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Drag reduction in transitional linearized channel flow using distributed control

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This work focuses on feedback control of incompressible transitional Newtonian channel flow described by the two-dimensional linearized Navier–Stokes equations. The control objective is to use distributed feedback to achieve stabilization of the parabolic velocity profile, for values of the Reynolds number for which this profile is unstable, and therefore to reduce the frictional drag exerted on the lower channel wall compared to the open-loop values. The control system uses measurements of shear stresses on the lower channel wall and the control actuation is assumed to be in the form of electromagnetic Lorentz forces applied to the flow near the bottom wall. Galerkin's method is initially used to derive a high-order discretization of the linearized flow field that captures the flow instability and accounts for the effect of control actuation on all the modes. Then, a low-order approximation of the linearized flow field is derived and used for the synthesis of a linear output feedback controller that enforces stability in the high-order closed-loop system. The controller is applied to a simulated transitional linearized channel flow and is shown to stabilize the flow field at the parabolic profile and significantly reduce the drag on the lower channel wall.

1. Introduction

The problem of trying to influence a flow field to behave in a desirable way has received significant attention in the past (see, for example, Gad-el-Hak (1994) for results in this area and reference lists). This problem has been motivated by many practical engineering applications, including reduction of the drag which is generated by a turbulent flow passing over a surface, by preventing the transition from laminar to turbulent flow inside the boundary layer through active feedback control. Drag reduction through active feedback control may have a very significant impact on the design and operation of underwater vehicles, airplanes and automobiles. In general, fluid flow control can be classified in two categories: passive and active. Passive control typically involves some kind of design modification of the surface (e.g. wall-mounted, streamwise ribs or riblets) and requires no auxiliary power, while active control involves continuous adjustment of a variable that affects the flow based on measurements of quantities of the flow field (feedback). Methods for active fluid flow control have included injection of polymers (Lumley 1973, Virk 1975), mass transport through porous walls (e.g. blowing/suction) (Choi *et al.* 1994, Carlson and Lumley 1996) and application of electromagnetic forcing (Crawford and Karniadakis 1997, Singh and Bandyopadhyay 1997).

Over the last decade, several efforts have been made on the design and implementation of feedback control systems on various fluid flows. The approach followed for controller design typically involves the derivation of low-order ordinary differential equation (ODE) approximations of the Navier–Stokes equations which describe the flow field using advanced discretization schemes including linear and non-linear Galerkin methods (e.g. Titi 1990, Deane *et al.* 1991, Baker *et al.* 2000) and reduced basis methods (e.g. Ito and Ravindran 1997). These ODE systems are subsequently used for the design of low-order output feedback controllers. This approach has led to the design of robust optimal controllers for flow in a driven cavity (Burns and Qu 1994, Burns and King 1994, King and Qu 1995), linear optimal and robust controllers for channel flow using boundary (blowing and suction) control actuation (Joshi *et al.* 1995, Cortelezzi and Speyer 1998, Cortelezzi *et al.* 1998), linear controllers for flow over flat plate using distributed (electromagnetic forcing) control actuation (Singh and Bandyopadhyay 1997), and linear and non-linear controllers for suppression of wavy behaviour exhibited by fluid dynamic systems described by the Korteweg–de Vries–Burgers (Armaou and Christofides 2000 b) and Kuramoto–Sivashinsky (Armaou and Christofides 2000 a, Christofides and Armaou 2000, Armaou and Christofides 2000 b) equations. An alternative approach to controller design is based on the concept of designing a feedback controller so that the time-derivative of an appropriate Lyapunov functional along the trajectories of the closed-loop system is negative definite and has been used to design controllers for channel flow (Kang and Ito 1992, Balogh *et al.* 2001) and Kuramoto–Sivashinsky (Liu and Krstic 2001) equations. Other results include the solution of the optimal control problem for the Navier–Stokes equations with distributed control (Desai and Ito 1994, Hou and Yan 1997).

In this work, we focus on feedback control of incompressible transitional Newtonian channel flow described by the two-dimensional linearized Navier–Stokes equa-

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tions. The control objective is to use distributed feedback to achieve stabilization of the parabolic velocity profile, for values of the Reynolds number for which this profile is unstable, and therefore to reduce the frictional drag exerted on the lower channel wall compared to the open-loop values. The control system uses measurements of shear stresses on the lower channel wall and the control actuation is assumed to be in the form of electromagnetic Lorentz forces applied to the flow near the bottom wall. Galerkin's method is initially used to derive a high-order discretization of the linearized flow field that captures the flow instability and accounts for the effect of control actuation on all the modes. Then, a low-order approximation of the linearized flow field is derived and used for the synthesis of a linear output feedback controller that enforces stability in the high-order closed-loop system. The controller is applied to a simulated transitional linearized channel flow and is shown to stabilize the flow field at the parabolic profile and significantly reduce the drag on the lower channel wall.

2. Two-dimensional channel flow with distributed control

We consider a two-dimensional channel flow and address the problem of stabilizing a transitional flow field at the parabolic velocity profile using feedback control. The control actuation is assumed to be in the form of electromagnetic Lorentz forces applied to the flow near the bottom wall (Crawford and Karniadakis 1997, Singh and Bandyopadhyay 1997). The Lorentz forces are due to embedded permanent magnets and electrodes in the bottom channel wall. In the presence of a conducting fluid, the electrodes induce a current near the bottom wall. Since the conducting fluid near the wall is also subjected to magnetic fields, due to the embedded magnets, a Lorentz force is generated which acts as a body force on the fluid; see figure 1 for a schematic of the flow field and the control actuation.

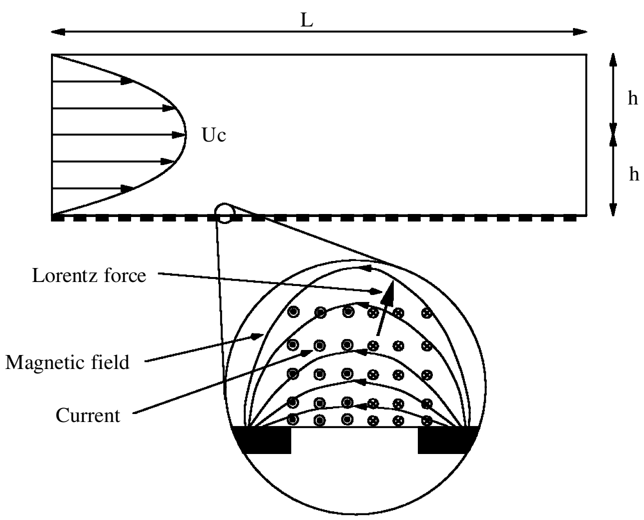

Figure 1. Channel flow with electromagnetic forcing on the bottom wall.

To present the various equations that describe the flow, we use the characteristic time $t = h/U_c$ where h is the half-channel height and U_c is the centre-channel velocity, as well as the Reynolds number $Re = U_c h/\nu$ where ν is the kinematic viscosity. In two dimensions, the Navier–Stokes equations take the following form

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} = 0 \tag{1}$$

$$\left.\begin{aligned} \frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} &= -\frac{\partial p^*}{\partial x} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2}\right) \\ &\quad + \tilde{b}_1(x,y)\bar{u}(t) \\ \frac{\partial v^*}{\partial t} + u^* \frac{\partial v^*}{\partial x} + v^* \frac{\partial v^*}{\partial y} &= -\frac{\partial p^*}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2}\right) \\ &\quad + \tilde{b}_2(x,y)\bar{u}(t) \end{aligned}\right\} \tag{2}$$

where u^* and v^* are the components of the velocity along the x (parallel to the wall) and y (normal to the wall) axes, respectively, p^* is the pressure, $\bar{u}(t)$ is the vector of manipulated inputs and $\tilde{b}_1(x,y)$ and $\tilde{b}_2(x,y)$ are the distribution functions of the control actuators. Equation (2) is subject to the following no-slip, no-penetration boundary conditions at the top and bottom walls of the channel

$$\left.\begin{aligned} u^*(x,y=-1,t) = u^*(x,y=+1,t) = 0 \\ v^*(x,y=-1,t) = v^*(x,y=+1,t) = 0 \end{aligned}\right\} \tag{3}$$

Boundary conditions in the x-direction are taken to be periodic

$$\left.\begin{aligned} u^*(x=0,y,t) &= u^*(x=L,y,t) \\ v^*(x=0,y,t) &= v^*(x=L,y,t) \\ \frac{\partial u^*}{\partial x}(x=0,y,t) &= \frac{\partial u^*}{\partial x}(x=L,y,t) \\ \frac{\partial v^*}{\partial x}(x=0,y,t) &= \frac{\partial v^*}{\partial x}(x=L,y,t) \end{aligned}\right\} \tag{4}$$

The actuator distribution functions, $\tilde{b}_1(x,y)$ and $\tilde{b}_2(x,y)$, are trigonometric functions in the x-direction and decay exponentially in the y-direction and take the form (see also Crawford and Karniadakis (1997), Singh and Bandyopadhyay (1997) for similar actuator distribution functions)

$$\left.\begin{aligned}\tilde{b}_1(x,y)\bar{u} &= \sum_{i=1}^{\tilde{N}}\sum_{j=0}^{\tilde{M}}[\tilde{b}_{1_{ij}}\cos(i\,\alpha_0\,x)\exp(-q_j(y+1))]\bar{u}_{ij}\\ \tilde{b}_2(x,y)\bar{u} &= \sum_{i=1}^{\tilde{N}}\sum_{j=0}^{\tilde{M}}[\tilde{b}_{2_{ij}}\cos(i\,\alpha_0\,x)\exp(-q_j(y+1))]\bar{u}_{ij}\end{aligned}\right\} \tag{5}$$

where $\alpha_0 = 2\pi/L$ is the fundamental wave number, $(\tilde{N}\times\tilde{M})$ is the total number of control actuators, $\tilde{b}_{1_{ij}}, \tilde{b}_{2_{ij}}$ are constants and q_j are positive constants related to the decay (in the y-direction) of the Lorentz force of the i, jth actuator.

Since we are interested in stabilizing the flow at the parabolic base flow, we assume that the flow field and the pressure field can be decomposed into a primary component plus a perturbation

$$\left.\begin{aligned}u^*(x,y) &= U(y)+u(x,y)\\ v^*(x,y) &= V(x,y)+v(x,y)\\ p^*(x,y) &= P(x,y)+p(x,y)\end{aligned}\right\} \tag{6}$$

where u and v are the velocity perturbations in the x and y directions, respectively, and $U(y) = U_c(1-y^2)$, the parabolic profile corresponding to the base channel flow. Also, note that $V(x,y)$ and $P(x,y)$, the base velocity field normal to the wall and the base pressure field, respectively, are equal to zero. Substituting (6) into (2), we obtain

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial x}+U'v+v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x}+\frac{1}{\text{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+U''\right)+\tilde{b}_1(x,y)\bar{u}(t) \tag{7}$$

$$\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y}+\frac{1}{\text{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\tilde{b}_2(x,y)\bar{u}(t) \tag{8}$$

where U' and U'' are the first and second-order derivatives of the base flow, $U(y)$, with respect to y. Utilizing the perturbation stream function, ψ, which satisfies

$$u = \frac{\partial\psi}{\partial y}, \qquad v = -\frac{\partial\psi}{\partial x} \tag{9}$$

taking the partial derivative of (7) with respect to y, and subtracting the partial derivative of (8) with respect to x, these two equations can be combined into a single equation for $\psi(x,y,t)$, of the form

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\Delta\psi - U''\frac{\partial\psi}{\partial x} = \frac{1}{\text{Re}}(\Delta\Delta\psi)+b(x,y)\bar{u}(t) \tag{10}$$

where Δ is the Laplacian in two dimensions and $b(x,y)$ is a non-linear function of the form

$$b(x,y)\bar{u}(t) = \sum_{i=1}^{\tilde{N}}\sum_{j=0}^{\tilde{M}}[(b_{1_{ij}}\cos(i\,\alpha_0\,x) + b_{2_{ij}}\sin(i\,\alpha_0\,x))\exp(-q_j(y+1))]\bar{u}_{ij}(t) \tag{11}$$

Note that in the formulation of (10) the continuity constraint is automatically satisfied. Equation (10) is subjected to the following no-slip boundary conditions at $y=+1$ and $y=-1$

$$\left.\begin{aligned}\psi(x,y=-1,t) &= \psi(x,y=+1,t)=0\\ \frac{\partial\psi}{\partial y}(x,y=-1,t) &= \frac{\partial\psi}{\partial y}(x,y=+1,t)=0\end{aligned}\right\} \tag{12}$$

and the following periodic boundary conditions at $x=0$ and $x=L$

$$\left.\begin{aligned}\psi(x=0,y,t) &= \psi(x=L,y,t)\\ \frac{\partial\psi}{\partial x}(x=0,y,t) &= \frac{\partial\psi}{\partial x}(x=L,y,t)\end{aligned}\right\} \tag{13}$$

where $L=2\pi$ is the length of the domain in the streamwise direction.

3. Solution of the flow field

We use Galerkin's method to solve the system of equations (10)–(12)–(13). To this end, we assume that the stream function, $\psi(x,y,t)$, can be written in the form

$$\psi(x,y,t) = \sum_{n=1}^{N}\sum_{m=0}^{M}[a_{nm}(t)\cos(n\,\alpha_0\,x) + b_{nm}(t)\sin(n\,\alpha_0\,x)]L_m(y) \tag{14}$$

where $L_m(y)$ are linear combinations of Chebyshev polynomials that are orthogonal, and satisfy the boundary conditions at $y=+1$ and $y=-1$, namely,

$$\left.\begin{aligned}L_m(y=-1) &= L_m(y=+1)=0\\ \frac{\mathrm{d}L_m}{\mathrm{d}y}(y=-1) &= \frac{\mathrm{d}L_m}{\mathrm{d}y}(y=+1)=0\end{aligned}\right\} \tag{15}$$

Substituting the series expansion of (14) into the PDE of (10), and taking the inner product with $\cos(n\,\alpha_0\,x)$ and $\sin(n\,\alpha_0\,x)$ for $n=1,2,\ldots N$, $x\in[0,2\pi]$, results in a system of differential equations for $a_{nm}(t)$ and $b_{nm}(t)$, respectively. Picking appropriately the constants $b_{1_{ij}}$ and $b_{2_{ij}}$ and defining

$$\left.\begin{aligned} L_{0nm} &= (L_m''(y) - ((n\,\alpha_0)^2\, L_m(y))) \\ L_{1nm} &= (L_m''''(y) - 2(n\,\alpha_0)^2\, L_m''(y) + (n\,\alpha_0)^4\, L_m(y)) \\ L_{2nm} &= (-U''(y)\,(n\,\alpha_0)\, L_m(y) + U(y)\,(n\,\alpha_0)\, L_m''(y) \\ &\quad - U(y)(n\,\alpha_0)^3\, L_m(y)) \end{aligned}\right\} \tag{16}$$

the linearization of the system of (10) takes the form

$$\left.\begin{aligned} \sum_{m=0}^{M} \dot{a}_{nm} L_{0nm} &= -\sum_{m=0}^{M} b_{nm} L_{2nm} + \frac{1}{Re} \sum_{m=0}^{M} a_{nm} L_{1nm} + \tilde{B}_{n1}\,\bar{u} \\ \sum_{m=0}^{M} \dot{b}_{nm} L_{0nm} &= \sum_{m=0}^{M} a_{nm} L_{2nm} + \frac{1}{Re} \sum_{m=0}^{M} b_{nm} L_{1nm} + \tilde{B}_{n2}\,\bar{u} \end{aligned}\right\} \tag{17}$$

Projecting the system of (17) onto the orthogonal (with respect to $(1-y^2)^{-1/2}$) set of $M+1$ linear combinations of Chebyshev polynomials, $L_m(y)$, we obtain the following system of ODEs:

$$\left.\begin{aligned} \dot{a}_n &= \frac{1}{Re}\, M_{n0}^{-1}\, M_{n1}\, a_n - M_{n0}^{-1}\, M_{n2}\, b_n + M_{n0}^{-1}\, B_{n1}\,\bar{u} \\ \dot{b}_n &= M_{n0}^{-1}\, M_{n2}\, a_n + \frac{1}{Re}\, M_{n0}^{-1}\, M_{n1}\, b_n + M_{n0}^{-1}\, B_{n2}\,\bar{u} \end{aligned}\right\} \tag{18}$$

where $n = 1, 2, \ldots N$ and M_{n0}, M_{n1} and M_{n2} are matrices of inner products of $L_m(y)$ and the system of (17) of the form

$$\left.\begin{aligned} M_{n0} &= [(M_{n0})_{lm}] = [(L_l, L_{0nm})] \\ M_{n1} &= [(M_{n1})_{lm}] = [(L_l, L_{1nm})] \\ M_{n2} &= [(M_{n2})_{lm}] = [(L_l, L_{2nm})] \end{aligned}\right\} \tag{19}$$

where $[(M_{ni})_{lm}]$ denotes the lmth element of matrix M_{ni}, which is the inner product of L_l and L_{inm}.

The time-integration of the system of ODEs of (18) is performed by using a fourth-order Runge–Kutta scheme. For Re = 6500 and random initial conditions of the form

$$\left.\begin{aligned} &a_{nm} = O(0.01/n^2), \quad b_{nm} = O(0.01/n^2) \\ &n = 1, \ldots, N,\ m = 0, \ldots, M \end{aligned}\right\} \tag{20}$$

we found that $N = 20$ and $M = 10$ yield a numerically stable discretization of the PDE system (further increase on N and/or M led to identical simulation results). Under these conditions, figure 2 (dashed line) shows the sum of the squares of the modes of the open-loop system ($S = \sum_{n=1}^{20} \sum_{m=0}^{10} (a_{nm}^2(t) + b_{nm}^2(t))$; this quantity is closely related to the energy of the system) which increases with time. This indicates that the flow field, as expected from linear stability theory calculations, is in the transition (unstable) regime.

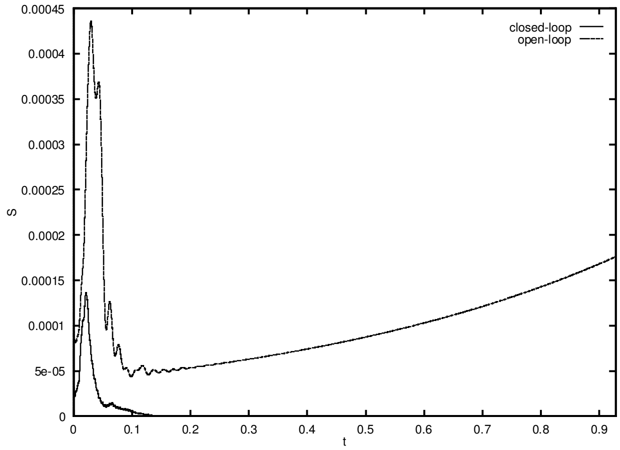

Figure 2. Profiles of S for open-loop (dashed line) and closed-loop system under output feedback control (solid line).

4. Controller design – closed-loop simulations

We synthesize and implement a linear output feedback controller to stabilize the perturbed flow field for Re = 6500 at the parabolic steady-state profile. To this end, a reduced-order ODE approximation of the PDE of (10) was initially obtained via Galerkin's method with $N = 2$ and $M = 20$, of the general form

$$\left.\begin{aligned} \dot{x} &= Ax + B\bar{u} \\ y &= Cx \end{aligned}\right\} \tag{21}$$

where x is the state vector consisting of the a_{nm} and b_{nm} mode amplitudes, A is the matrix resulting from the discretization via Galerkin's method, B is a matrix which describes the interaction between the actuators and the modes, u is the vector of manipulated inputs, y is the vector of measured outputs and C is a matrix describing the interaction between the shear measurement sensors and the modes. The detailed expression of these matrices are omitted for brevity. To achieve the stabilization of the flow field, each state of the reduced-order ODE model with $n = 1, 2$ and $m = 0, 1, 2, \ldots, 20$ was considered as a controlled output and a linear feedback controller of the form

$$\bar{u} = Kx \tag{22}$$

was designed using geometric methods (see, for example, Isidori (1989) for the design of state feedback controllers for ODE systems with controlled outputs), to stabilize the unstable open-loop system. To be able to regulate all the outputs, $2 \times 20 \times 2$ control actuators distributing

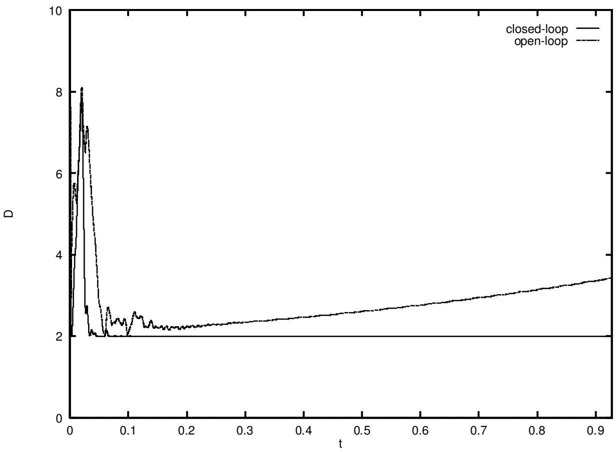

Figure 3. Profiles of the average drag along the bottom of the channel for open-loop (dashed line) and closed-loop system under output feedback control (solid line).

body forces in the near vicinity of the bottom wall by means of Lorentz forces were assumed to be available. To implement the state feedback controller of (22), the system of (21) was used as the basis for the design of a state observer of the general form

$$\dot{\eta} = A\eta + B\bar{u} + L(y - C\eta) \tag{23}$$

where the observer gain matrix, L, was calculated using MATLAB, in order to make the matrix $(A - LC)$ stable. The output feedback controller resulting from the combination of the state feedback controller of (22) and the state observer of (23) takes the form

$$\left.\begin{aligned} \dot{\eta} &= A\eta + BK\eta + L(y - C\eta) \\ \bar{u} &= K\eta \end{aligned}\right\} \tag{24}$$

The above controller was implemented on the high-order discretization of the flow field described in § 3. All the results shown below are for Re = 6500 for which the flow field is unstable.

Figure 2 (solid line) shows the profile of S for the linearized two-dimensional channel flow under the output feedback controller. The controller successfully stabilizes the perturbations (solid line) in the closed-loop system, while the energy of the uncontrolled flow steadily increases due to linear instability of the flow field (dashed line). Figure 3 (solid line) shows the average drag at the bottom of the channel for the linearized two-dimensional channel flow under the output feedback controller. The controller reduces the drag to that of laminar channel flow (solid line), while the drag of the unstable uncontrolled flow steadily increases with time.

Finally, we to note that we have tested the developed output feedback controller for different sets of initial conditions for the flow field and several Reynolds numbers in the regime (6000–10 000) and have found similar results to the ones shown in figures 2 and 3 for the behaviour of the open- and closed-loop systems; these results are omitted here for brevity. The reader may also refer to Baker *et al*. (2000) for results on distributed non-linear control of fully non-linear channel flow for Re = 500 (for which the flow field is naturally stable) which demonstrate that improved convergence rates to the steady-state can be achieved when non-linear control is used.

5. Conclusions

In this work, we presented some results on distributed feedback control of incompressible transitional Newtonian channel flow described by two-dimensional linearized Navier–Stokes equations. The developed feedback control system uses measurements of shear stresses on the lower channel wall and the control actuation is assumed to be in the form of electromagnetic Lorentz forces applied to the flow near the bottom wall. The control objective is to use distributed feedback to achieve stabilization of the parabolic velocity profile and therefore to reduce the frictional drag exerted on the lower channel wall compared to the open-loop values. Galerkin's method was initially used to derive a high-order discretization of the linearized flow field that captures the flow instability and accounts for the effect of control actuation on all the modes. Then, a low-order approximation of the linearized flow field was derived and used for the synthesis of a linear output feedback controller that enforces stability in the high-order closed-loop system. The controller was applied to a simulated transitional (Re = 6500) linearized channel flow and was shown to stabilize the flow field at the parabolic profile and significantly reduce the drag on the lower channel wall. These results motivate applying the developed output feedback controller to fully non-linear, transitional channel flow to achieve stabilization of the parabolic profile using electromagnetic Lorentz forces and this will be the subject of future research.

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