satisfies the constraint on the maximum time that the system operates in open-loop, with a data loss rate lower than p. In the simulations different values of p and n_o have been used (see below for precise values).

A sampling time of $0.02min$ is used to implement the LMPC controller. The parameters of the objective function of the LMPC controller are chosen as $Q_c = R_c = I$. The constrained optimization problem is solved using MATLAB function `fmincon` and the set of ODEs is integrated using the Euler method with a fixed integration step of $0.0002min$. A new value for the uncertainty is obtained from a uniform distribution at each integration step. The prediction horizon is set to $N = 12$ while $N_R = n_o$ for each simulation. The proposed LMPC controller is compared with the LMPC scheme that does not take into account data losses in the controller design, that is, $N_R = 1$. In this case, even if the input is updated using the optimal manipulated input trajectory of a previous sampling time, the controller may not inherit the robustness properties of the Lyapunov-based controller $h(x)$ implemented following (3).

With the chosen sampling time, uncertainty bounds and controller and system parameters, extensive simulations were carried out using the Lyapunov-based controller $h(x)$ implemented following (3) to evaluate the robustness of this control law with respect to data losses. One set of simulations was carried out to estimate the maximum time N_{ux} that the system can operate in open-loop without leaving Ω_{ux} and satisfying the state and input constraints for all times. A second set of simulations was carried out to estimate the maximum time N_u that the system can operate in open-loop violating the state constraints, but staying inside Ω_u. Taking into account that $\Omega_{ux} \in \Omega_u$, it follows that $N_{ux} \le N_u$. For this example, $N_{ux} = 4$ and $N_u = 12$.

We first demonstrate the implementation of the Lyapunov-based model predictive controller of Theorem 1 assuming hard constraints in the states and inputs. We compare this controller with the standard LMPC implementation with $N_R = 1$. We consider an initial condition inside Ω_{ux} and sensor data losses defined by $p = 0.66$ and $n_o = 3$. The resulting data losses have a data loss rate of $r = 0.6059$. The initial condition is given by $C_A = 0.7045kmol/m^3$ and $T = 392.6374K$. Figure 2 shows the trajectories of the LMPC controller with $N_R = 12$ (solid line) and $N_R = 1$ (dashed line). Following Theorem 1, the state of the closed-loop system under the LMPC controller with $N_R = 12$ remains bounded in Ω_{ux} because $n_o \le N_R$. This is not the case with the standard LMPC implementation. It can be seen that if the Lyapunov contractive constraint is not defined taking into account data losses, the closed-loop state leaves Ω_{ux}, and moreover, violates the state constraints.

For the second simulation the initial condition lies outside Ω_{ux} but inside Ω_u. In this case, the state constraints are violated. The sensor data losses are de-

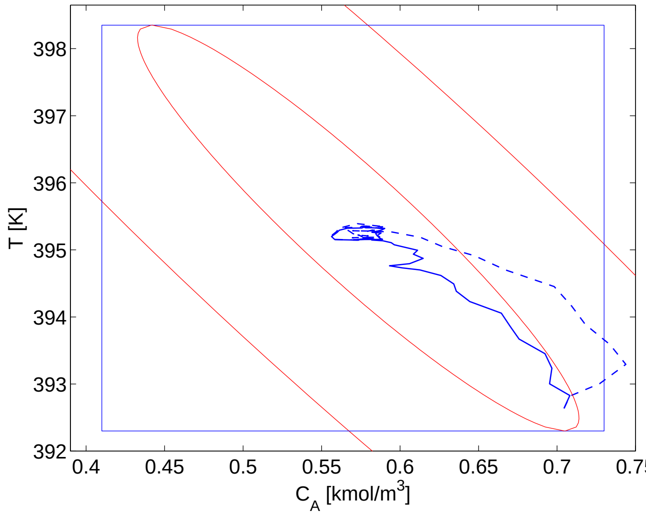

Figure 2: Closed-loop system states under the LMPC controller with $N_R = 12$ (solid line) and $N_R = 1$ (dashed line).

fined by $p = 0.9167$ and $n_o = 12$. The resulting data loss rate is $r = 0.8812$. The initial condition is given by $C_A = 0.8766kmol/m^3$ and $T = 387.3743K$. Figure 3 shows the trajectories of the LMPC controller with $N_R = 12$ (solid line) and $N_R = 1$ (dashed line). Note that for this simulation the hybrid implementation is not taken into account. It can be seen that the state of the closed-loop system under the proposed LMPC controller with $N_R = n_o$ remains inside Ω_u for all times, while the state of the closed-loop system under the implementation with $N_R = 1$ leaves the stability region and goes to another equilibrium point.

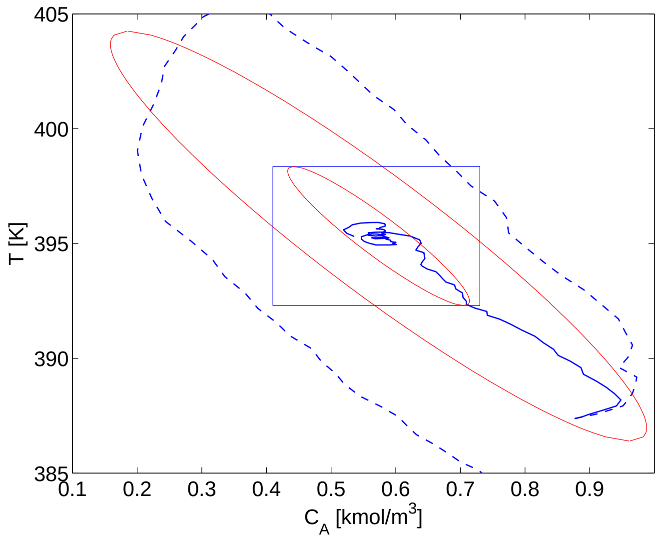

Figure 3: Closed-loop system states under the LMPC controller with $N_R = 12$ (solid line) and $N_R = 1$ (dashed line).

The last simulation compares the LMPC controller $K_L(x, i)$ with the hybrid controller $K_H(x, i)$. Both formulations are subject to the same set of constraints in the optimization problem. The only difference is the cost

DCDIS A Supplement, Advances in Dynamical Systems, Vol. 14(S2) 39--45
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function. In the hybrid formulation, the objective is to minimize the Lyapunov function at the end of the prediction horizon. In this way, faster convergence to Ω_{ux} is achieved in general. The initial condition is given by $C_A = 0.1923kmol/m^3$ and $T = 403.5146K$. The sensor data losses are defined by $p = 0.75$ and $n_o = 4$. The resulting data loss rate is $r = 0.6733$. Figure 3 shows the trajectories of the Lyapunov function for the LMPC controller and the hybrid controller. The horizontal dashed line is set at c_{ux}^{max}. It can be seen that the hybrid controller reaches Ω_{ux} before the LMPC controller. Note that the Lyapunov function increases at certain times. This is due to the fact that when the system operates in open-loop for a sufficiently long time, the error between the predicted state and the actual state becomes too high and the Lyapunov function starts increasing because the control action is computed based on wrong state estimates.

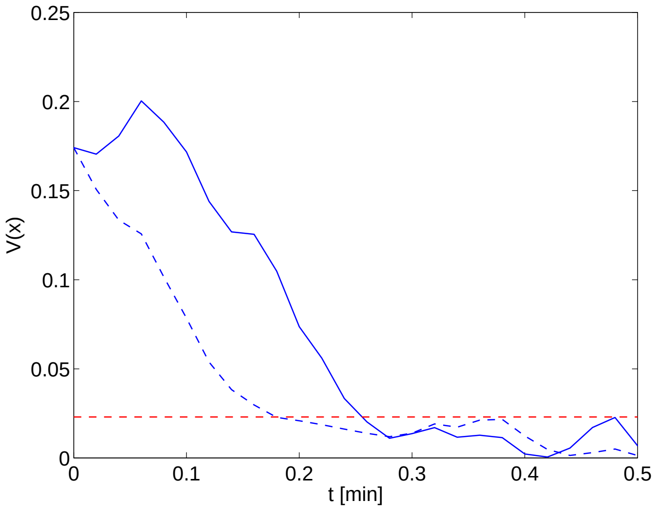

Figure 4: Profiles of $V(x)$ for the closed-loop system under the LMPC controller with $N_R = 12$ (solid line) and the hybrid controller with $N_R = 12$ (dashed line).

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