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Event-triggered filtering and intermittent fault detection for time-varying systems with stochastic parameter uncertainty and sensor saturation

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Summary
In this paper, the event-triggered filtering and intermittent fault detection problems are investigated for a class of time-varying systems with stochastic parameter uncertainty and sensor saturation. Due to the existence of event-triggered mechanism, the measured signal could be transmitted only when it satisfies the triggering condition. An event-triggered filter is developed, which takes the event-triggered mechanism, parameter uncertainty, and sensor saturation into full consideration but does not depend on any specific uncertainty structure. By utilizing the inductive and stochastic analysis technique, the filter gain is designed to ensure that the upper bound of the estimation error covariance is minimized at each time step. Based on the proposed filter, a residual is generated and the corresponding evaluation function and detection threshold are given to achieve fault detection. At last, two simulation studies are carried out to demonstrate the effectiveness and applicability of the proposed method.

KEYWORDS
event-triggered filtering, fault detection, parameter uncertainty, sensor saturation, stochastic time-varying systems

1 | INTRODUCTION

With the increasing complexity and higher safety demand of modern engineering systems, the filtering and fault detection problems have attracted tremendous attention from both the academia and the industry.[1-4] The basic idea of filtering is to estimate the system state based on the information of sensor measurement and system model. The basic idea of fault detection is to construct a residual based on the system model and then evaluate it to determine whether there is a fault in the system. The model uncertainty and time delay[5,6] are inevitable in practice, considerable research efforts have been devoted to studying robust filtering and fault detection problems.[7-14]

Meanwhile, the event-triggered mechanism has received enhanced research interest due to the rapid development of network and communication technology in recent years. For the conventional periodic mechanism, the signal will be transmitted at each sampling time. However, for the event-triggered mechanism, the signal will be transmitted only when the triggering condition is satisfied. It has the advantage of reduced signal transmission, which could thus save energy and extend service life. This has aroused much research attention on event-triggered filtering and fault detection recently.[15-23]

On the other hand, the sensor saturation phenomenon is very common in real-world situations due to physical and technological limitations. Therefore, some research efforts have been devoted to filtering and fault detection problems with sensor saturation.[24-30] In the available literature, the saturation is usually treated as sector-bounded nonlinearity. However, the sensor saturation sometimes may occur in a stochastic way due to environmental changes, sensor failures, and so on.[31-33] As a result, it is of vital importance to investigate the filtering and fault detection problems with stochastic sensor saturation.

Summarizing the aforementioned discussion, it can be concluded that, although the filtering and fault detection problems have attracted much research interest, the common event-triggered filtering and intermittent fault detection problems for time-varying systems with stochastic uncertainty and sensor saturation have not been investigated. It may be due to difficulties in dealing with parameter uncertainty and imperfect measurement simultaneously. Besides, most available methods usually require to assume that the parameter uncertainty has a specific form of matrix structure and both the structure and structural parameters of it are also required to be known beforehand. Unfortunately, it sometimes may be impossible to obtain these information and there still lacks of a different method based on statistical information rather than structural information.

Motivated by these considerations, this work aims to develop a novel event-triggered filtering and fault detection method to solve the aforementioned problems. The main contributions of this paper can be summarized as follows. *(1) A comprehensive model is established, which covers event-triggered transmission, stochastic parameter uncertainty, and sensor saturation. (2) A novel event-triggered filter is proposed, whose upper bound estimation error covariance is obtained and minimized. (3) A recursive online fault detection strategy is presented.* The rest of this paper is organized as follows. In Section 2, the event-triggered filtering and fault detection problems are formulated with some assumptions. In Section 3, the event-triggered filtering method is proposed and applied to the fault detection. Simulation results are provided and discussed in Section 4. In the end, some concluding remarks are given in Section 5.

Notation. Except where otherwise stated, the notations of this paper are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $\mathbf{I}_{n\times n}$ is the identity matrix with n rows and n columns (1 at the (i,i)th entry and 0 elsewhere). $\mathcal{S}(m,n)$ represents the set $\{m, m+1, \dots, n\}, (m \in \mathbb{Z}, n \in \mathbb{Z}, m \le n)$. Given a matrix $\mathbf{X} = \left[x_{ij}\right] \in \mathbb{R}^{n\times m}$, $\text{row}\{\mathbf{X}, i\}$ denotes the vector $\left[x_{i1} \;\; x_{i2} \;\; \cdots \;\; x_{im}\right]^{\mathrm{T}}$. Given a set $\mathcal{V} = \{n_1, n_2, \dots, n_m\}, (n_1 \le n_2 \le \cdots \le n_m)$, $\text{col}_{i\in\mathcal{V}}\{\mathbf{X}_i\}$ and $\text{diag}_{i\in\mathcal{V}}\{\mathbf{A}_i\}$ represent $\left[\mathbf{X}_{n_1}{}^{\mathrm{T}} \;\; \mathbf{X}_{n_2}{}^{\mathrm{T}} \;\; \cdots \;\; \mathbf{X}_{n_m}{}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\text{diag}\left\{\mathbf{A}_{n_1}, \mathbf{A}_{n_2}, \dots, \mathbf{A}_{n_m}\right\}$, respectively. $\mathbb{E}\{\mathbf{X}\}$ is the expectation of a stochastic variable $\mathbf{X}$. $\text{p}_{\text{row}}(\mathbf{X})$, $\boldsymbol{\mu}_{\mathbf{X}}$, and $\boldsymbol{\Sigma}_{\mathbf{X}}$ denote $\text{col}_{i\in\mathcal{S}(1,n)}\{\text{row}\{\mathbf{X}, i\}\}$, $\mathbb{E}\{\mathbf{X}\}$, and $\mathbb{E}\left\{\text{p}_{\text{row}}(\mathbf{X})\,\text{p}_{\text{row}}(\mathbf{X})^{\mathrm{T}}\right\}$, respectively. If the dimensions of matrices are not explicitly stated, they are assumed to be compatible for algebraic operations.

2 | PROBLEM FORMULATION

Consider a class of stochastic time-varying systems described by the following state-space equations:

$$\mathbf{x}(k+1) = (\mathbf{A}_c(k) + \mathbf{A}_\delta(k))\,\mathbf{x}(k) + (\mathbf{B}_c(k) + \mathbf{B}_\delta(k))\,\mathbf{u}(k) + \mathbf{w}(k), \tag{1}$$

$$\mathbf{y}(k) = \left(\mathbf{I}_{n_{\mathrm{y}}\times n_{\mathrm{y}}} - \boldsymbol{\Lambda}(k)\right)(\mathbf{C}_c(k) + \mathbf{C}_\delta(k))\,\mathbf{x}(k) + \boldsymbol{\Lambda}(k)\,\mathrm{g}\left((\mathbf{C}_c(k) + \mathbf{C}_\delta(k))\,\mathbf{x}(k)\right) + \mathbf{f}(k) + \mathbf{v}(k), \tag{2}$$

where $\mathbf{x}(k) \in \mathbb{R}^{n_{\mathrm{x}}}$ is the system state, $\mathbf{u}(k) \in \mathbb{R}^{n_{\mathrm{u}}}$ is the control input, and $\mathbf{y}(k) \in \mathbb{R}^{n_{\mathrm{y}}}$ is the measurement output. $\mathbf{w}(k) \in \mathbb{R}^{n_{\mathrm{x}}}$ is the process noise, and $\mathbf{v}(k) \in \mathbb{R}^{n_{\mathrm{y}}}$ is the measurement noise. $\mathbf{A}_c(k) \in \mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{x}}}$, $\mathbf{B}_c(k) \in \mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{u}}}$, and $\mathbf{C}_c(k) \in \mathbb{R}^{n_{\mathrm{y}}\times n_{\mathrm{x}}}$ are known deterministic parameters, and $\mathbf{A}_\delta(k) \in \mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{x}}}$, $\mathbf{B}_\delta(k) \in \mathbb{R}^{n_{\mathrm{x}}\times n_{\mathrm{u}}}$, and $\mathbf{C}_\delta(k) \in \mathbb{R}^{n_{\mathrm{y}}\times n_{\mathrm{x}}}$ are unknown stochastic parameter uncertainties. $\boldsymbol{\Lambda}(k) \in \mathbb{R}^{n_{\mathrm{y}}\times n_{\mathrm{y}}}$ is the saturation coefficient, and $\mathrm{g}(\cdot) : \mathbb{R}^{n_{\mathrm{y}}} \to \mathbb{R}^{n_{\mathrm{y}}}$ is the saturation function. $\mathbf{f}(k) \in \mathbb{R}^{n_{\mathrm{y}}}$ is the sensor fault. The aforementioned system is called nominal system if the fault is identically equal to zero. Otherwise, it is called faulty system.

The saturation coefficient $\boldsymbol{\Lambda}(k) \in \mathbb{R}^{n_{\mathrm{y}}\times n_{\mathrm{y}}}$ is a diagonal matrix whose ith entry $\lambda_i(k), i \in \mathcal{S}\left(1, n_{\mathrm{y}}\right)$ is the Bernoulli white process taking the value 1 or 0 with

$$\begin{cases} \text{Prob}\{\lambda_i(k) = 1\} = \lambda_{c,i}(k), \\ \text{Prob}\{\lambda_i(k) = 0\} = 1 - \lambda_{c,i}(k). \end{cases} \tag{3}$$

Denoting $\Lambda_c(k) = \text{diag}_{i\in\mathcal{S}(1,n_y)}\left\{\lambda_{c,i}(k)\right\}$, then it follows immediately that $\mathbb{E}\left\{\Lambda(k)\right\} = \Lambda_c(k)$ and $\Sigma_{\Lambda(k)} = \Lambda_c(k) \otimes \mathbf{I}_{n_y\times n_y}$, where $\otimes$ is the Kronecker product. The saturation coefficient describes the phenomenon of stochastic sensor saturation.

Given a vector $\mathbf{q} = \text{col}_{i\in\mathcal{S}(1,n_y)}\left\{q_i\right\}$, the saturation function $\text{g}(\cdot) : \mathbb{R}^{n_y} \to \mathbb{R}^{n_y}$ is defined as

$$\text{g}(\mathbf{q}) = \left[\text{g}_1(q_1) \quad \text{g}_2(q_2) \quad \cdots \quad \text{g}_{n_y}(q_{n_y})\right]^{\text{T}}, \tag{4}$$

with $\text{g}_i(q_i) = \text{sgn}(q_i)\min(s_i, \|q_i\|_2)$, where $\text{sgn}(\cdot) : \mathbb{R}^1 \to \mathbb{R}^1$ is the signum function and $\mathbf{s} = \text{col}_{i\in\mathcal{S}(1,n_y)}\left\{s_i\right\}$ is the saturation level.

In this paper, the send-on-delta transmission strategy is considered. The current ith measurement $y_i(k)$ will be transmitted as long as it satisfies the following triggering condition:

$$\|y_i(k) - y_i(k-l)\|_2 > \delta_i, \tag{5}$$

where $y_i(k-l)$ is the previously transmitted ith measurement and $\boldsymbol{\delta} = \text{col}_{i\in\mathcal{S}(1,n_y)}\left\{\delta_i\right\}$ is the triggering threshold. Let $k_{t,1}, k_{t,2}, \ldots$ denote the triggering sequence, then the available measurement signal can be written as $\mathbf{y}_t(k) = \text{col}_{i\in\mathcal{S}(1,n_y)}\{y_{t,i}(k)\}$, whose ith entry satisfies $y_{t,i}(k) = y_i(k_{t,l}), k \in \mathcal{S}(k_{t,l}, k_{t,l+1}-1)$.

The matrices are independent of each other if and only if their entries are independent of each other. The following assumptions are made throughout this paper, which give the statistical information of uncertainties.

Assumption 1. The initial state $\mathbf{x}(0)$ has the mean $\boldsymbol{\mu}_{\mathbf{x}(0)}$, covariance $\Sigma_{(\mathbf{x}(0)-\mu_{\mathbf{x}(0)})}$, and second moment $\Sigma_{\mathbf{x}(0)}$. The process noise $\mathbf{w}(k)$ and measurement noise $\mathbf{v}(k)$ are zero-mean white processes with covariances $\Sigma_{\mathbf{w}(k)}$ and $\Sigma_{\mathbf{v}(k)}$.

Assumption 2. The parameter uncertainties $\mathbf{A}_\delta(k), \mathbf{B}_\delta(k), \mathbf{C}_\delta(k)$ are zero-mean white processes with

$$\begin{aligned}
\mathbb{E}\left\{\text{p}_{\text{row}}\left(\mathbf{A}_\delta(k)\right)\text{p}_{\text{row}}\left(\mathbf{A}_\delta(k)\right)^{\text{T}}\right\} &= \Sigma_{\mathbf{A}_\delta(k)},\\
\mathbb{E}\left\{\text{p}_{\text{row}}\left(\mathbf{B}_\delta(k)\right)\text{p}_{\text{row}}\left(\mathbf{B}_\delta(k)\right)^{\text{T}}\right\} &= \Sigma_{\mathbf{B}_\delta(k)},\\
\mathbb{E}\left\{\text{p}_{\text{row}}\left(\mathbf{C}_\delta(k)\right)\text{p}_{\text{row}}\left(\mathbf{C}_\delta(k)\right)^{\text{T}}\right\} &= \Sigma_{\mathbf{C}_\delta(k)}.
\end{aligned} \tag{6}$$

Besides, the initial state, process noise, measurement noise, and parameter uncertainties are mutually independent of each other.

The main purpose of this paper is to develop a filter to estimate the system state and utilize it to detect the fault in the time-varying system. It is worth mentioning that our proposed method takes event-triggered transmission, parameter uncertainty, and sensor saturation into full consideration, thus can achieve favorable performance.

3 | MAIN RESULTS

Before proceeding further, we firstly introduce a lemma that will be utilized in the subsequent analysis.

Lemma 1. (See the work of Hu et al[34])
Given any two matrices $\mathbf{X} \in \mathbb{R}^{n\times m}$ *and* $\mathbf{Y} \in \mathbb{R}^{n\times m}$ *and positive scalar* α*, the following inequality holds:*

$$\mathbf{X}\mathbf{Y}^{\text{T}} + \mathbf{Y}\mathbf{X}^{\text{T}} \le \alpha\mathbf{X}\mathbf{X}^{\text{T}} + \alpha^{-1}\mathbf{Y}\mathbf{Y}^{\text{T}}. \tag{7}$$

The recursive structure of our proposed filter is as follows:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}_c(k-1)\hat{\mathbf{x}}(k-1) + \mathbf{B}_c(k-1)\mathbf{u}(k-1), \tag{8}$$

$$\mathbf{r}(k|k-1) = \mathbf{y}_t(k) - \Lambda_c(k)\text{g}\left(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)\right) - \left(\mathbf{I}_{n_y\times n_y} - \Lambda_c(k)\right)\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1), \tag{9}$$

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\mathbf{r}(k|k-1), \tag{10}$$

where $\hat{\mathbf{x}}(k|k-1)$ is the predicted state estimate, $\mathbf{r}(k|k-1)$ is the innovation, $\hat{\mathbf{x}}(k)$ is the state estimate, and $\mathbf{K}(k)$ is the filter gain to be designed. It is worth mentioning that the dimensions of the fault, output, and innovation are all the same. Let $\tilde{\mathbf{x}}(k|k-1) = \mathbf{x}(k) - \hat{\mathbf{x}}(k|k-1)$ be the predicted estimation error, and $\tilde{\mathbf{x}}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$ be the estimation error. Then, the corresponding predicted estimation error covariance $\Sigma_{\tilde{\mathbf{x}}(k|k-1)} = \mathbb{E}\{\tilde{\mathbf{x}}(k|k-1)\tilde{\mathbf{x}}(k|k-1)^{\text{T}}\}$ and estimation error covariance $\Sigma_{\tilde{\mathbf{x}}(k)} = \mathbb{E}\{\tilde{\mathbf{x}}(k)\tilde{\mathbf{x}}(k)^{\text{T}}\}$ can be recursively calculated by the following theorem.

Theorem 1. *Consider the nominal system (1)-(2) with filter (8)-(10), the predicted estimation error covariance is*

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k|k-1)} = \mathbf{A}_c(k-1)\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k-1)}\mathbf{A}_c(k-1)^{\mathrm{T}} + \boldsymbol{\Sigma}_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} + \boldsymbol{\Sigma}_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} + \boldsymbol{\Sigma}_{\mathbf{w}(k-1)}, \tag{11}$$

and the estimation error covariance is

$$\begin{aligned}
\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k)} = {} & \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k|k-1)} \times \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}} \\
& + \mathbf{K}(k)\left(\boldsymbol{\Sigma}_{(\mathbf{y}_t(k)-\mathbf{y}(k))} + \boldsymbol{\Sigma}_{\mathbf{C}_\delta(k)\mathbf{x}(k)} + \boldsymbol{\Sigma}_{\mathbf{v}(k)}\right)\mathbf{K}(k)^{\mathrm{T}} + \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\Sigma}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\Sigma}_{\boldsymbol{\Lambda}(k)(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\mathbf{K}(k)^{\mathrm{T}} \\
& - \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\boldsymbol{\mu}_{\tilde{\mathbf{x}}(k|k-1)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} \\
& - \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\tilde{\mathbf{x}}(k|k-1)^{\mathrm{T}}}\left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}} \\
& - \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right) \\
& \times \boldsymbol{\mu}_{\tilde{\mathbf{x}}(k|k-1)(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} - \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))\tilde{\mathbf{x}}(k|k-1)^{\mathrm{T}}} \\
& \times \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}} - \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right) \\
& \times \boldsymbol{\mu}_{\tilde{\mathbf{x}}(k|k-1)(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} \\
& - \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\tilde{\mathbf{x}}(k|k-1)^{\mathrm{T}}} \\
& \times \left(\mathbf{I}_{n_x\times n_x} - \mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}} + \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))(\mathbf{C}_\delta(k)\mathbf{x}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} + \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{C}_\delta(k)\mathbf{x}(k))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\mathbf{v}(k)^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} + \mathbf{K}(k)\boldsymbol{\mu}_{\mathbf{v}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{C}_\delta(k)\mathbf{x}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}} \\
& + \mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{C}_\delta(k)\mathbf{x}(k))(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}.
\end{aligned} \tag{12}$$

Proof. From (1) and (8), we have

$$\tilde{\mathbf{x}}(k|k-1) = \mathbf{A}_c(k-1)\tilde{\mathbf{x}}(k-1) + \mathbf{w}(k-1) + \mathbf{A}_\delta(k-1)\mathbf{x}(k-1) + \mathbf{B}_\delta(k-1)\mathbf{u}(k-1). \tag{13}$$

It then follows immediately from (13) that

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k|k-1)} = \mathbf{A}_c(k-1)\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k-1)}\mathbf{A}_c(k-1)^{\mathrm{T}} + \boldsymbol{\Sigma}_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} + \boldsymbol{\Sigma}_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} + \boldsymbol{\Sigma}_{\mathbf{w}(k-1)}. \tag{14}$$

Now, we will continue to prove (12). By resorting to (1), (10), and (13), it can be obtained that

$$\tilde{\mathbf{x}}(k) = \tilde{\mathbf{x}}(k|k-1) - \mathbf{K}(k)\left(\mathbf{y}_t(k) - \boldsymbol{\Lambda}_c(k)\mathrm{g}\left(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)\right) - \left(\mathbf{I}_{n_y\times n_y} - \boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)\right). \tag{15}$$

Adding the following zero term

$$\mathbf{K}(k)\mathbf{y}(k)-\mathbf{K}(k)\mathbf{y}(k)+\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\mathbf{x}(k)-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\mathbf{x}(k) \tag{16}$$

to the right-hand side of (15), it can be further derived that

$$\begin{aligned}\tilde{\mathbf{x}}(k)=&\,\tilde{\mathbf{x}}(k|k-1)-\mathbf{K}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))-\mathbf{K}(k)\boldsymbol{\Lambda}(k)\,\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\\&-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k)+\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\,\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1))\\&+\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)+\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\mathbf{x}(k)\\&-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\mathbf{x}(k)-\mathbf{K}(k)\mathbf{v}(k)\\=&\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\tilde{\mathbf{x}}(k|k-1)-\mathbf{K}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))-\mathbf{K}(k)\mathbf{v}(k)\\&-\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\left(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1))\right)-\mathbf{K}(k)\boldsymbol{\Lambda}(k)\left(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\right.\\&\left.-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k)\right)-\mathbf{K}(k)\mathbf{C}_\delta(k)\mathbf{x}(k).\end{aligned} \tag{17}$$

Then, (12) can be directly got from (17). The proof is thus completed. □

Next, we will give the upper bound matrices of the predicted estimation error covariance and estimation error covariance in the following theorem.

Theorem 2. *Consider the nominal system (1)-(2) with filter (8)-(10), positive scalars $\alpha_l, l\in\mathcal{S}(1,11)$, and initial condition $\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(0)}=\boldsymbol{\Sigma}_{(\mathbf{x}(0)-\boldsymbol{\mu}_{\mathbf{x}(0)})}$, the upper bound of the predicted estimation error covariance is*

$$\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}=\mathbf{A}_c(k-1)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k-1)}\mathbf{A}_c(k-1)^{\mathrm{T}}+\boldsymbol{\Sigma}_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)}+\boldsymbol{\Sigma}_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)}+\boldsymbol{\Sigma}_{\mathbf{w}(k-1)}, \tag{18}$$

and the upper bound of the estimation error covariance is

$$\begin{aligned}\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)}=&\,\mathbf{K}(k)\mathbf{S}(k)\mathbf{K}(k)^{\mathrm{T}}+\left(1+\sum_{i=3}^{5}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}-\left(1+\sum_{i=3}^{5}\alpha_i\right)\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\\&-\left(1+\sum_{i=3}^{5}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{19}$$

where

$$\begin{aligned}\mathbf{S}(k)=&\left(1+\alpha_{10}+\alpha_4^{-1}+\alpha_6^{-1}\right)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\\&+\left(1+\alpha_5^{-1}+\alpha_7^{-1}+\alpha_{10}^{-1}+\alpha_{11}^{-1}\right)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\\&+\left(1+\sum_{i=3}^{5}\alpha_i\right)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\left(\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\&+\left(1+\alpha_3^{-1}+\sum_{i=6}^{9}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}+\left(1+\alpha_{11}+\alpha_8^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{C}_\delta(k)\mathbf{x}(k)}+\left(1+\alpha_9^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{v}(k)},\end{aligned} \tag{20}$$

$$\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}=\|\boldsymbol{\delta}\|_2^2\,\mathbf{I}_{n_y\times n_y}, \tag{21}$$

$$\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}=\|\mathbf{s}\|_2^2\,\mathbf{I}_{n_y\times n_y}, \tag{22}$$

$$\bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}=(1+\alpha_1)\|\mathbf{s}\|_2^2\,\mathbf{I}_{n_y\times n_y}+\left(1+\alpha_1^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{C}_c(k)\mathbf{x}(k)}, \tag{23}$$

$$\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}=(1+\alpha_2)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}+\left(1+\alpha_2^{-1}\right)\boldsymbol{\Sigma}_{(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k)}. \tag{24}$$

Proof. The theorem can be proved by mathematical induction. Based on the initial condition, we have $\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(0)}\geq\boldsymbol{\Sigma}_{(\mathbf{x}(0)-\boldsymbol{\mu}_{\mathbf{x}(0)})}$. Then, assume that the theorem holds for the integers from 1 to $k-1$, it can be obtained from Theorem 1 that

$$\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k|k-1)}\leq\mathbf{A}_c(k-1)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k-1)}\mathbf{A}_c(k-1)^{\mathrm{T}}+\boldsymbol{\Sigma}_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)}+\boldsymbol{\Sigma}_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)}+\boldsymbol{\Sigma}_{\mathbf{w}(k-1)}, \tag{25}$$

which completes the proof of (18). Before proving (19), we will firstly give some preliminary results that will be utilized in the subsequent proof. According to (1)-(2), (8)-(10), and Lemma 1, we have

$$\boldsymbol{\Sigma}_{(\mathbf{y}_t(k)-\mathbf{y}(k))} \le \|\boldsymbol{\delta}\|_2^2 \mathbf{I}_{n_y\times n_y} = \bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}, \tag{26}$$

$$\boldsymbol{\Sigma}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))} \le \|\mathbf{s}\|_2^2 \mathbf{I}_{n_y\times n_y} = \bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}, \tag{27}$$

$$\boldsymbol{\Sigma}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))} \le (1+\alpha_1)\|\mathbf{s}\|_2^2 \mathbf{I}_{n_y\times n_y} + \left(1+\alpha_1{}^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{C}_c(k)\mathbf{x}(k)} = \bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}, \tag{28}$$

$$\begin{aligned}\boldsymbol{\Sigma}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))} &\le (1+\alpha_2)\|\mathbf{s}\|_2^2 \mathbf{I}_{n_y\times n_y}\\ &+ \left(1+\alpha_2{}^{-1}\right)\boldsymbol{\Sigma}_{(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k)} = \bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))},\end{aligned} \tag{29}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{\Lambda}(k)(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))} \le \boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}, \tag{30}$$

$$\begin{aligned}&-\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\boldsymbol{\mu}_{\tilde{\mathbf{x}}(k|k-1)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}\\ &-\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\tilde{\mathbf{x}}(k|k-1)^{\mathrm{T}}}\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\ &\le \alpha_3\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\\ &\times\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}+\alpha_3{}^{-1}\mathbf{K}(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{31}$$

$$\begin{aligned}&-\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\times\boldsymbol{\mu}_{\tilde{\mathbf{x}}(k|k-1)(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &-\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))\tilde{\mathbf{x}}(k|k-1)^{\mathrm{T}}}\times\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\ &\le\alpha_4\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\times\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\ &+\alpha_4{}^{-1}\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{32}$$

$$\begin{aligned}&-\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\times\boldsymbol{\mu}_{\tilde{\mathbf{x}}(k|k-1)(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &-\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\tilde{\mathbf{x}}(k|k-1)^{\mathrm{T}}}\times\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\ &\le\alpha_5\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\bar{\mathbf{P}}(k|k-1)\times\left(\mathbf{I}_{n_x\times n_x}-\mathbf{K}(k)\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\ &+\alpha_5{}^{-1}\mathbf{K}(k)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{33}$$

$$\begin{aligned}&\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &+\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}\\ &\le\alpha_6\mathbf{K}(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\mathbf{K}(k)^{\mathrm{T}}+\alpha_6{}^{-1}\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{34}$$

$$\begin{aligned}&\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &+\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}\\ &\le\alpha_7\mathbf{K}(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\mathbf{K}(k)^{\mathrm{T}}+\alpha_7{}^{-1}\mathbf{K}(k)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{35}$$

$$\begin{aligned}&\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))(\mathbf{C}_\delta(k)\mathbf{x}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}+\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{C}_\delta(k)\mathbf{x}(k))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}\\ &\le\alpha_8\mathbf{K}(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\mathbf{K}(k)^{\mathrm{T}}+\alpha_8{}^{-1}\mathbf{K}(k)\boldsymbol{\Sigma}_{\mathbf{C}_\delta(k)\mathbf{x}(k)}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{36}$$

$$\begin{aligned}&\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\mathbf{v}(k)^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}+\mathbf{K}(k)\boldsymbol{\mu}_{\mathbf{v}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}\\ &\le\alpha_9\mathbf{K}(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\mathbf{K}(k)^{\mathrm{T}}+\alpha_9{}^{-1}\mathbf{K}(k)\boldsymbol{\Sigma}_{\mathbf{v}(k)}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{37}$$

$$\begin{aligned}&\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &+\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &\le\alpha_{10}\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &+\alpha_{10}{}^{-1}\mathbf{K}(k)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}},\end{aligned} \tag{38}$$

$$\begin{aligned}&\mathbf{K}(k)\boldsymbol{\Lambda}_c(k)\boldsymbol{\mu}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{C}_\delta(k)\mathbf{x}(k))^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}\\ &+\mathbf{K}(k)\boldsymbol{\mu}_{(\mathbf{C}_\delta(k)\mathbf{x}(k))(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}\\ &\le\alpha_{11}\mathbf{K}(k)\boldsymbol{\Sigma}_{\mathbf{C}_\delta(k)\mathbf{x}(k)}\mathbf{K}(k)^{\mathrm{T}}+\alpha_{11}{}^{-1}\mathbf{K}(k)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\mathbf{K}(k)^{\mathrm{T}}.\end{aligned} \tag{39}$$

Now, we are in a position to show that $\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k)} \le \bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)}$. It follows from (26)-(39) and Theorem 1 that

$$
\begin{aligned}
\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}(k)} \le \mathbf{K}(k)\Bigg(&\left(\left(1+\alpha_3{}^{-1}+\sum_{i=6}^{9}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\right.\\
&+\left(1+\alpha_{10}+\alpha_4{}^{-1}+\alpha_6{}^{-1}\right)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{(\mathbf{C}_c(k)\mathbf{x}(k)-\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1)))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\\
&+\left(1+\alpha_5{}^{-1}+\alpha_7{}^{-1}+\alpha_{10}{}^{-1}+\alpha_{11}{}^{-1}\right)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{(\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))-(\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\\
&+\left(1+\sum_{i=3}^{5}\alpha_i\right)\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\left(\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\\
&\left.+\left(1+\alpha_{11}+\alpha_8{}^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{C}_\delta(k)\mathbf{x}(k)}+\left(1+\alpha_9{}^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{v}(k)}\right)\mathbf{K}(k)^{\mathrm{T}}\\
&-\left(1+\sum_{i=3}^{5}\alpha_i\right)\mathbf{K}(k)\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\\
&-\left(1+\sum_{i=3}^{5}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\left(\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}+\left(1+\sum_{i=3}^{5}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}.
\end{aligned}
\tag{40}
$$

The proof of this theorem is thus completed. □

It can be easily seen that the upper bound of the estimation error covariance is less than $\mathbf{K}(k)\mathbf{S}(k)\mathbf{K}(k)^{\mathrm{T}}+1.3\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}$ $-1.3\mathbf{K}(k)\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}$ $-1.3\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)^{\mathrm{T}}\mathbf{K}(k)^{\mathrm{T}}$. Due to the existence of both stochastic parameter uncertainty and sensor saturation, the estimation error covariance is too complex to be minimized. Now, we are in a position to give the important theorem showing how to design the recursive filter gain to minimize the upper bound of the estimation error covariance.

Theorem 3. *Consider the nominal system (1)-(2) with filter (8)-(10), positive scalars $\alpha_l, l \in \mathcal{S}(1,11)$, and initial condition $\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(0)} = \boldsymbol{\Sigma}_{(\mathbf{x}(0)-\boldsymbol{\mu}_{\mathbf{x}(0)})}$, the upper bound of the estimation error covariance is minimized if and only if*

$$
\mathbf{K}(k) = \left(1+\sum_{i=3}^{5}\alpha_i\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)^{\mathrm{T}}\mathbf{S}(k)^{-1}. \tag{41}
$$

The corresponding minimum upper bound of the predicted estimation error covariance is

$$
\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)} = \mathbf{A}_c(k-1)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k-1)}\mathbf{A}_c(k-1)^{\mathrm{T}} + \boldsymbol{\Sigma}_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} + \boldsymbol{\Sigma}_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} + \boldsymbol{\Sigma}_{\mathbf{w}(k-1)}, \tag{42}
$$

and the corresponding minimum upper bound of the estimation error covariance is

$$
\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)} = \left(1+\sum_{i=3}^{5}\alpha_i\right)\left(\mathbf{I}_{n_{\mathbf{x}}\times n_{\mathbf{x}}}-\mathbf{K}(k)\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\right)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k|k-1)}, \tag{43}
$$

where $\mathbf{S}(k)$ is defined in (20).

Proof. It can be directly derived from the results in Theorem 2, thus omitted here. □

Based on the proposed filter, we can generate the recursive residual as follows:

$$
\mathbf{r}(k) = \mathbf{y}_t(k) - \boldsymbol{\Lambda}_c(k)\,\mathrm{g}\left(\mathbf{C}_c(k)\hat{\mathbf{x}}(k)\right) - \left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}_c(k)\right)\mathbf{C}_c(k)\hat{\mathbf{x}}(k). \tag{44}
$$

The detection statistic has the following square form:

$$
T_D(k) = \sum_{n=k-n_w+1}^{k}\|\mathbf{r}(n)\|_2^2, \tag{45}
$$

where n_w is the sliding window length. The basic property of the residual is given in the following theorem.

Theorem 4. *Consider the nominal system (1)-(2) with residual (44) and positive scalars $\alpha_l, l \in \mathcal{S}(1,11)$, $\beta_m, m \in \mathcal{S}(1,13)$, the expectation of the detection statistic (45) satisfies*

$$
\mu_{\|\mathbf{r}(k)\|_2^2} \le J_{th}(k), \tag{46}
$$

where $\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}$, $\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}$, *and* $\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)}$ *are defined in (21), (24), and (43), respectively.* $J_{th}(k)$ *is as follows:*

$$\begin{aligned} J_{th}(k) = \operatorname{tr}\Bigg(&\left(1+\sum_{m=1}^{5}\beta_m\right)\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} + \left(1+\beta_1^{-1}+\sum_{m=6}^{8}\beta_m\right)\boldsymbol{\Sigma}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)} \\ &+\left(1+\beta_2^{-1}+\beta_6^{-1}+\beta_9+\beta_{10}\right)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}} \\ &+\left(1+\beta_3^{-1}+\beta_7^{-1}+\beta_9^{-1}+\beta_{11}+\beta_{12}\right)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}} \\ &+\left(1+\beta_4^{-1}+\beta_8^{-1}+\beta_{10}^{-1}+\beta_{11}^{-1}+\beta_{13}\right)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))} \\ &+\left(1+\beta_5^{-1}+\beta_{12}^{-1}+\beta_{13}^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{v}(k)}\Bigg). \end{aligned} \tag{47}$$

Proof. Adding the zero term $\mathbf{y}(k) - \mathbf{y}(k)$ to the right hand of (44), we have

$$\begin{aligned} \mathbf{r}(k) = &\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k) + \left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k) \\ &+\boldsymbol{\Lambda}(k)\mathrm{g}\left(\left(\mathbf{C}_c(k)+\mathbf{C}_\delta(k)\right)\mathbf{x}(k)\right)+\mathbf{v}(k)-\boldsymbol{\Lambda}_c(k)\,\mathrm{g}\left(\mathbf{C}_c(k)\hat{\mathbf{x}}(k)\right)+\left(\mathbf{y}_t(k)-\mathbf{y}(k)\right). \end{aligned} \tag{48}$$

It then follows immediately from (48), Lemmas 1, and Theorem 3 that

$$\begin{aligned} &\mathbb{E}\left\{\mathbf{r}(k)^{\mathrm{T}}\mathbf{r}(k)\right\} = \mathbb{E}\left\{\operatorname{tr}\left(\mathbf{r}(k)\mathbf{r}(k)^{\mathrm{T}}\right)\right\} \\ &= \operatorname{tr}\Big(\boldsymbol{\Sigma}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k)} + \boldsymbol{\Sigma}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)} + \boldsymbol{\Sigma}_{\boldsymbol{\Lambda}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))} \\ &+\boldsymbol{\Sigma}_{\boldsymbol{\Lambda}_c(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))} + \boldsymbol{\Sigma}_{(\mathbf{y}_t(k)-\mathbf{y}(k))} + \boldsymbol{\Sigma}_{\mathbf{v}(k)} + \boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k)\mathbf{x}(k)^{\mathrm{T}}\mathbf{C}_\delta(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} \\ &+\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)\tilde{\mathbf{x}}(k)^{\mathrm{T}}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} + \boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}\boldsymbol{\Lambda}(k)^{\mathrm{T}}} \\ &+\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k)\mathbf{v}(k)^{\mathrm{T}}} + \boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\tilde{\mathbf{x}}(k)^{\mathrm{T}}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} + \boldsymbol{\mu}_{\mathbf{v}(k)\tilde{\mathbf{x}}(k)^{\mathrm{T}}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))^{\mathrm{T}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}} + \boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\tilde{\mathbf{x}}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\boldsymbol{\Lambda}_c(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))\tilde{\mathbf{x}}(k)^{\mathrm{T}}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} + \boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\tilde{\mathbf{x}}(k)^{\mathrm{T}}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} \\ &+\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}\boldsymbol{\Lambda}(k)^{\mathrm{T}}} + \boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\mathbf{x}(k)^{\mathrm{T}}\mathbf{C}_\delta(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))^{\mathrm{T}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}} + \boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\boldsymbol{\Lambda}_c(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))\mathbf{x}(k)^{\mathrm{T}}\mathbf{C}_\delta(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} + \boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\mathbf{x}(k)^{\mathrm{T}}\mathbf{C}_\delta(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))^{\mathrm{T}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}} - \boldsymbol{\mu}_{\boldsymbol{\Lambda}_c(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}\boldsymbol{\Lambda}(k)^{\mathrm{T}}} \\ &+\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}} + \boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))^{\mathrm{T}}\boldsymbol{\Lambda}(k)^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\boldsymbol{\Lambda}_c(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))\mathbf{v}(k)^{\mathrm{T}}} - \boldsymbol{\mu}_{\mathbf{v}(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))^{\mathrm{T}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}} + \boldsymbol{\mu}_{\mathbf{v}(k)(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}} \\ &-\boldsymbol{\mu}_{\boldsymbol{\Lambda}_c(k)\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))(\mathbf{y}_t(k)-\mathbf{y}(k))^{\mathrm{T}}} - \boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\mathrm{g}(\mathbf{C}_c(k)\hat{\mathbf{x}}(k))^{\mathrm{T}}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}} + \boldsymbol{\mu}_{(\mathbf{y}_t(k)-\mathbf{y}(k))\mathbf{v}(k)^{\mathrm{T}}}\Big) \\ &\le \operatorname{tr}\Bigg(\left(1+\sum_{m=1}^{5}\beta_m\right)\boldsymbol{\mu}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}} + \left(1+\beta_1^{-1}+\sum_{m=6}^{8}\beta_m\right)\boldsymbol{\Sigma}_{\left(\mathbf{I}_{n_y\times n_y}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_\delta(k)\mathbf{x}(k)} \\ &+\left(1+\beta_2^{-1}+\beta_6^{-1}+\beta_9+\beta_{10}\right)\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}} \\ &+\left(1+\beta_3^{-1}+\beta_7^{-1}+\beta_9^{-1}+\beta_{11}+\beta_{12}\right)\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_\delta(k))\mathbf{x}(k))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}} \\ &+\left(1+\beta_4^{-1}+\beta_8^{-1}+\beta_{10}^{-1}+\beta_{11}^{-1}+\beta_{13}\right)\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))} + \left(1+\beta_5^{-1}+\beta_{12}^{-1}+\beta_{13}^{-1}\right)\boldsymbol{\Sigma}_{\mathbf{v}(k)}\Bigg), \end{aligned} \tag{49}$$

which completes the proof of this theorem. □

It is obvious that the upper bound of the expectation of the detection statistic is less than $4\mathrm{tr}\left(\boldsymbol{\Sigma}_{\mathbf{v}(k)}\right)+2.3\mathrm{tr}\left(\boldsymbol{\Sigma}_{\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_{\delta}(k)\mathbf{x}(k)}\right) + 4.2\mathrm{tr}\left(\boldsymbol{\Lambda}_c(k)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_{\delta}(k))\mathbf{x}(k))}\boldsymbol{\Lambda}_c(k)^{\mathrm{T}}\right) + 5.1\mathrm{tr}\left(\mathrm{tr}\left(\bar{\boldsymbol{\Sigma}}_{(\mathbf{y}_t(k)-\mathbf{y}(k))}\right)\right.$ $\left.+1.5\mathrm{tr}\left(\boldsymbol{\mu}_{\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}(k)\right)\mathbf{C}_c(k)\bar{\boldsymbol{\Sigma}}_{\tilde{\mathbf{x}}(k)}\mathbf{C}_c(k)^{\mathrm{T}}\left(\mathbf{I}_{n_{\mathbf{y}}\times n_{\mathbf{y}}}-\boldsymbol{\Lambda}(k)\right)^{\mathrm{T}}}\right) + 3.2\mathrm{tr}\left(\boldsymbol{\mu}_{\boldsymbol{\Lambda}(k)\bar{\boldsymbol{\Sigma}}_{\mathrm{g}((\mathbf{C}_c(k)+\mathbf{C}_{\delta}(k))\mathbf{x}(k))}\boldsymbol{\Lambda}(k)^{\mathrm{T}}}\right)\right.$.

The detection threshold is set as

$$J_D(k) = \sum_{n=k-n_w+1}^{k} J_{th}(n), \tag{50}$$

then we can detect the fault according to the following detection logic:

$$\begin{cases} T_D(k) > J_D(k) \Rightarrow \text{with fault}, \\ T_D(k) \le J_D(k) \Rightarrow \text{no fault}. \end{cases} \tag{51}$$

4 | SIMULATION EXAMPLES

4.1 | Example A

In this section, a numerical example is given to illustrate the validity of the proposed method. The parameters of the stochastic time-varying system are as follows:

$$\mathbf{A}_c(k) = \begin{bmatrix} 0.83 + 0.02\sin(0.15k) & 0.42 \\ 0.18 & -0.76 + 0.01\cos(0.12k) \end{bmatrix},$$

$$\mathbf{B}_c(k) = \begin{bmatrix} 0.14 + 0.01\sin(0.12k) \\ 0.16 + 0.01\cos(0.15k) \end{bmatrix},$$

$$\mathbf{C}_c(k) = \begin{bmatrix} 0.86 & 0.63 + 0.02\cos(0.15k) \\ 0.75 + 0.01\sin(0.12k) & 0.87 \end{bmatrix}.$$

In our simulation, the initial state, process noise, and measurement noise are mutually independent zero-mean Gaussian white processes with covariances $\boldsymbol{\Sigma}_{\mathbf{x}(0)} = 1.2\mathbf{I}_{2\times 2}\times 10^{-6}$, $\boldsymbol{\Sigma}_{\mathbf{w}(k)} = 1.3\mathbf{I}_{2\times 2}\times 10^{-6}$, and $\boldsymbol{\Sigma}_{\mathbf{v}(k)} = 1.5\mathbf{I}_{2\times 2}\times 10^{-6}$, respectively. The triggering threshold is $\begin{bmatrix} 0.026 & 0.028 \end{bmatrix}^{\mathrm{T}}$, and the sliding window length is $n_w = 5$. The saturation level is $\begin{bmatrix} 0.1 & 0.1 \end{bmatrix}^{\mathrm{T}}$, and the saturation coefficient is a Bernoulli white process with expectation $\mathrm{diag}(0.1, 0.2)$. The parameter uncertainties are mutually independent zero-mean white processes with covariances

$$\boldsymbol{\Sigma}_{\mathbf{A}_{\delta}(k)} = \begin{bmatrix} 1.3 + 0.1\sin(0.15k) & & & \\ & 1.1 & & \\ & & 1.4 + 0.1\cos(0.12k) & \\ & & & 1.5 \end{bmatrix} \times 10^{-6},$$

$$\boldsymbol{\Sigma}_{\mathbf{B}_{\delta}(k)} = \begin{bmatrix} 1.4 + 0.1\cos(0.12k) & \\ & 1.2 + 0.1\sin(0.15k) \end{bmatrix} \times 10^{-6},$$

$$\boldsymbol{\Sigma}_{\mathbf{C}_{\delta}(k)} = \begin{bmatrix} 1.2 & & & \\ & 1.4 + 0.1\sin(0.12k) & & \\ & & 1.5 + 0.2\cos(0.15k) & \\ & & & 1.1 \end{bmatrix} \times 10^{-6}.$$

Various fault cases are all taken into consideration to fully demonstrate the effectiveness of our proposed method.

Case 1. The fault is positive

$$\mathbf{f}(k) = \begin{cases} \begin{bmatrix} 0.52 & 0.47 \end{bmatrix}^{\mathrm{T}}, k \in [66, 95], \\ \begin{bmatrix} 0.48 & 0.53 \end{bmatrix}^{\mathrm{T}}, k \in [171, 210], \\ \begin{bmatrix} 0.49 & 0.54 \end{bmatrix}^{\mathrm{T}}, k \in [286, 335], \\ \begin{bmatrix} 0 & 0 \end{bmatrix}^{\mathrm{T}}, \text{otherwise}. \end{cases}$$

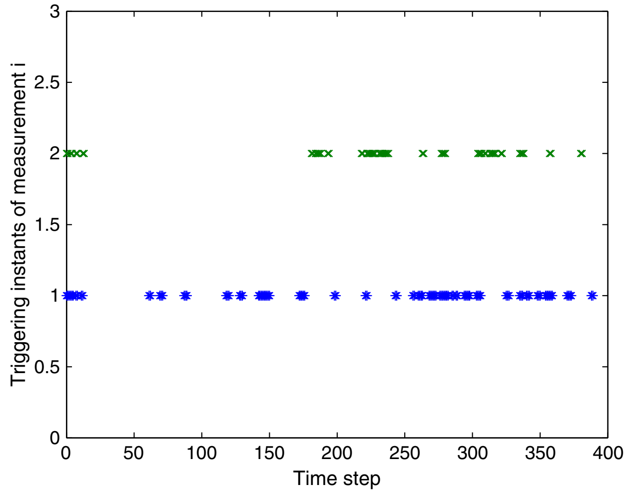

FIGURE 1 Example A: Triggering sequence [Colour figure can be viewed at wileyonlinelibrary.com]

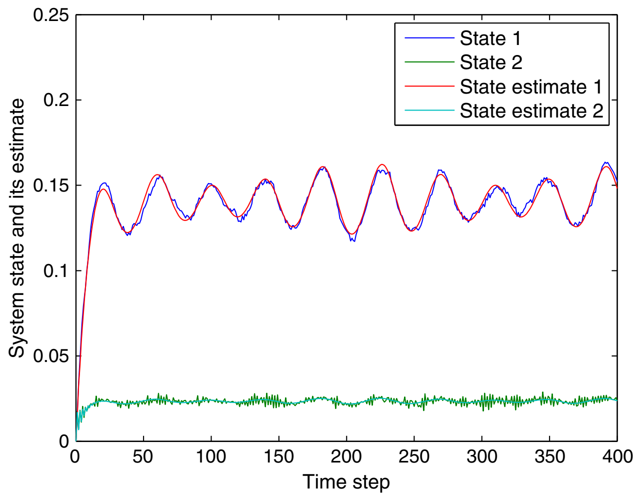

FIGURE 2 Example A: System state and its estimate [Colour figure can be viewed at wileyonlinelibrary.com]

Case 2. The fault is negative

$$\mathbf{f}(k)=\begin{cases}\begin{bmatrix}-0.48 & -0.51\end{bmatrix}^{\mathrm{T}}, k\in[66,95],\\ \begin{bmatrix}-0.46 & -0.54\end{bmatrix}^{\mathrm{T}}, k\in[171,210],\\ \begin{bmatrix}-0.52 & -0.47\end{bmatrix}^{\mathrm{T}}, k\in[286,335],\\ \begin{bmatrix}0 & 0\end{bmatrix}^{\mathrm{T}}, \text{otherwise}.\end{cases}$$

Simulation results are shown in Figures 1-5. For the fault-free case, Figure 1 depicts the triggering sequence, presenting that the transmission times are greatly reduced in comparison with periodic mechanism, which indicates the advantage of the event-triggered mechanism. The systems state and its estimate are presented in Figure 2. Besides, the mean square estimation error of system state is given in Figure 3 to show the effectiveness of our proposed robust filter more objectively. It can be seen that our proposed method can estimate the state very well. For the faulty case, Figures 4-5 show the fault detection results from which we can observe that our proposed method has a favorable detection performance for all cases.

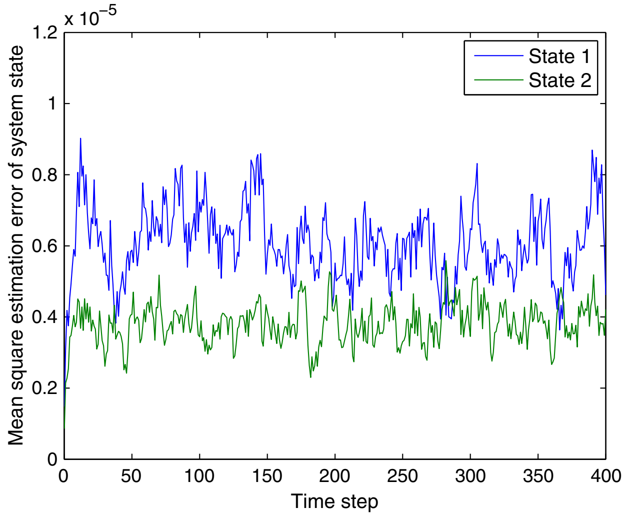

FIGURE 3 Example A: Mean square estimation error of system state [Colour figure can be viewed at wileyonlinelibrary.com]

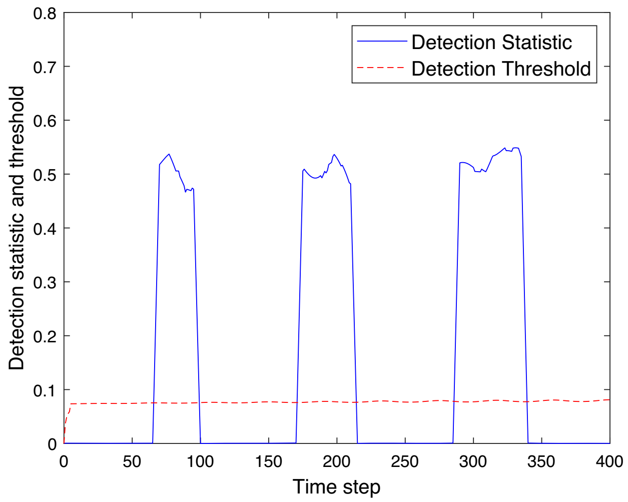

FIGURE 4 Example A: Fault detection result of case 1 [Colour figure can be viewed at wileyonlinelibrary.com]

4.2 | Example B

$$\mathbf{x}(k+1) = \begin{bmatrix} -\frac{R_aT-L_a}{L_a} & -\frac{C_v(k)T}{L_a} \\ \frac{C_m(k)T}{M_i} & -\frac{F_cT-M_i}{M_i} \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} \frac{T}{L_a} \\ 0 \end{bmatrix} u(k) + \mathbf{w}(k),$$

$$\mathbf{y}(k) = (\mathbf{I}_{2\times 2} - \mathbf{\Lambda}(k)) \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \mathbf{x}(k) + \mathbf{\Lambda}(k)\, \mathrm{g}\left(\begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \mathbf{x}(k) \right) + \mathbf{f}(k) + \mathbf{v}(k),$$

where the system state is the armature current and angular velocity. The control input is the armature voltage. The measurement output is the armature current and angular velocity.

In our simulation, the deterministic parameters are $R_a = 1.2$, $L_a = 0.05$, $M_i = 0.1352$, $T = 0.01$, and $F_c = 0.3$. The stochastic parameters $C_v(k)$ and $C_m(k)$ are mutually independent white processes with means 0.6 and 0.6 and covariances 0.18 and 0.12. The initial state is zero and the control input is 0.01. The process and measurement noise are mutually independent zero-mean white processes with covariances $\mathbf{\Sigma}_{\mathbf{w}(k)} = 2.5\mathbf{I}_{2\times 2} \times 10^{-3}$ and $\mathbf{\Sigma}_{\mathbf{v}(k)} = 2.5\mathbf{I}_{2\times 2} \times 10^{-3}$. The triggering threshold is $\begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^{\mathrm{T}}$, and the sliding window length is $n_w = 5$. The saturation level is $\begin{bmatrix} 0.008 & 0.006 \end{bmatrix}^{\mathrm{T}}$, and the saturation coefficient is a Bernoulli white process with expectation $\mathrm{diag}(0.1, 0.1)$. We consider the following fault cases.

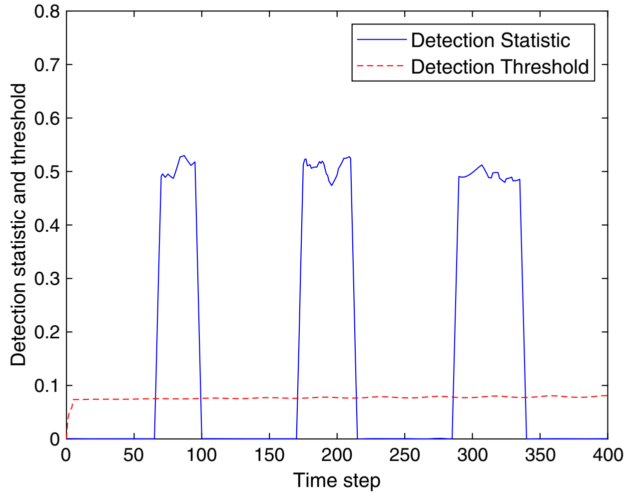

FIGURE 5 Example A: Fault detection result of case 2 [Colour figure can be viewed at wileyonlinelibrary.com]

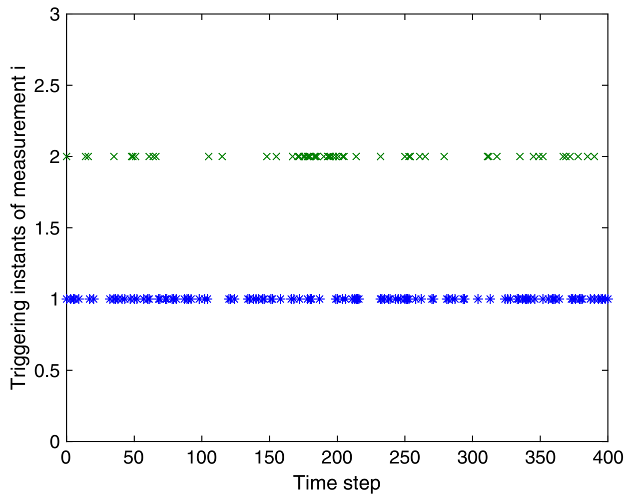

FIGURE 6 Example B: Triggering sequence [Colour figure can be viewed at wileyonlinelibrary.com]

Case 3. The fault is positive

$$\mathbf{f}(k)=\begin{cases}\begin{bmatrix}0.46 & 0.51\end{bmatrix}^{\mathrm{T}}, k\in[66,95],\\ \begin{bmatrix}0.52 & 0.48\end{bmatrix}^{\mathrm{T}}, k\in[171,210],\\ \begin{bmatrix}0.56 & 0.43\end{bmatrix}^{\mathrm{T}}, k\in[286,335],\\ \begin{bmatrix}0 \; 0\end{bmatrix}^{\mathrm{T}}, \text{otherwise.}\end{cases}$$

Case 4. The fault is negative

$$\mathbf{f}(k)=\begin{cases}\begin{bmatrix}-0.52 & -0.48\end{bmatrix}^{\mathrm{T}}, k\in[66,95],\\ \begin{bmatrix}-0.49 & -0.51\end{bmatrix}^{\mathrm{T}}, k\in[171,210],\\ \begin{bmatrix}-0.53 & -0.43\end{bmatrix}^{\mathrm{T}}, k\in[286,335],\\ \begin{bmatrix}0 & 0\end{bmatrix}^{\mathrm{T}}, \text{otherwise.}\end{cases}$$

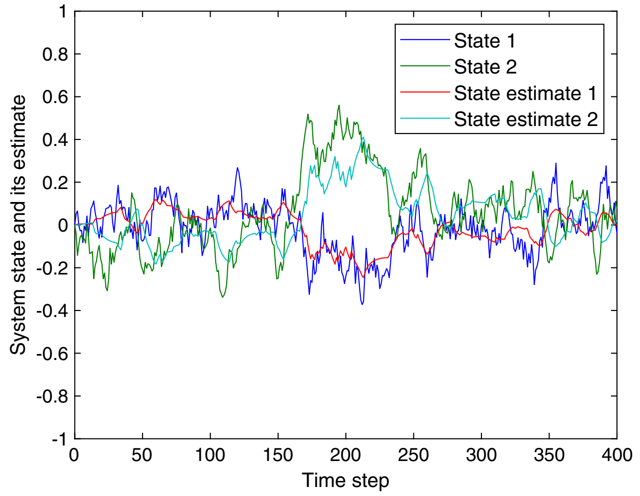

FIGURE 7 Example B: System state and its estimate [Colour figure can be viewed at wileyonlinelibrary.com]

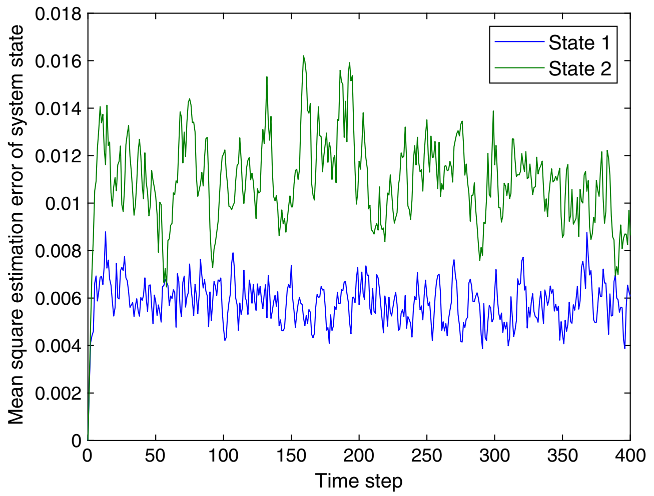

FIGURE 8 Example B: Mean square estimation error of system state [Colour figure can be viewed at wileyonlinelibrary.com]

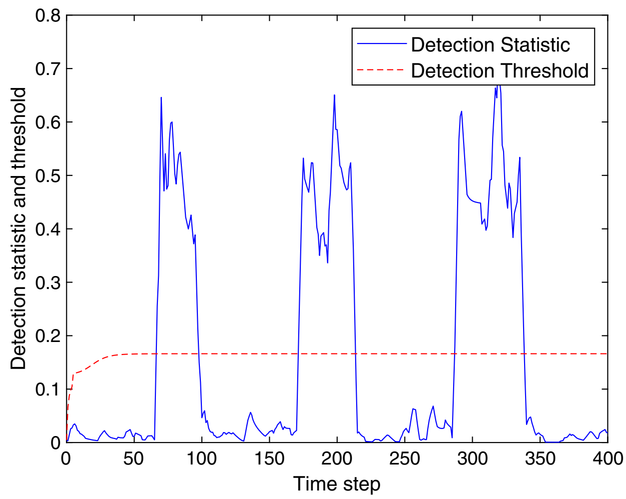

FIGURE 9 Example B: Fault detection result of case 1 [Colour figure can be viewed at wileyonlinelibrary.com]

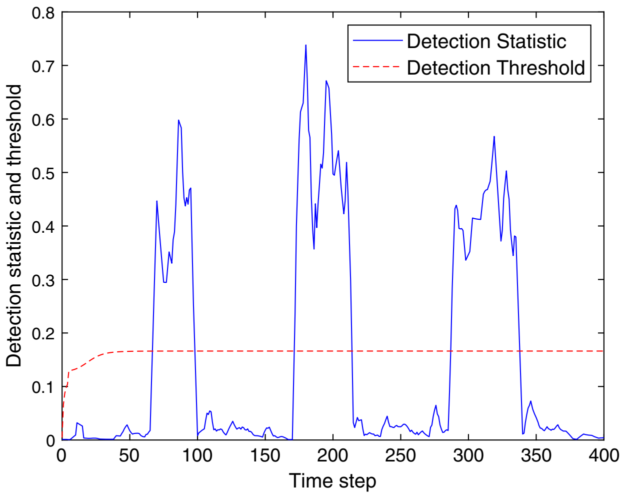

FIGURE 10 Example B: Fault detection result of case 2 [Colour figure can be viewed at wileyonlinelibrary.com]

Simulation results are shown in Figures 6-10. It can be seen that our proposed method still has a favorable performance for direct current motor system.

5 | CONCLUSION

This work has addressed the event-triggered filtering and fault detection problems of stochastic uncertain systems subject to sensor saturation and event-triggered transmission, which is of wide practical use. An event-triggered filter was firstly proposed at the criterion of minimum upper bound of the estimation error covariance. Then, based on it, we developed an effective fault detection strategy to detect the fault. Finally, we exploited two simulation examples to illustrate the validity and effectiveness of our developed method.

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