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Lyapunov-based model predictive control of nonlinear systems subject to time-varying measurement delays

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SUMMARY

In this work, we focus on model predictive control of nonlinear systems subject to time-varying measurement delays. The motivation for studying this control problem is provided by networked control problems and the presence of time-varying delays in measurement sampling in chemical processes. We propose a Lyapunov-based model predictive controller that is designed taking time-varying measurement delays explicitly into account, both in the optimization problem formulation and in the controller implementation. The proposed predictive controller allows for an explicit characterization of the stability region and guarantees that the closed-loop system in the presence of time-varying measurement delays is ultimately bounded in a region that contains the origin if the maximum delay is smaller than a constant that depends on the parameters of the system and the Lyapunov-based controller that is used to formulate the optimization problem. The application of the proposed Lyapunov-based model predictive control method is illustrated using a nonlinear chemical process example with asynchronous, delayed measurements and its stability and performance properties are illustrated to be superior to the ones of two existing Lyapunov-based model predictive controllers. Copyright © 2008 John Wiley & Sons, Ltd.

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1. INTRODUCTION

The problem of designing feedback control systems for nonlinear systems subject to time-varying measurement delays is a fundamental one and its solution can find significant application in a number of control engineering problems including, for example, design of networked control systems (NCS). NCS are control systems that operate over a communication network (wired or wireless) and can lead to significant improvements in the efficiency, flexibility, robustness and fault-tolerance of industrial control systems as well as to reduction of the installation, reconfiguration and maintenance costs. However, the design of NCS has to account for the dynamics introduced by the communication network, which may include time-varying delays, data quantization or data losses. In addition to NCS, another source of time-varying delays in the feedback loop is measurement sensor delays, which are particularly important during the measurement of species concentrations and particle size distributions in process control applications.

Model predictive control (MPC) is a technique used to compute control actions that are optimal with respect to a meaningful performance index and satisfy suitable control actuator and state variable constraints. In MPC, the optimization problem is solved at each sampling time using a process model to predict the future evolution of the system from the current state along a given prediction horizon. Once the optimization problem is solved, only the first input is implemented by the control actuator, the rest of the input trajectory is discarded and the optimization is repeated at the next sampling step (the so-called 'receding horizon scheme'); see, for example, [1, 2] for a review of results in this area. In order to guarantee stability of the closed-loop system, MPC schemes must include a set of stability constraints. Different MPC schemes can be found in the literature, see [3] for a review on MPC stability results. In a recent series of papers, we proposed Lyapunov-based model predictive control (LMPC) schemes for nonlinear systems [4, 5] based on uniting receding horizon control with control Lyapunov functions (CLFs) as a way of guaranteeing closed-loop stability. The main idea is to formulate appropriate constraints in the predictive controller's optimization problem based on an existing Lyapunov-based controller, in such a way that the MPC controller inherits the robustness and stability properties of the Lyapunov-based controller. The proposed LMPC schemes allow for an explicit characterization of the stability region and lead to reduced complexity optimization problems. LMPC has been applied with success to constrained nonlinear systems, switched systems and to fault-tolerant control problems [4–7]. However, the available results on LMPC do not account for the effect of time-varying measurement delays, and when time-varying measurement delays are taken into account, these schemes are not guaranteed to maintain the desired closed-loop stability properties. In terms of other research work pertaining to the control problem studied in this paper, we note that most of the available results on MPC of systems with delays deal with linear systems (e.g. [8, 9]). Finally, the importance of time delays in the context of NCS has motivated significant research effort in modeling such delays and designing control systems to deal with them, primarily in the context of linear systems (e.g. [10–15]).

Motivated by the above, this study deals with the design of predictive controllers for nonlinear system subject to time-varying measurement delay in the feedback loop. In particular, we propose to design the controller in an LMPC scheme. While there are several LMPC schemes that have been proposed in the literature (see, for example, [16]), in this study we design the proposed LMPC based on the results developed in [4–6] by our group. In the LMPC scheme proposed in the present study, when measurement delays occur, the nominal model of the system is used together with the delayed measurement to estimate the current state, and the resulting estimate is used to evaluate the LMPC controller; at sampling times where no measurements are available due to the delay,

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instead of setting the control input to zero or to the last available value, the actuator implements the last optimal input trajectory evaluated by the controller (this requires that the controller must store in memory the last evaluated optimal control input trajectory). The proposed LMPC scheme inherits the stability and robustness properties in the presence of uncertainty and time-varying delay of the Lyapunov-based controller, while taking into account optimality considerations. Specifically, the proposed LMPC scheme allows for an explicit characterization of the stability region and guarantees that the closed-loop system in the presence of time-varying measurement delays is ultimately bounded in a region that contains the origin if the maximum delay is smaller than a constant that depends on the parameters of the system and the Lyapunov-based controller that is used to formulate the optimization problem. The theoretical results are illustrated through a chemical process example.

2. PRELIMINARIES

2.1. Nonlinear systems and Lyapunov-based controller

In this work, we consider a nonlinear system subject to disturbances with the following state-space description

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{1}$$

where $x(t) \in R^{n_x}$ denotes the vector of state variables, $u(t) \in R^{n_u}$ denotes the vector of manipulated input variables, $w(t) \in R^{n_w}$ denotes the vector of disturbance variables and f is a locally Lipschitz vector function on $R^{n_x} \times R^{n_u} \times R^{n_w}$. The disturbance vector is bounded, i.e. $w(t) \in W$ where

$$W := \{w \in R^{n_w} \text{ s.t.} |w| \leqslant \theta, \theta > 0\}^{\ddagger}$$

with θ being a known positive real number. The vector of uncertain variables is introduced into the model in order to account for the occurrence of uncertainty in the values of the process parameters and the influence of disturbances in practical control applications.

We assume that the nominal closed-loop system (system (1) with $w(t) \equiv 0$ for all t) has an asymptotically stable equilibrium at the origin $x = 0$ for a given feedback control $h: R^{n_x} \rightarrow R^{n_u}$, which satisfies $h(0) = 0$ (this assumption is equivalent to the existence of a CLF for the system $\dot{x} = f(x, u, 0)$). This feedback law will be used in the design of the LMPC controller. Using converse Lyapunov theorems (see [17]), this assumption implies that there exist functions $\alpha_i(\cdot), i = 1, 2, 3, 4$ of class $\mathcal{K}$§ and a Lyapunov function V for the nominal closed-loop system (system (1) with $u(t) = h(x(t))$ and $w(t) \equiv 0$), which is continuous and bounded in R^{n_x}, that satisfy the following inequalities:

$$\begin{aligned} \alpha_1(|x|) \leqslant V(x) &\leqslant \alpha_2(|x|) \\ \frac{\partial V(x)}{\partial x} f(x, h(x), 0) &\leqslant -\alpha_3(|x|) \\ \left| \frac{\partial V(x)}{\partial x} \right| &\leqslant \alpha_4(|x|) \end{aligned} \tag{2}$$

‡ $|\cdot|$ denotes Euclidean norm of a vector.

§ Class $\mathcal{K}$ functions $\alpha_i(\cdot)$ are strictly increasing functions of their argument and satisfy $\alpha_i(0) = 0$.

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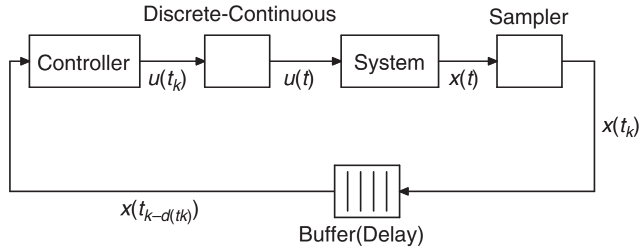

Figure 1. Sampled-data closed-loop system subject to time-varying measurement delay.

for all $x \in O \subseteq R^{n_x}$ where O is any open neighborhood of the origin, see [17]. We denote the region $\Omega_\rho^{\P} \subseteq O$ as the stability region of the closed-loop system under controller $h(x)$. Note that explicit stabilizing control laws that provide explicitly defined regions of attraction for the closed-loop system have been developed using Lyapunov techniques for specific classes of nonlinear systems, particularly input-affine nonlinear systems; the reader may refer to [18–23] for results in this area. In Section 4, a method such as the one presented in [18] is used for the design of $h(x)$. In the remainder, we will refer to the controller $h(x)$ as the Lyapunov-based controller.

By continuity and the local Lipschitz property assumed for the vector field $f(x,u,w)$ and the continuous differentiable property of the control Lyapunov function V, there exist positive constants M, L_w, L_x and L'_x such that

$$|f(x,h(x),w)| \leqslant M \tag{3}$$

$$|f(x,h(x),w)-f(x',h(x),0)| \leqslant L_w|w|+L_x|x-x'| \tag{4}$$

$$\left|\frac{\partial V}{\partial x} f(x,h(x),0)-\frac{\partial V}{\partial x} f(x',h(x),0)\right| \leqslant L'_x|x-x'| \tag{5}$$

for all $x, x' \in \Omega_\rho$ and $w \in W$. These constants will be used in the proof of the main results of this work.

2.2. Model of measurement delay

Although system (1) is defined in continuous time, we focus on a sample-and-hold implementation of the controller subject to time-varying measurement delay because we work within an MPC framework. State measurements are obtained with a sampling time Δ at times $t_k = t_0 + k\Delta$ where t_0 is the initial time and $k = 0, 1\ldots$. However, due to the time-varying measurement delays, the controller may not receive the latest measurement, but a previous one. We assume that each measurement is time-labeled; hence, the controller is able to discard non-relevant information; i.e. the controller discards earlier measurements if it has already received more recent information. We do not consider delays in the computation and implementation of the control actions. This model is of relevance to systems subject to asynchronous delayed measurements and to networked control systems, where the delay is introduced by the communication network connecting the sensor and the controller. Figure 1 shows a schematic of the closed-loop system for the control problem considered.

To model measurement delay, an auxiliary random variable $d(t_k)$ is introduced to indicate the number of sampling times that the measurement received at time t_k is delayed. That is, at

¶We use Ω_r to denote the set $\Omega_r := \{x \in R^{n_x} | V(x) \leqslant r\}$.

sampling time t_k, the controller receives the delayed measurement $x(t_{k-d(t_k)})$. When more than one measurements are received, the controller only uses the latest one and discards the rest. For example, if at sampling time t_k, three new measurements are received with a delay of 5, 4 and 3 sampling times, respectively, the controller only uses the measurement with a delay of 3 sampling times (recall that the measurements are time-labeled). The sequence $\{d(t_k)\geqslant 0\}$ characterizes the time needed to obtain a new measurement in the case of asynchronous measurements or the quality of the feedback link in the case of NCS. Because of the delay, it is possible that there exist sampling times t_k in which a new measurement is not received. In this case, the controller must decide the control action in open loop, for example, setting the control input to zero or to the last implemented value. In the following section, we present an LMPC controller in which time-varying delays are taken into account explicitly both in the controller design and in the implementation procedure.

In general, if the sequence $\{d(t_k)\geqslant 0\}$ is modeled using a random process, there exists the possibility of arbitrarily large delays. In this case, it is not possible to provide guaranteed stability properties, because there exists a non-zero probability that the controller operates in open loop for a period of time large enough for the state to leave the stability region or even escape to infinity (i.e. finite escape time). In order to study the stability properties in a deterministic framework, in this paper we consider systems where there exists an upper bound D on the delay of the measurement that we receive at each sampling time, i.e. $d(t_k)\leqslant D, k=0,1\ldots$.

3. LYAPUNOV-BASED MODEL PREDICTIVE CONTROL

3.1. LMPC design

LMPC is based on uniting receding horizon control with CLFs and computes the control input trajectory solving a finite horizon constrained optimal control problem. The control input trajectory (i.e. the free variable of the LMPC optimization problem) is constrained to belong to the family of piecewise constant functions $S(\Delta)$ with sampling period Δ and length equal to the prediction horizon. As mentioned in the introduction, LMPC is characterized by a set of constraints based on an existing Lyapunov-based controller.

Previous LMPC schemes (see [4–6] for standard results and [7] for systems subject to asynchronous measurements and data losses) do not consider time-varying measurement delays. Under certain assumptions, these controllers guarantee practical stability of the closed-loop system; however, when time-varying measurement delays are present, these results do not hold. In this section, a Lyapunov-based model predictive controller for system (1), which takes into account time-varying measurement delay explicitly, both in the constraints imposed in the optimization problem and in the implementation procedure, is proposed. The proposed controller guarantees (as we will prove in Section 3.2) that, under certain conditions, the closed-loop system subject to time-varying measurement delays is ultimately bounded in a set that contains the origin.

A controller for a system subject to time-varying measurement delays must take into account two important issues. First, when a new measurement is received, this measurement may not correspond to the current state of the system. This implies that in this case, the controller has to take a decision using an estimate of the current state. Second, because the delays are time-varying, the controller may not receive new information every sampling time. This implies that in this case, the controller has to operate in open loop using the last received measurements. In order to deal with these two issues, we propose to take advantage of the MPC scheme to decide the control

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input based on a prediction obtained using the nominal model of the system. This prediction is used both for estimating the current state from previous measurements and for deciding the input when the controller does not receive new information.

The proposed LMPC that takes into account time-varying measurement delay in an explicit way is based on the following finite horizon constrained optimal control problem:

$$u_k^*(t) = \arg\min_{u_k \in S(\Delta)} \int_{t_{k-d(t_k)}}^{t_{k+N}} [\tilde{x}(\tau)^{\mathrm{T}} Q_c \tilde{x}(\tau) + u_k(\tau)^{\mathrm{T}} R_c u_k(\tau)]\, \mathrm{d}\tau \tag{6a}$$

$$\text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u_k(t), 0) \tag{6b}$$

$$u_k(t) = u_{k-1}^*(t) \quad \forall t \in [t_{k-d(t_k)}, t_k) \tag{6c}$$

$$\tilde{x}(t_{k-d(t_k)}) = x(t_{k-d(t_k)}) \tag{6d}$$

$$\dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_j)), 0), \quad t \in [t_j, t_{j+1}), \quad j = k, \ldots, k+N-1 \tag{6e}$$

$$\hat{x}(t_k)) = \tilde{x}(t_k) \tag{6f}$$

$$V(\tilde{x}(t)) \leqslant V(\hat{x}(t)) \quad \forall t \in [t_k, t_{k+D+1-d(t_k)}] \tag{6g}$$

where $S(\Delta)$ is the family of piecewise constant functions with sampling period Δ, $\tilde{x}(t)$ is the predicted sampled trajectory of the nominal system for the input trajectory computed by the LMPC (6), $x(t_{k-d(t_k)})$ is the delayed measurement that is received at t_k, $u_{k-1}^*(t)$ is the optimal control input trajectory computed at time t_{k-1}, $\hat{x}(t)$ is the nominal sampled trajectory under the Lyapunov-based controller $u = h(\hat{x}(t))$ along the prediction horizon with initial state the estimated state $\tilde{x}(t_k)$, Q_c, R_c are weight matrices that define the cost and N is the prediction horizon.

If at a sampling time, a new measurement $x(t_{k-d(t_k)})$ is received, an estimate of the current state $\tilde{x}(t_k)$ is obtained using the nominal model of the system (constraint (6b)) and the control inputs applied to the system from $t_{k-d(t_k)}$ to t_k (constraint (6c)) with the initial condition being $\tilde{x}(t_{k-d(k)}) = x(t_{k-d(t_k)})$ (constraint (6d)). Note that this implies that the controller has to store the past control input trajectory. The estimated state $\tilde{x}(t_k)$ is then used to obtain the optimal future control input trajectory solving the finite horizon constrained optimal control problem (6). The LMPC scheme uses the nominal model to predict the future trajectory $\tilde{x}(t)$ for a given input trajectory $u_k(t) \in S(\Delta)$ with $t \in [t_k, t_{k+N}]$. A cost function is minimized (Equation (6)) while assuring that the value of the Lyapunov function along the predicted trajectory $\tilde{x}(t)$ satisfies a Lyapunov-based contractive constraint (constraint (6g)) where $\hat{x}(t)$ is the state trajectory corresponding to the nominal system in closed loop with the Lyapunov-based controller (constraint (6e)) with the initial condition being $\hat{x}(t_k) = \tilde{x}(t_k)$ (constraint (6f)). Note that the contractive constraint (6g) depends on the current delay $d(t_k)$. If the controller does not receive any new measurement at a sampling time, it keeps implementing the last evaluated optimal trajectory. This strategy is a receding horizon strategy, which takes time-varying measurement delays explicitly into account.

This scheme is motivated by the fact that when we have a delayed measurement or no measurement, a reasonable estimate of the current state and the future evolution is given by the nominal model. And the receding horizon scheme can be summarized as follows:

1. If a new measurement is received, then solve (6) and obtain $u_k^*(t)$, else $u_k^*(t) = u_{k-1}^*(t)$.
2. Apply $u(t) = u_k^*(t)$ for all $t \in [t_k, t_{k+1})$.
3. Obtain a new sample and go to 1.

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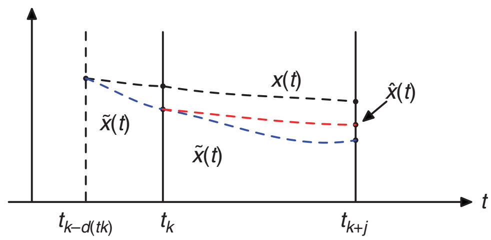

Figure 2. Possible scenario of the measurements received by the LMPC and the corresponding state trajectories defined in problem (6).

This modification states that if no measurement is received, then the previous optimal control input trajectory is applied. This is equivalent to using the model to predict the future evolution of the state of the system and update the input accordingly as in [7, 11, 24–26]. In addition, the modified receding horizon scheme and constraint (6c) guarantee that $u_{k-1}(t)$ stores the control input applied to the system from $t_{k-d(t_k)}$ to t_k.

Figure 2 shows a possible scenario for a system of dimension 1. A delayed measurement $x(t_{k-d(t_k)})$ is received at time t_k and the next new measurement is not obtained until t_{k+j}. This implies that at time t_k we solve problem (6) and we apply the optimal input $u_k^*(t)$ from t_k to t_{k+j}. The solid vertical lines are used to indicate sampling times in which a new measurement is obtained (that is, t_k and t_{k+j}) and the dashed vertical line is used to indicate the time corresponding to the measurement obtained in t_k (that is, $t_{k-d(t_k)}$).

Remark 1
The LMPC scheme proposed above optimizes a cost function, subject to a set of constraints defined by the state trajectory corresponding to the nominal system in closed loop with the Lyapunov-based controller. This allows us to formulate an LMPC problem that does not depend on the uncertainty and so it is of manageable computational complexity.

3.2. LMPC stability subject to measurement delay

In this section, we present the stability properties of the proposed LMPC controller for systems subject to time-varying measurement delays. To this end, we need to introduce several auxiliary results that will be used in the proof of the main theorem of this paper. We first investigate the properties of the Lyapunov-based controller $h(x)$ applied in a sample-and-hold fashion without considering uncertainty or time-varying measurement delay. These properties are important because the proposed LMPC scheme is based on the nominal model of system (1).

Proposition 1 (*cf. Muñoz de la Peña and Christofides* [7])
Consider the nominal sampled trajectory $\hat{x}(t)$ of system (1) for a controller $h(x)$ that satisfies (2) obtained by solving recursively

$$\dot{\hat{x}}(t)=f(\hat{x}(t),h(\hat{x}(t_k)),0),\quad t\in[t_k,t_{k+1})$$

where $t_k=t_0+k\Delta, k=0,1,\ldots$. Let $\Delta,\varepsilon_s>0$ and $\rho>\rho_s>0$ satisfy

$$-\alpha_3(\alpha_2^{-1}(\rho_s))+\alpha_4(\alpha_1^{-1}(\rho))L'_x M\Delta\leqslant-\varepsilon_s/\Delta \tag{7}$$

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Then, if $\rho_{\min} \leqslant \rho$ where

$$\rho_{\min} = \max\{V(\hat{x}(t+\Delta)) : V(\hat{x}(t)) \leqslant \rho_s\}$$

and $\hat{x}(t_0) \in \Omega_\rho$, the following inequalities hold:

$$\begin{aligned} V(\hat{x}(t_k)) &\leqslant \max\{V(\hat{x}(t_0)) - k\varepsilon_s, \rho_{\min}\} \\ V(\hat{x}(t)) &\leqslant \max\{V(\hat{x}(t_k)), \rho_{\min}\} \quad \forall t \in [t_k, t_{k+1}] \end{aligned} \tag{8}$$

Proposition 1 guarantees that if system (1) with $w(t) \equiv 0$ for all t under the control law $u = h(x)$, implemented in a sample-and-hold fashion, starts in $\Omega_\rho / \Omega_{\rho_s}$, then it is ultimately bounded in $\Omega_{\rho_{\min}}$. The following proposition provides an upper bound on the deviation of the state trajectory obtained using the nominal model, from the real-state trajectory when the same control input trajectory is applied.

Proposition 2
Consider the following state trajectories:

$$\begin{aligned} \dot{x}_a(t) &= f(x_a(t), u(t), w(t)) \\ \dot{x}_b(t) &= f(x_b(t), u(t), 0) \end{aligned} \tag{9}$$

with initial states $x_a(t_0) = x_b(t_0) \in \Omega_\rho$, then the following inequality holds,

$$|x_a(t) - x_b(t)| \leqslant f_W(t - t_0) \quad \forall x_a(t), x_b(t) \in \Omega_\rho \text{ and } w(t) \in W \tag{10}$$

where

$$f_W(\tau) = \frac{L_w \theta}{L_x}(\mathrm{e}^{L_x \tau} - 1)$$

Proof
Define the error vector as $e(t) = x_a(t) - x_b(t)$. The time derivative of the error is given by

$$\dot{e}(t) = f(x_a(t), u(t), w(t)) - f(x_b(t), u(t), 0)$$

Applying (4), the following inequality holds

$$|\dot{e}(t)| \leqslant L_w |w(t) - 0| + L_x |x_a(t) - x_b(t)| \leqslant L_w \theta + L_x |e(t)|$$

for all $x_a(t), x_b(t) \in \Omega_\rho$ and $w(t) \in W$. Integrating $|\dot{e}(t)|$ with initial condition $e(t_0) = 0$ (recall that $x_a(t_0) = x_b(t_0)$), the following bound on the norm of the error vector is obtained:

$$|e(t)| \leqslant \frac{L_w \theta}{L_x}(\mathrm{e}^{L_x (t - t_0)} - 1)$$

This implies that (10) holds for

$$f_W(\tau) = \frac{L_w \theta}{L_x}(\mathrm{e}^{L_x \tau} - 1) \qquad \square$$

The following proposition bounds the difference between the magnitudes of the Lyapunov function of two different states in Ω_ρ. This proposition is used together with Propositions 1 and 2 to obtain an upper bound on the value of the Lyapunov function of the real state if a given control input trajectory is applied.

Proposition 3
Consider the Lyapunov function $V(\cdot)$ of system (1). Given any positive constant $\rho>0$, there exists a quadratic function $f_V(\cdot)$ such that

$$V(x) \leqslant V(\hat{x}) + f_V(|x-\hat{x}|) \tag{11}$$

for all $x, \hat{x} \in \Omega_\rho$.

Proof
Because the Lyapunov function $V(x)$ is continuous and bounded on compact sets, we can find a positive constant β such that a Taylor series expansion of V around $\hat{x}$ yields

$$V(x) \leqslant V(\hat{x}) + \frac{\partial V}{\partial x}|x-\hat{x}| + \beta |x-\hat{x}|^2 \quad \forall x, \hat{x} \in \Omega_\rho$$

Note that the term $\beta|x-\hat{x}|^2$ bounds the high-order terms of the Taylor series of $V(x)$ for all $x, \hat{x} \in \Omega_\rho$. Taking into account (2), the following bound for $V(x)$ is obtained:

$$V(x) \leqslant V(\hat{x}) + \alpha_4(\alpha_1^{-1}(\rho))|x-\hat{x}| + \beta|x-\hat{x}|^2 \quad \forall x, \hat{x} \in \Omega_\rho$$

This implies that (11) holds for $f_V(x) = \alpha_4(\alpha_1^{-1}(\rho))x + \beta x^2$. □

Theorem 1 provides sufficient conditions under which the LMPC scheme (6) guarantees stability of the nonlinear closed-loop system in the presence of time-varying measurement delays.

Theorem 1
Consider system (1) in closed loop with the LMPC scheme (6) based on a controller $h(x)$ that satisfies (2). Let constants $\varepsilon_s, \rho_s > 0$, Δ, θ and D satisfy (7) and the following constraint:

$$-\varepsilon_s + f_V(f_W(D\Delta)) + f_V(f_W(D+1)\Delta) < 0 \tag{12}$$

If $D+1 \leqslant N$, $x(t_0) \in \Omega_\rho$ and $d(t_0) = 0$, then $x(t)$ is ultimately bounded in Ω_{ρ_c} where

$$\rho_c = \rho_{\min} + f_V(f_W(D\Delta)) + f_V(f_W(D+1)\Delta)$$

Proof
In order to prove that system (1) in closed loop with the proposed LMPC is ultimately bounded in a region that contains the origin, we will prove that the Lyapunov function $V(x)$ is a decreasing function of time with a lower bound on its magnitude. We assume that the delayed measurement $x(t_{k-d(t_k)})$ is received at time t_k and that a new measurement is not obtained until t_{k+j}. The optimization problem (6) is solved at t_k and the optimal input $u_k^*(t)$ is applied from t_k to t_{k+j}.

The trajectory $\hat{x}(t)$ corresponds to the nominal system in closed loop with the Lyapunov-based controller implemented in a sample-and-hold fashion with initial condition $\tilde{x}(t_k)$ (constraints (6e) and (6f)). Applying Proposition 1, we obtain the following inequality:

$$V(\hat{x}(t_{k+j})) \leqslant \max\{V(\hat{x}(t_k)) - j\varepsilon_s, \rho_{\min}\}$$

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The contractive constraint (6g) of the proposed LMPC guarantees that

$$V(\tilde{x}(\tau))\leqslant V(\hat{x}(\tau)) \quad \forall\tau\in[t_k, t_{k+D+1-d(t_k)}]$$

and constraint (6f) guarantees that $V(\hat{x}(t_k))=V(\tilde{x}(t_k))$. This implies that

$$V(\tilde{x}(t_{k+j}))\leqslant \max\{V(\tilde{x}(t_k))-j\varepsilon_s, \rho_{\min}\}$$

Assuming that $x(t)\in\Omega_\rho$ for all times, we can apply Proposition 3 to obtain the following inequalities:

$$V(\tilde{x}(t_k))\leqslant V(x(t_k))+f_V(|x(t_k)-\tilde{x}(t_k)|)$$

and

$$V(x(t_{k+j}))\leqslant V(\tilde{x}(t_{k+j}))+f_V(|x(t_{k+j})-\tilde{x}(t_{k+j})|)$$

This assumption is automatically satisfied if the system is proved to be ultimately bounded. Constraints (6b), (6d), (6c) and the implementation procedure allow us to apply Proposition 2 because it is guaranteed that the real state $x(t)$ and the state estimated using the nominal model $\tilde{x}(t)$ are obtained using the same input trajectory. Applying Proposition 2 we obtain the following upper bounds on the deviation of $\tilde{x}(t)$ from $x(t)$:

$$|x(t_k)-\tilde{x}(t_k)|\leqslant f_W(d(t_k)\Delta)$$

$$|x(t_{k+j})-\tilde{x}(t_{k+j})|\leqslant f_W((d(t_k)+j)\Delta)$$

Using these inequalities the following upper bound on $V(x(t_{k+j}))$ is obtained:

$$V(x(t_{k+j}))\leqslant \max\{V(x(t_k))-j\varepsilon_s, \rho_{\min}\}+f_V(f_W(d(t_k)\Delta))+f_V(f_W((d(t_k)+j)\Delta)) \tag{13}$$

In order to prove that for all possible sequences $d(t_k)$ the Lyapunov function is guaranteed to decrease between two consecutive new measurements until a lower bound is obtained, we will consider the worst-case scenario; that is, after a new measurement is obtained at sampling time t_k, the controller has to operate in open loop for the maximum possible time due to the time-varying delay. Taking into account that the maximum allowable delay is D, it holds that the maximum number of sampling times in which the system will operate in open loop is $D+1-d(t_k)$. This implies that the worst case is given for $j=D+1-d(t_k)$ and the following bound can be written:

$$\begin{aligned}V(x(t_{k+D+1-d(t_k)})) \leqslant{} & \max\{V(x(t_k))-(D+1-d(t_k))\varepsilon_s, \rho_{\min}\}\\ & + f_V(f_W(d(t_k)\Delta))+f_V(f_W((D+1)\Delta))\end{aligned}$$

In order to prove that the Lyapunov function is decreasing between two consecutive new measurements for the worst possible case the following inequality must hold:

$$(D+1-d(t_k))\varepsilon_s > f_V(f_W(d(t_k)\Delta))+f_V(f_W((D+1)\Delta))$$

for all $d(t_k)=0,\dots,D$. The worst possible case is $d(t_k)=D$. This implies that if condition (12) is satisfied, then for all $d(t_k)=0,1,\dots,D$ and all $j=1,\dots,D+1-d(t_k)$ (where j indicates when a new measurement is received after sampling time t_k) there exists $\varepsilon_w>0$ such that the following inequality holds

$$V(x(t_{k+j}))\leqslant \max\{V(x(t_k))-\varepsilon_w, \rho_c\} \tag{14}$$

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which implies that when $x(t_k)\in\Omega/\Omega_{\rho_c}^{\|}$, $V(x(t))$ will decrease until the state converges to Ω_{ρ_c} for all $t\in[t_k, t_{k+j}]$, and when $x(t_k)\in\Omega_{\rho_c}$, it remains inside Ω_{ρ_c} for all $t\in[t_k, t_{k+j}]$.

If $x(t_0)\in\Omega$ and it is known, using (14) recursively, it is proved that the closed-loop trajectories of system (1) subject to time-varying measurements delays satisfy

$$\limsup_{t\to\infty} V(x(t))\leqslant\rho_c$$

for all possible sequences $\{d(t_k)\}$. This proves that the closed-loop system is ultimately bounded in Ω_{ρ_c}. □

Remark 2

Theorem 1 is very important from an MPC point of view because if constraints (7) and (12) are satisfied and the initial condition is in the stability region, the state of system (1) is guaranteed to be inside the stability region, thereby yielding a feasible optimization problem for all times.

Remark 3

In this work, no input or state constraints have been considered but the proposed approach can be extended to handle these issues by modifying the Lyapunov-based controller and its closed-loop stability region to account for input constraints and by restricting the closed-loop stability region further to satisfy the state constraints.

Remark 4

It is also important to remark that when measurement delay is present in the control system, standard MPC formulations do not provide guaranteed closed-loop stability results unless the delay is infinitesimally small. For any MPC scheme, in order to obtain guaranteed closed-loop stability results even in the case where the initial feasibility of the optimization problem is given, the formulation of the optimization problem has to be modified accordingly to take into account measurement delay in an explicit way.

Remark 5

When time-varying measurement delays are not present and new measurements of $x(t)$ are fed into the controller every sampling time $t_k,\ k=0,1,2,\ldots$, the LMPC scheme (6) may be simplified to the following form [4–6]:

$$u_k^*(t)=\arg\min_{u_k\in S(\Delta)} \int_{t_k}^{t_{k+N}} [\tilde{x}(\tau)^{\mathrm{T}} Q_c \tilde{x}(\tau)+u_k(\tau)^{\mathrm{T}} R_c u_k(\tau)]\,\mathrm{d}\tau \tag{15a}$$

$$\text{s.t.}\quad \dot{\tilde{x}}(t)=f(\tilde{x}(t),u_k(t),0) \tag{15b}$$

$$\tilde{x}(t_k)=x(t_k) \tag{15c}$$

$$\dot{\hat{x}}(t)=f(\hat{x}(t),h(\hat{x}(t_k)),0),\quad t\in[t_k,t_{k+1}) \tag{15d}$$

$$\hat{x}(t_{k)})=\tilde{x}(t_k) \tag{15e}$$

$$V(\tilde{x}(t))\leqslant V(\hat{x}(t))\quad \forall t\in[t_k,t_{k+1}] \tag{15f}$$

‖We use the operator '/' to denote set substraction, i.e. $A/B:=\{x\in R^{n_x}\,|\,x\in A, x\notin B\}$.

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Comparing the above scheme with the one of (6), the difference is that the constraint (6g) has to hold only for t_{k+1}. This implies that even if the same implementation procedure is used, and the same optimization problem is solved (in order to estimate the current state), if the contractive constraint is not changed, stability cannot be proved. This point will be illustrated in the example in the following section.

Remark 6
When asynchronous measurements are fed into the control system, the following LMPC scheme was proposed in [7]:

$$u_k^*(t) = \arg \min_{u_k \in S(\Delta)} \int_{t_k}^{t_{k+N}} [\tilde{x}(\tau)^{\mathrm{T}} Q_c \tilde{x}(\tau) + u_k(\tau)^{\mathrm{T}} R_c u_k(\tau)]\,\mathrm{d}\tau \tag{16a}$$

$$\text{s.t.} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u_k(t), 0) \tag{16b}$$

$$\tilde{x}(t_k) = x(t_k) \tag{16c}$$

$$\dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_j)), 0), \quad t \in [t_j, t_{j+1}), \quad j = k, \ldots, k+N-1 \tag{16d}$$

$$\hat{x}(t_{k)}) = \tilde{x}(t_k) \tag{16e}$$

$$V(\tilde{x}(t)) \leqslant V(\hat{x}(t)) \quad \forall t \in [t_k, t_{k+N}] \tag{16f}$$

where the contractive constraint has to hold for the entire prediction horizon, which is equal or bigger than the maximum time without new measurement. This constraint makes the computed control action more conservative (and thus less optimal) because the controller may have to satisfy the contractive constraint over unnecessarily large horizons. If the controller (16) is implemented using the proposed procedure, it will be, in general, less optimal than (6). This point will also be illustrated in the example in the following section.

4. APPLICATION TO A CHEMICAL REACTOR

4.1. Process description and modeling

Consider a well mixed, non-isothermal continuous stirred tank reactor where three parallel irreversible elementary exothermic reactions take place of the form $A \to B$, $A \to C$ and $A \to D$. B is the desired product and C and D are byproducts. The feed to the reactor consists of pure A at flow rate F, temperature T_{A0} and molar concentration $C_{A0} + \Delta C_{A0}$, where ΔC_{A0} is an unknown time-varying uncertainty. Owing to the non-isothermal nature of the reactor, a jacket is used to remove/provide heat to the reactor. Using first principles and standard modeling assumptions, the following mathematical model of the process is obtained [27]:

$$\begin{aligned} \frac{\mathrm{d}T}{\mathrm{d}t} &= \frac{F}{V_r}(T_{A0} - T) - \sum_{i=1}^{3} \frac{\Delta H_i}{\sigma c_p} k_{i0} \mathrm{e}^{-E_i/RT} C_A + \frac{Q}{\sigma c_p V_r} \\ \frac{\mathrm{d}C_A}{\mathrm{d}t} &= \frac{F}{V_r}(C_{A0} + \Delta C_{A0} - C_A) + \sum_{i=1}^{3} k_{i0} \mathrm{e}^{-E_i/RT} C_A \end{aligned} \tag{17}$$

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Table I. Process parameters.

F	4.998 (m^3/h)	k_{10}	$3*10^6$ (h^{-1})
V_r	1 (m^3)	k_{20}	$3*10^5$ (h^{-1})
R	8.314 (kJ/kmol K)	k_{30}	$3*10^5$ (h^{-1})
T_{A0}	300 (K)	E_1	$5*10^4$ (kJ/kmol)
C_{A0}	4 (kmol/m^3)	E_2	$7.53*10^4$ (kJ/kmol)
ΔH_1	$-5.0*10^4$ (kJ/kmol)	E_3	$7.53*10^4$ (kJ/kmol)
ΔH_2	$-5.2*10^4$ (kJ/kmol)	σ	1000 (kg/m^3)
ΔH_3	$-5.4*10^4$ (kJ/kmol)	c_p	0.231 (kJ/kg K)

where C_A denotes the concentration of the reactant A, T denotes the temperature of the reactor, Q denotes the rate of heat input/removal, V_r denotes the volume of the reactor, $\Delta H_i, k_{i0}, E_i, i=1,2,3$ denote the enthalpies, pre-exponential constants and activation energies of the three reactions, respectively, and c_p and σ denote the heat capacity and the density of the fluid in the reactor. The values of the process parameters are shown in Table I.

System (17) has three steady states (two locally asymptotically stable and one unstable). The control objective is to stabilize the system at the open-loop unstable steady state $T_s = 388\,\text{K}$, $C_{As} = 3.59\,\text{mol/l}$. The manipulated input is the rate of heat input Q and the allowable input is bounded by $|Q| \leqslant 10^5\,\text{kJ/h}$. We consider a time-varying uncertainty in the concentration of the inflow $|\Delta C_{A0}| \leqslant 0.2\,\text{mol/l}$. The control system is subject to time-varying measurement delay in the measurements of the concentration of the reactant, C_A, and in the measurements of the temperature, T. Note that we do not consider the possible different sampling rates of temperature and concentration sensors in this work. Note also that the delay in the measurements could be regarded as the total time needed for online sensors to get a sample, analyze the sample and transmit the data to the controller.

4.2. Lyapunov-based controller design

To illustrate the theoretical results, we first design a Lyapunov-based feedback law using the method presented in [18]. System (17) belongs to the following class of nonlinear systems:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + w(x(t))\theta(t)$$

where $x^{\mathrm{T}} = [T - T_s \, C_A - C_{As}]$ is the state, $u = Q$ is the input and $\theta = \Delta C_{A0}$ is a time-varying bounded disturbance. Consider the CLF $V(x) = x^{\mathrm{T}} P x$ with

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 10^4 \end{bmatrix}$$

The values of the weights have been chosen to account for the different range of numerical values for each state. The following feedback law [18] asymptotically stabilizes the open-loop unstable

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steady state of the nominal system (i.e. $\theta(t)=0$) and is of the form (2):

$$h(x)=\begin{cases} -\dfrac{L_f V+\sqrt{(L_f V)^2+(L g V)^4}}{L_g V} & \text{if } L_g V\neq 0 \\ 0 & \text{if } L_g V=0 \end{cases} \tag{18}$$

where $L_f V=(\partial V(x)/\partial x) f$ and $L_g V=(\partial V(x)/\partial x) g$ denote the Lie derivatives of the scalar function V with respect to the vectors fields f and g, respectively. This controller will be used to design the LMPC controller. The stability region Ω_ρ is defined as $V(x)\leqslant 700$, i.e. $\rho=700$.

4.3. LMPC design

In order to choose an appropriate sampling time, we resort to extensive off-line closed-loop simulations under the Lyapunov-based controller of (18). After trying different sampling times, we choose $\Delta=0.025$ h. For this sampling time, the closed-loop system without measurement delay under $u=h(x)$ is practically stable and the performance is similar to the closed-loop system with continuous measurements. For this sampling time, we chose the maximum allowable measurement delay equal to 6Δ (i.e. $D=6$). The cost function is defined by the weight matrices $Q_c=P$ and $R_c=10^{-6}$. The values of the weights have been tuned in such a way that the values of the control inputs are comparable to the ones computed by the Lyapunov-based controller.

We will first compare the proposed LMPC scheme (6) with the original LMPC scheme (15) in the case where no time-varying measurement delays are present. For this simulation, we choose the prediction horizon of the two LMPC controllers N equal to 7 ($N\geqslant D+1$). The same weights, sampling time and prediction horizon are used. We implement the original LMPC scheme using the same approach employed in the implementation of the proposed LMPC scheme, that is, the current state is estimated using the nominal model of system (17) when a delayed measurement is received and the last optimal input is applied when no new measurement is received. In Figure 3, the trajectories of system (17) under both controllers are shown assuming no measurement delay is present, that is, the state $x(t_k)$ is available every sampling time. It can be seen that both closed-loop systems are practically stable and the trajectories remain in the stability region Ω_ρ.

In order to simulate the process in the presence of measurement delay, we use a random process to generate the delay sequence $d(t_k)$, and the delay sequence $d(t_k)$ in which the control system is subjected to is shown in Figure 4. In this figure, we see the time-varying nature of the measurement delays and the largest delays are equal to the maximum allowable delay $D=6$. Note that when $d(t_{k+1})=d(t_k)+1$, the controller does not receive any new measurement.

When time-varying measurement delays are present, the proposed LMPC is more robust. The stability region is invariant for the closed-loop system if $D+1\leqslant N$. This is not the case with the original LMPC scheme (15). In Figure 5, the trajectories of the closed-loop system under both controllers are shown in the presence of measurement delay with $D=6$. It can be seen that the original LMPC controller cannot stabilize the system at the desired open-loop unstable steady state and the trajectories leave the stability region, while the proposed LMPC scheme keeps the trajectories inside the stability region. When measurement delay is present, in order to provide stability guarantees, the constraints must be modified to take into account measurement delay as in the proposed LMPC controller (6).

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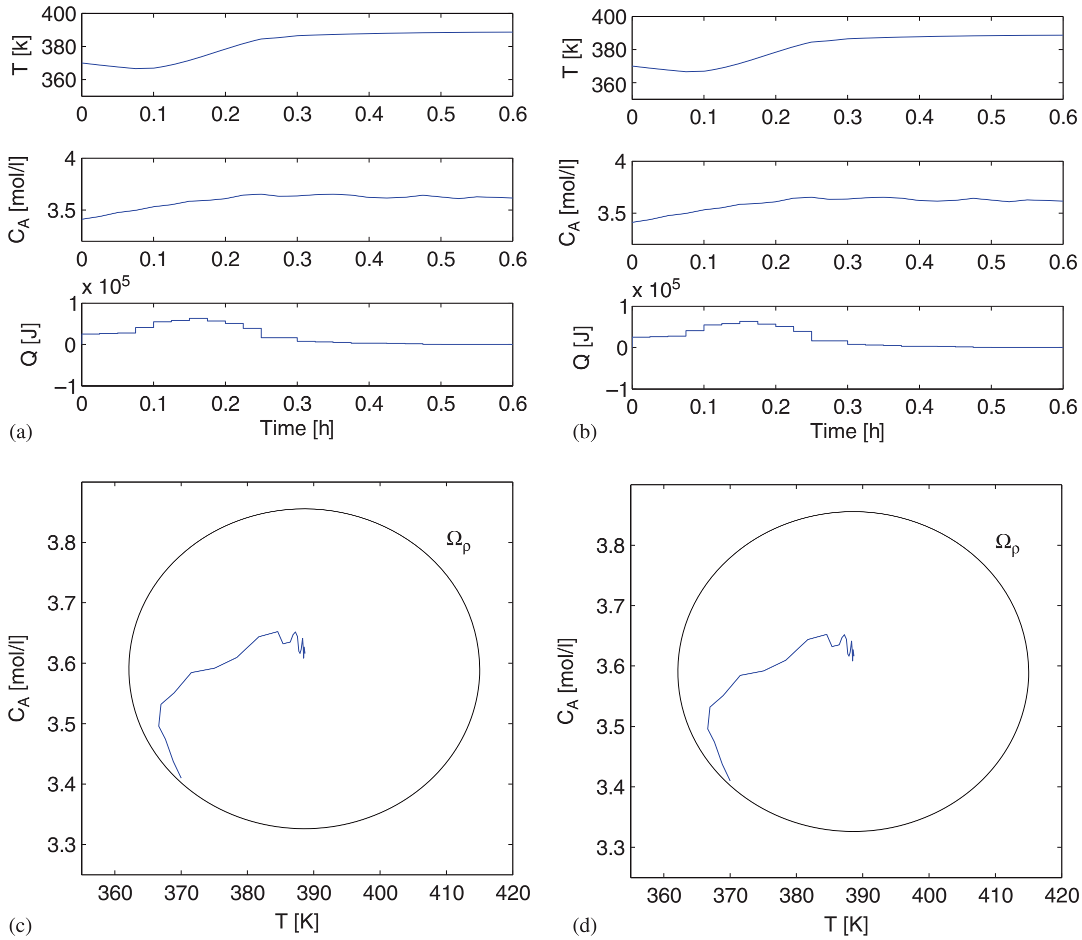

Figure 3. (a) (c) Trajectories of system (17) with the proposed LMPC scheme (6) when no measurement delay is present. (b) (d) Trajectories of system (17) with the original LMPC scheme (15) when no measurement delay is present.

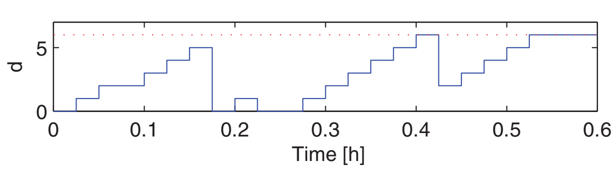

Figure 4. Delay sequence $d(t_k)$ used in the simulation shown in Figure 5.

4.4. Performance comparison

We have also carried out a set of simulations to compare the proposed LMPC scheme with the LMPC scheme (16) for nonlinear systems subject to data losses from a performance index point

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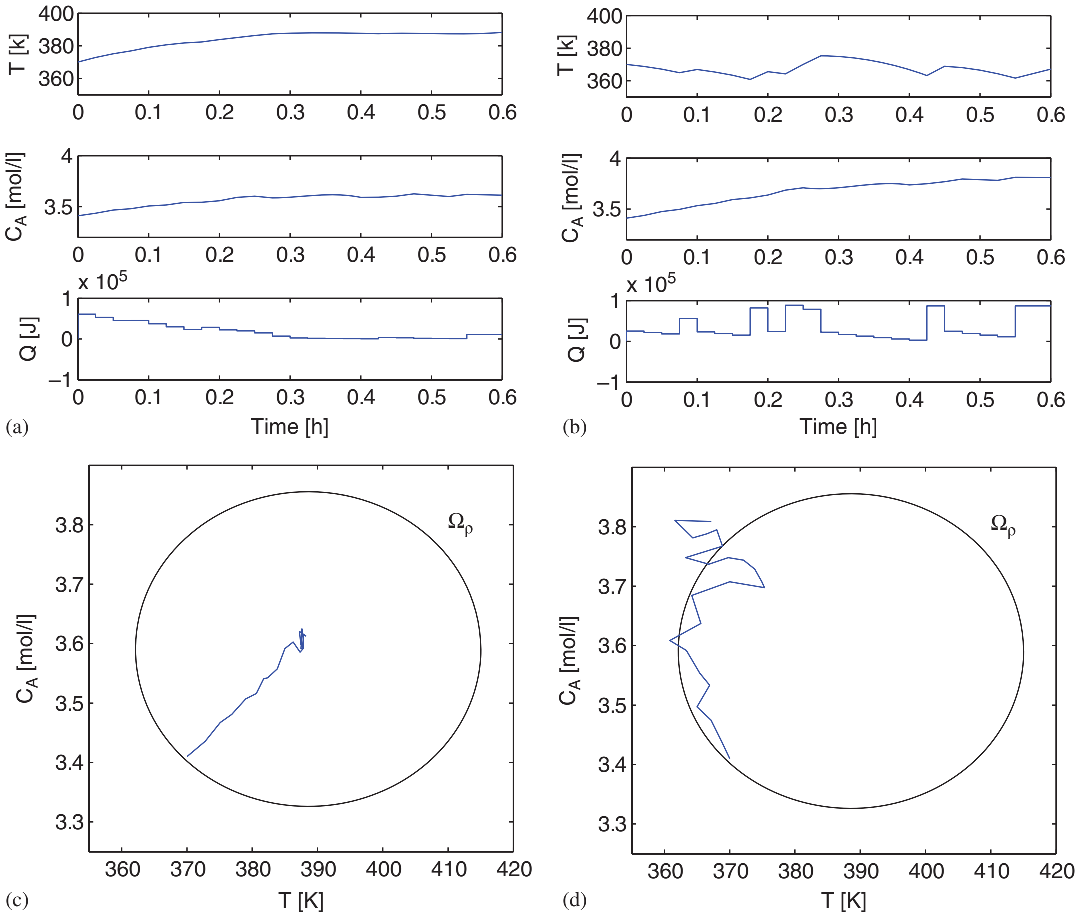

Figure 5. (a) (c) Trajectories of system (17) with the proposed LMPC scheme (6) when the maximum allowable measurement delay D is 6. (b) (d) Trajectories of system (17) with the original LMPC scheme (15) when the maximum allowable measurement delay D is 6.

of view. We also implement the LMPC scheme (16) using the same approach employed in the implementation of the proposed LMPC scheme. Table II shows the total cost computed for 20 different closed-loop simulations under the proposed LMPC and the LMPC for systems subject to data losses. To carry out this comparison, we have computed the total cost of each simulation based on the following performance index

$$\sum_{i=0}^{M} x(t_i)^{\mathrm{T}} Q_c x(t_i) + u(t_i)^{\mathrm{T}} R_c u(t_i)$$

where t_0 is the initial time of the simulations and $t_M = 2$ h is the final simulation time. The prediction horizon in this set of simulations is $N = 10$. For each pair of simulations (one for each controller) a different initial state inside the stability region, a different uncertainty trajectory and a different random measurement delay sequence are chosen. As it can be seen in Table II, the proposed

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Table II. Performance costs along the closed-loop trajectories.

sim.	Proposed LMPC	LMPC for data losses
1	1.8295×10^4	2.4428×10^4
2	4.2057×10^4	6.0522×10^4
3	3.2481×10^3	1.0428×10^4
4	7.4328×10^2	7.3961×10^2
5	1.4229×10^3	2.7798×10^5
6	4.9435×10^4	6.1596×10^4
7	3.2519×10^4	3.4319×10^4
8	2.7590×10^4	4.7075×10^4
9	9.4216×10^2	9.4866×10^2
10	5.4505×10^2	5.4322×10^2
11	1.9723×10^4	3.1282×10^4
12	2.7235×10^4	3.8772×10^4
13	1.8671×10^3	1.9200×10^3
14	3.7789×10^4	4.0050×10^4
15	2.1839×10^3	2.1392×10^3
16	4.2920×10^4	4.4594×10^4
17	1.5153×10^2	1.7190×10^2
18	4.9955×10^3	9.9094×10^3
19	3.2086×10^4	4.8838×10^4
20	1.5420×10^3	1.5197×10^3

LMPC controller has a cost lower than the corresponding total cost under the LMPC controller designed for system subject to data losses in 16 out of 20 simulations (see also Remark 6). This illustrates that the proposed LMPC controller is, in general, more optimal. This is because the LMPC controller designed for system subject to data losses requires the contractive constraint (6g) to be satisfied along the whole prediction horizon (that is, $t\in[t_k, t_{k+N}]$), which yields a more conservative controller from a performance point of view.

4.5. *Effects of the maximum delay D*

We have also carried out a set of simulations to study the dependence on the value of the maximum delay D of the set in which the trajectory of system (17) under the proposed LMPC scheme is ultimately bounded. In order to estimate the size of each set for a given D, we start the system very close to the equilibrium state and run it for a sufficiently long time. In this set of simulations, we set $\Delta C_{A0}=0.1\,\text{kmol/m}^3$ and $N=7$. The simulation time is 25 h. Figure 6 shows the location of the states, (C_A, T), at each sampling time and the estimated regions for $D=2,4,6$. Three ellipses are used to estimate the boundaries of the sets, and they are chosen to be as small as possible but still include all the corresponding points indicating the states. From Figure 6, we see that the size of these sets becomes larger as D increases. The results are expected because the size of the sets is not only dependent on the system and the controller, but it also depends on the maximum measurement delay. The longer the size of the delay, the further the system can move away from the steady state, which means a larger set (if the state is still in the stability region Ω_ρ). Note that

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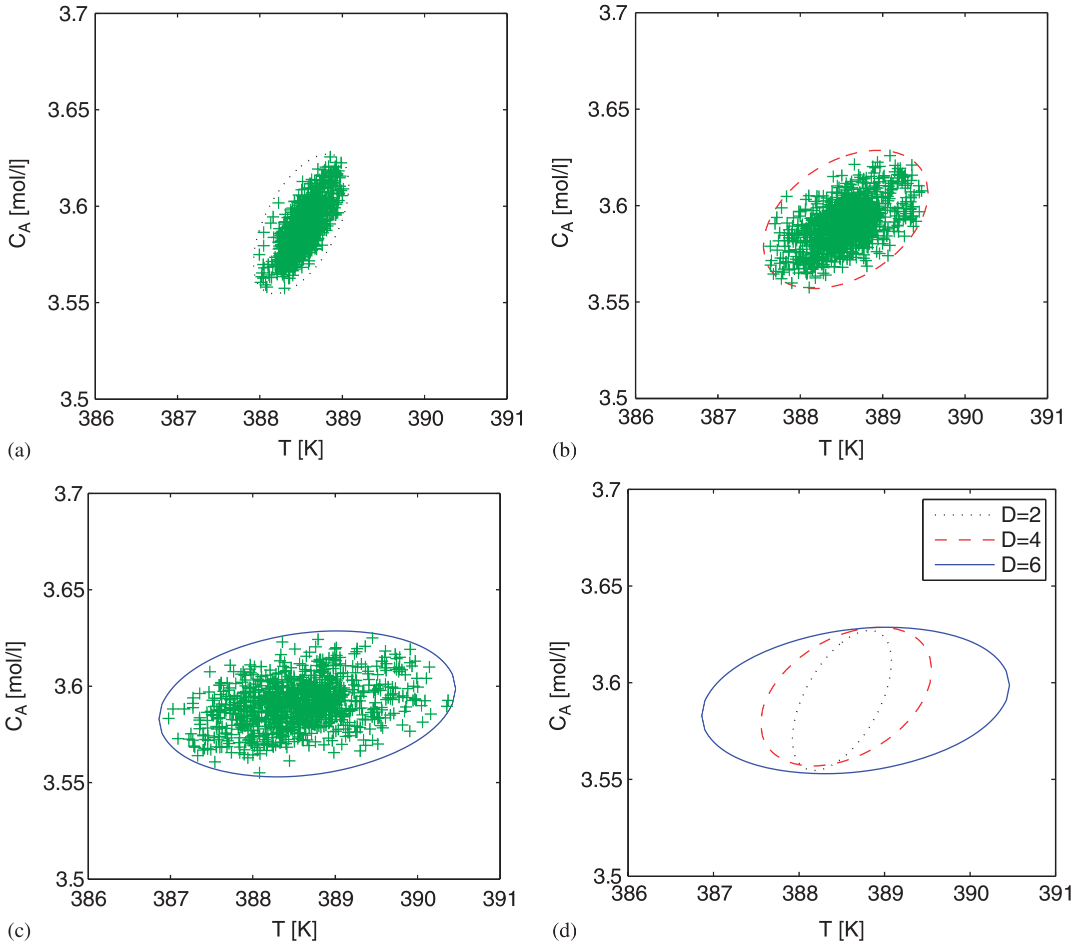

Figure 6. (a) Estimate of the set in which the trajectories of the system (17) with the proposed LMPC scheme are ultimately bounded when the maximum allowable measurement delay D is 2; (b) estimate of the set in which the trajectories of the system (17) with the proposed LMPC scheme are ultimately bounded when the maximum allowable measurement delay D is 4; (c) estimate of the set in which the trajectories of the system (17) with the proposed LMPC scheme are ultimately bounded when the maximum allowable measurement delay D is 6; and (d) comparison of the three sets.

all the sets for $D=2,4,6$ are included in the stability region of the closed-loop system under the proposed LMPC (Ω_ρ, $\rho=700$).

5. CONCLUSION

In this work, a Lyapunov-based model predictive controller was proposed for control of a broad class of nonlinear uncertain systems in the presence of measurement delay. The main idea is that in order to provide guaranteed stability results in the presence of time-varying measurement delays,

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the constraints that define the LMPC optimization problem as well as the implementation procedure have to be modified to account for measurement delay. The proposed LMPC controller allows for an explicit characterization of the stability region, guarantees practical stability in the presence of measurement delay and guarantees that the stability region is an invariant set if the maximum delay is shorter than a constant that depends on the parameters of the system and the controller used to design the LMPC. The application of the proposed LMPC method was illustrated using a nonlinear chemical process example with asynchronous, delayed measurements and its stability and performance properties were found to be superior to the ones of two existing LMPC algorithms.

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