

Robust detection of intermittent multiplicative sensor fault

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Abstract

In this paper, the detection problem of intermittent multiplicative sensor fault is investigated for stochastic uncertain systems. A robust optimal filter is designed according to the criterion of minimum estimation error covariance. Then, based on this, a residual generator is constructed, and the quantitative effect of the fault on it is discussed in detail. After that we design the evaluation function and detection threshold to achieve intermittent fault detection. Our proposed strategy has a recursive form and only includes simple arithmetic operations, thus it is suitable for real-time online applications. Finally, a simulation example is given to demonstrate the effectiveness of the proposed strategy.

KEYWORDS

fault detection, intermittent fault (IF), multiplicative sensor fault, stochastic uncertain systems

1 | INTRODUCTION

There is increasing demand for quality and efficient production when integrating higher automation into modern engineering systems [1–3]. As an essential part of automation systems, the sensor is an important tool to obtain information, thus crucial for both system performance and safety [4]. If the sensor fails, it would be likely to lead to a series of serious problems and even disasters. For example, on February 23, 2008, a B-2 Spirit bomber crashed on the

runway shortly after takeoff from Andersen Air Force Base in Guam [5]. It caused a direct economic loss up to US 1.4 billion dollars. The main cause of the crash was attributed to the occurrence of a sensor fault that distorted the information for the flight control system [6]. Therefore, it is of great significance to develop effective sensor fault detection methods to ensure the safety and reliability of modern engineering systems.

Compared with hardware redundancy methods, analytical redundancy methods are more economical and

appealing. To date, there have been various research results on analytical redundancy-based sensor fault detection. They can be roughly classified into two general categories: model-based methods and data-driven methods. The model-based methods primarily contain: state estimation-based methods [7–13], parameter estimation-based methods [14–16], and parity space-based methods [17–20]. And the data-driven methods mainly include: multivariate statistics-based methods [21–24], signal processing-based methods [25–27], and machine learning-based methods [28–30].

Meanwhile, the intermittent fault (IF) has also drawn increasing attention in recent years. The IF is referred to as a kind of non-permanent fault that lasts for a limited period of time and then disappears without any external corrective action; it is often recurrent [31]. In practice, many systems are very vulnerable to IF, such as aerospace systems, electronic systems, and so on. The IF may not only cause some unnecessary corrective actions, but also have the potential to result in a system crash. Consequently, there is an urgent need to investigate the issue of intermittent fault detection, and some initial efforts have been devoted to it [32–39].

However, so far, to the best of our knowledge, the robust intermittent multiplicative sensor fault detection problem for stochastic uncertain linear time-varying systems has yet not been fully studied. Due to the requirement to detect both the occurrence and disappearance of the fault, the IF detection itself is very difficult, not to mention the challenge brought from multiplicative parameter uncertainties and additive noise. Furthermore, the intermittent and multiplicative characteristic of the fault and the presence of stochastic model uncertainties add extra difficulties.

It is worth mentioning that most previous works on robust fault detection usually assume that the model uncertainties have a specific form of matrix structure and both the structure and corresponding structural parameters of it are known beforehand. Unfortunately, sometimes it may be impossible to obtain this information and there is still no robust detection method based on statistical information rather than structural information. As such, we are motivated to propose a novel robust intermittent multiplicative sensor fault detection method. The main contributions of this paper can be summarized as follows. First, a robust residual is designed that takes system dynamic and model uncertainties into full consideration. Second, the quantitative effect of the fault on the residual is presented. Third, a detection strategy is given that is recursive and only includes simple arithmetic operations thus suitable for real-time online applications.

The remainder of this paper is organized as follows. In Section 2, the robust detection problem of intermittent multiplicative sensor fault is formulated with some assumptions. In Section 3, the IF detection strategy is pro-

posed utilizing our robust optimal filter. Simulation results are given in Section 4. Section 5 concludes the paper.

Notations. Except where otherwise stated, the notations used throughout this paper are fairly standard. \mathbb{R}^n and $\mathbb{R}^{n \times m}$ represent the n dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $0_{n \times m}$ denotes the null matrix with n rows and m columns (0 at all entries), and $\mathbf{I}_{n \times n}$ denotes the identity matrix with n rows and n columns (1 at the (i, i) th entry and 0 elsewhere). $S(m, n)$ denotes the set $\{m, m + 1, \dots, n\}$, ($m \in \mathbb{Z}, n \in \mathbb{Z}, m \leq n$). Given a matrix $\mathbf{X} = [x_{ij}] \in \mathbb{R}^{n \times m}$, $\text{row}\{\mathbf{X}, i\}$ denotes the vector $[x_{i1} \ x_{i2} \ \dots \ x_{im}]^T$. Given a set $\mathcal{V} = \{n_1, n_2, \dots, n_m\}$, ($n_1 \leq n_2 \leq \dots \leq n_m$), $\text{col}_{i \in \mathcal{V}}\{\mathbf{X}_i\}$ and $\text{diag}_{i \in \mathcal{V}}\{\mathbf{X}_i\}$ represent the block-column matrix $[X_{n_1}^T \ X_{n_2}^T \ \dots \ X_{n_m}^T]^T$ and block-diagonal matrix $\text{diag}\{\mathbf{X}_{n_1}, \mathbf{X}_{n_2}, \dots, \mathbf{X}_{n_m}\}$, respectively. $\mathbb{E}\{\mathbf{X}\}$ is the mathematical expectation of a stochastic variable \mathbf{X} . $\text{p}_{\text{row}}\mathbf{X}, \mu_{\mathbf{X}}, \Sigma_{\mathbf{X}}$ denote $\text{col}_{i \in S(1, n)}\{\text{row}\{\mathbf{X}, i\}\}$, $\mathbb{E}\{\mathbf{X}\}$, $\mathbb{E}\{\text{p}_{\text{row}}(\mathbf{X})\text{p}_{\text{row}}(\mathbf{X})^T\}$, respectively. If the dimensions of matrices are not explicitly stated, they are assumed to be compatible for algebraic operations.

2 | PROBLEM FORMULATION AND PRELIMINARIES

Consider the stochastic uncertain systems with multiplicative sensor fault:

$$\begin{aligned} \mathbf{x}(k+1) &= (\mathbf{A}_c(k) + \mathbf{A}_\delta(k))\mathbf{x}(k) \\ &\quad + (\mathbf{B}_c(k) + \mathbf{B}_\delta(k))\mathbf{u}(k) + \mathbf{w}(k), \end{aligned} \quad (1)$$

$$\mathbf{y}(k) = (\mathbf{I}_{n_y} \times n_y + \mathbf{F}(k))(\mathbf{C}_c(k) + \mathbf{C}_\delta(k))\mathbf{x}(k) + \mathbf{v}(k), \quad (2)$$

where $\mathbf{x}(k) \in \mathbb{R}^{n_x}$ is the system state, $\mathbf{u}(k) \in \mathbb{R}^{n_u}$ is the control input, and $\mathbf{y}(k) \in \mathbb{R}^{n_y}$ is the measurement output. $\mathbf{w}(k) \in \mathbb{R}^{n_x}$ is the process noise, and $\mathbf{v}(k) \in \mathbb{R}^{n_y}$ is the measurement noise. $\mathbf{A}_c(k) \in \mathbb{R}^{n_x \times n_x}$, $\mathbf{B}_c(k) \in \mathbb{R}^{n_x \times n_u}$, $\mathbf{C}_c(k) \in \mathbb{R}^{n_y \times n_x}$ are known deterministic parameters, and $\mathbf{A}_\delta(k) \in \mathbb{R}^{n_x \times n_x}$, $\mathbf{B}_\delta(k) \in \mathbb{R}^{n_x \times n_u}$, $\mathbf{C}_\delta(k) \in \mathbb{R}^{n_y \times n_x}$ are unknown stochastic parameter uncertainties. $\mathbf{F}(k) \in \mathbb{R}^{n_y \times n_y}$ is the fault matrix of the following form:

$$\mathbf{F}(k) = \begin{bmatrix} f(k)^{(1)} & 0 & \dots & 0 \\ 0 & f(k)^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & f(k)^{(n_y)} \end{bmatrix}. \quad (3)$$

The above form of sensor fault is very general and common in the fault diagnosis community [40–45] and can be used to describe the abnormal measurement output of the sensor, for example, the liquid height sensor fault of the three-tank system [42], the speed sensor fault of the squirrel cage induction [43], and so on. The parameter uncertainty $\mathbf{C}_\delta(k)$ is essentially the stochastic multiplicative measurement noise, the statistical information of

which is assumed to be known beforehand. On the other hand, the fault $\mathbf{F}(k)$ is not assumed to have any specific statistical characteristic, thus is completely unknown.

The IF is described as follows:

$$\mathbf{F}(k) = \sum_{i=1}^{\infty} \mathbf{F}_i (\Gamma(k - k_{o,i}) - \Gamma(k - k_{d,i})), \quad (4)$$

where $\Gamma(\cdot)$ is the scalar unit step function, which is used to describe the occurrence and disappearance of the IF. F_i is the i th fault profile matrix, which is multiplied by $\Gamma(\cdot)$ and used to describe the magnitude and location of the IF. And $k_{o,i}, k_{d,i}$ are corresponding fault occurrence and disappearance time. $f(k)^{(j)}$ ($j \in S(1, n_y)$) is the (j, j) th entry of \mathbf{F}_i when $k \in S(k_{o,i}, k_{d,i} - 1)$, otherwise it is equal to zero. The above system is called a fault-free system if the fault matrix $\mathbf{F}(k)$ is identically equal to zero. Otherwise it is called a faulty system.

Thanks to the diagonal structure of the fault matrix, we can rewrite (1)–(2) as

$$\mathbf{x}(k+1) = (\mathbf{A}_c(k) + \mathbf{A}_\delta(k))\mathbf{x}(k) + (\mathbf{B}_c(k) \quad (5)$$

$$+ \mathbf{B}_\delta(k))\mathbf{u}(k) + \mathbf{w}(k), \quad (6)$$

$$\mathbf{y}(k) = (\mathbf{C}_c(k) + \mathbf{C}_\delta(k))\mathbf{x}(k) + \mathbf{v}(k) + \mathbf{E}_f(k)\mathbf{f}(k),$$

where

$$\mathbf{f}(k) = [f(k)^{(1)} \quad f(k)^{(2)} \quad \dots \quad f(k)^{(n_y)}]^T, \quad (7)$$

$$\mathbf{E}_f(k) = \text{diag}_{i \in S(1, n_y)} \{ \text{row}\{(\mathbf{C}_c(k) + \mathbf{C}_\delta(k))\mathbf{x}(k), i\} \}. \quad (8)$$

The matrices are independent of each other if and only if their entries are independent of each other. And the following assumptions are made throughout this paper which give the statistical information of uncertainties.

Assumption 1. The initial state $\mathbf{x}(0)$ has the mean $\bar{\mathbf{x}}_0$, covariance \mathbf{P}_0 , and second moment Σ_0 . The noise $\mathbf{w}(k), \mathbf{v}(k)$ are zero-mean white processes with positive definite covariances $\Sigma_{\mathbf{w}(k)}$ and $\Sigma_{\mathbf{v}(k)}$.

Assumption 2. The stochastic parameter uncertainties $\mathbf{A}_\delta(k), \mathbf{B}_\delta(k), \mathbf{C}_\delta(k)$ are zero-mean white processes with

$$\begin{aligned} \mathbb{E} \{ \text{p}_{\text{row}}(\mathbf{A}_\delta(k)) \text{p}_{\text{row}}(\mathbf{A}_\delta(k))^T \} &= \Sigma_{\mathbf{A}_\delta(k)}, \\ \mathbb{E} \{ \text{p}_{\text{row}}(\mathbf{B}_\delta(k)) \text{p}_{\text{row}}(\mathbf{B}_\delta(k))^T \} &= \Sigma_{\mathbf{B}_\delta(k)}, \\ \mathbb{E} \{ \text{p}_{\text{row}}(\mathbf{C}_\delta(k)) \text{p}_{\text{row}}(\mathbf{C}_\delta(k))^T \} &= \Sigma_{\mathbf{C}_\delta(k)}. \end{aligned} \quad (9)$$

Moreover, the initial state, noise, and parameter uncertainties are mutually independent of each other.

In this paper, our main goal is to develop an effective detection strategy to detect both the occurrence and disappearance of the intermittent multiplicative sensor fault. It is worth mentioning that our proposed strategy can also be extended to the case of non zero-mean noise and parameter uncertainties. But for the sake of brevity, it is thus omitted.

3 | MAIN RESULTS

In this section, we will firstly give the structure of our proposed robust optimal filter as follows:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{A}_c(k-1)\hat{\mathbf{x}}(k-1) + \mathbf{B}_c(k-1)\mathbf{u}(k-1), \quad (10)$$

$$\mathbf{r}(k|k-1) = \mathbf{y}(k) - \mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1), \quad (11)$$

$$\hat{\mathbf{x}}(k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}_x(k)\mathbf{r}(k|k-1), \quad (12)$$

where $\hat{\mathbf{x}}(k|k-1)$ is the predicted state estimate, $\mathbf{r}(k|k-1)$ is the innovation, $\hat{\mathbf{x}}(k)$ is the state estimate, and $\mathbf{K}_x(k)$ is the filter gain to be designed.

Before proceeding further, the following lemmas are introduced.

Lemma 1. Consider the fault-free system (1)–(2), the expectation and second moment of the system state are

$$\mu_{\mathbf{x}(k)} = \Phi_{\mathbf{A}_c}(k, 0)\bar{\mathbf{x}}_0 + \sum_{i=0}^{k-1} \Phi_{\mathbf{A}_c}(k, i+1)\mathbf{B}_c(i)\mathbf{u}(i), \quad (13)$$

$$\begin{aligned} \Sigma_{\mathbf{x}(k)} &= \mathbf{A}_c(k-1)\Sigma_{\mathbf{x}(k-1)}\mathbf{A}_c(k-1)^T + \Sigma_{\mathbf{w}(k-1)} \\ &+ \Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} + \Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} \\ &+ \mathbf{B}_c(k-1)\Sigma_{\mathbf{u}(k-1)}\mathbf{B}_c(k-1)^T \\ &+ \mathbf{A}_c(k-1)\mu_{\mathbf{x}(k-1)}\mathbf{u}(k-1)^T\mathbf{B}_c(k-1)^T \\ &+ \mathbf{B}_c(k-1)\mathbf{u}(k-1)\mu_{\mathbf{x}(k-1)}^T\mathbf{A}_c(k-1)^T, \end{aligned} \quad (14)$$

where

$$\Phi_{\mathbf{A}_c}(k, m) = \begin{cases} \prod_{i=1}^{k-m} \mathbf{A}_c(k-i), & k > m, \\ \mathbf{I}_{n_x \times n_x}, & k \leq m, k \in \mathbb{Z}, m \in \mathbb{Z}. \end{cases} \quad (15)$$

Proof. This lemma can be directly obtained by system dynamic and backstepping recursion, thus omitted here. \square

Lemma 2. Consider the fault-free system (1)–(2), the predicted estimation error covariance is

$$\begin{aligned} \mathbf{P}_x(k|k-1) &= \mathbb{E} \{ \tilde{\mathbf{x}}(k|k-1)\tilde{\mathbf{x}}(k|k-1)^T \} \\ &= \mathbf{A}_c(k-1)\mathbf{P}_x(k-1)\mathbf{A}_c(k-1)^T \\ &+ \Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} + \Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} + \Sigma_{\mathbf{w}(k-1)}, \end{aligned} \quad (16)$$

and the estimation error covariance is

$$\begin{aligned} \mathbf{P}_x(k) &= \mathbb{E} \{ \tilde{\mathbf{x}}(k)\tilde{\mathbf{x}}(k)^T \} \\ &= (\mathbf{I}_{n_x \times n_x} - \mathbf{K}_x(k)\mathbf{C}_c(k))\mathbf{P}_x(k|k-1) \\ &\times (\mathbf{I}_{n_x \times n_x} - \mathbf{K}_x(k)\mathbf{C}_c(k))^T \\ &+ \mathbf{K}_x(k)(\Sigma_{\mathbf{C}_\delta(k)\mathbf{x}(k)} + \Sigma_{\mathbf{v}(k)})\mathbf{K}_x(k)^T. \end{aligned} \quad (17)$$

Proof. This lemma is a direct result of (1)–(2), (10)–(12), thus omitted here. \square

Lemma 2 describes the relationship between the predicted estimation error covariance and estimation error covariance, which is a preliminary to derive Theorem 1 for designing the filter gain.

Theorem 1. Consider the fault-free system (1)–(2) with filter (10)–(12), then the gain (18) can make the filter achieve optimal estimation at the criterion of minimum estimation error covariance:

$$\mathbf{K}_x(k) = \mathbf{H}(k)\mathbf{C}_c(k)^T\mathbf{S}(k)^{-1}, \quad (18)$$

where

$$\begin{aligned} \mathbf{H}(k) &= \mathbf{A}_c(k-1)\mathbf{P}_x(k-1)\mathbf{A}_c(k-1)^T + \Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} \\ &+ \Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} + \Sigma_{\mathbf{w}(k-1)}, \end{aligned} \quad (19)$$

$$\mathbf{S}(k) = \mathbf{C}_c(k)\mathbf{H}(k)\mathbf{C}_c(k)^T + \Sigma_{\mathbf{C}_\delta(k)\Sigma_{\mathbf{x}(k)}} + \Sigma_{\mathbf{v}(k)}. \quad (20)$$

And the corresponding estimation error covariance is

$$\mathbf{P}_x(k) = (\mathbf{I}_{n_x \times n_x} - \mathbf{K}_x(k)\mathbf{C}_c(k))\mathbf{H}(k). \quad (21)$$

Proof. According to (1)–(2) and (10)–(12), we can firstly obtain the dynamics of estimation error as follows:

$$\begin{aligned} \tilde{\mathbf{x}}(k) &= \mathbf{x}(k) - \hat{\mathbf{x}}(k) \\ &= (\mathbf{A}_c(k-1) - \mathbf{K}_x(k)\mathbf{C}_c(k)\mathbf{A}_c(k-1))\tilde{\mathbf{x}}(k-1) \\ &+ (\mathbf{A}_\delta(k-1) - \mathbf{K}_x(k)(\mathbf{C}_c(k)\mathbf{A}_\delta(k-1) \\ &+ \mathbf{C}_\delta(k)\mathbf{A}_c(k-1) + \mathbf{C}_\delta(k)\mathbf{A}_\delta(k-1)))\mathbf{x}(k-1) \\ &+ (\mathbf{B}_\delta(k-1) - \mathbf{K}_x(k)(\mathbf{C}_c(k)\mathbf{B}_\delta(k-1) \\ &+ \mathbf{C}_\delta(k)\mathbf{B}_c(k-1) + \mathbf{C}_\delta(k)\mathbf{B}_\delta(k-1)))\mathbf{u}(k-1) \\ &+ (\mathbf{I}_{n_x \times n_x} - \mathbf{K}_x(k)(\mathbf{C}_c(k) + \mathbf{C}_\delta(k)))\mathbf{w}(k-1) \\ &- \mathbf{K}_x(k)\mathbf{v}(k). \end{aligned} \quad (22)$$

Based on the above result and Lemmas 1–2, we can get the following estimation error covariance:

$$\begin{aligned} \mathbf{P}_x(k) &= \mathbb{E}\{\tilde{\mathbf{x}}(k)\tilde{\mathbf{x}}(k)^T\} \\ &= \mathbf{K}_x(k)(\mathbf{C}_c(k)\mathbf{A}_c(k-1)\mathbf{P}_x(k-1)\mathbf{A}_c(k-1)^T\mathbf{C}_c(k)^T \\ &+ \mathbf{C}_c(k)\Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)}\mathbf{C}_c(k)^T + \Sigma_{\mathbf{C}_\delta(k)\mathbf{A}_c(k-1)\mathbf{x}(k-1)} \\ &+ \Sigma_{\mathbf{C}_\delta(k)\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} \\ &+ \mu_{\mathbf{C}_\delta(k)}(\mathbf{A}_c(k-1)\tilde{\mathbf{x}}(k-1)\mathbf{u}(k-1)^T\mathbf{B}_c(k-1)^T)\mathbf{C}_\delta(k)^T \end{aligned}$$

$$\begin{aligned} &+ \mu_{\mathbf{C}_\delta(k)}(\mathbf{B}_c(k-1)\mathbf{u}(k-1)\tilde{\mathbf{x}}(k-1)^T\mathbf{A}_c(k-1)^T)\mathbf{C}_\delta(k)^T \\ &+ \mathbf{C}_c(k)\Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)}\mathbf{C}_c(k)^T + \Sigma_{\mathbf{C}_\delta(k)\mathbf{B}_c(k-1)\mathbf{u}(k-1)} \\ &+ \Sigma_{\mathbf{C}_\delta(k)\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} \\ &+ \Sigma_{\mathbf{C}_\delta(k)\mathbf{w}(k-1)} + \mathbf{C}_c(k)\Sigma_{\mathbf{w}(k-1)}\mathbf{C}_c(k)^T + \Sigma_{\mathbf{v}(k)})\mathbf{K}_x(k)^T \\ &+ \mathbf{K}_x(k)(-\mathbf{C}_c(k)\mathbf{A}_c(k-1)\mathbf{P}_x(k-1)\mathbf{A}_c(k-1)^T \\ &- \mathbf{C}_c(k)\Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} \\ &- \mathbf{C}_c(k)\Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} - \mathbf{C}_c(k)\Sigma_{\mathbf{w}(k-1)}) \\ &+ (-\mathbf{A}_c(k-1)\mathbf{P}_x(k-1)\mathbf{A}_c(k-1)^T\mathbf{C}_c(k)^T \\ &- \Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)}\mathbf{C}_c(k)^T \\ &- \Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)}\mathbf{C}_c(k)^T - \Sigma_{\mathbf{w}(k-1)}\mathbf{C}_c(k)^T)\mathbf{K}_x(k)^T \\ &+ \mathbf{A}_c(k-1)\mathbf{P}_x(k-1)\mathbf{A}_c(k-1)^T + \Sigma_{\mathbf{A}_\delta(k-1)\mathbf{x}(k-1)} \\ &+ \Sigma_{\mathbf{B}_\delta(k-1)\mathbf{u}(k-1)} + \Sigma_{\mathbf{w}(k-1)} \\ &= \mathbf{K}_x(k)\mathbf{S}(k)\mathbf{K}_x(k)^T - \mathbf{K}_x(k)\mathbf{C}_c(k)\mathbf{H}(k) \\ &- \mathbf{H}(k)\mathbf{C}_c(k)^T\mathbf{K}_x(k)^T + \mathbf{H}(k) \\ &= (\mathbf{K}_x(k)\mathbf{S}(k) - \mathbf{H}(k)\mathbf{C}_c(k)^T)\mathbf{S}(k)^{-1} \\ &\times (\mathbf{K}_x(k)\mathbf{S}(k) - \mathbf{H}(k)\mathbf{C}_c(k)^T)^T \\ &+ \mathbf{H}(k) - \mathbf{H}(k)\mathbf{C}_c(k)^T\mathbf{S}(k)^{-1}\mathbf{C}_c(k)\mathbf{H}(k). \end{aligned} \quad (23)$$

It is important to note that $\mathbf{S}(k)$ is positive definite and thus invertible. It can be further seen that $\mathbf{P}_x(k)$ is minimized when

$$\mathbf{K}_x(k) = \mathbf{H}(k)\mathbf{C}_c(k)^T\mathbf{S}(k)^{-1}. \quad (24)$$

Moreover, we have

$$\mathbf{P}_x(k) = (\mathbf{I}_{n_x \times n_x} - \mathbf{K}_x(k)\mathbf{C}_c(k))\mathbf{H}(k). \quad (25)$$

This completes the proof of the theorem. \square

Based on the above state estimate, we can generate the following residual:

$$\mathbf{r}(k) = \mathbf{y}(k) - \mathbf{C}_c(k)\hat{\mathbf{x}}(k|k-1). \quad (26)$$

The quantitative relationship between the residual and the fault is analyzed and given in Theorems 2.

Theorem 2. Consider the faulty system (1)–(2), the relationship between residual (26) and fault is as follows:

$$\begin{aligned} \mathbf{r}(k) &= \mathbf{r}_o(k) + (\mathbf{I}_{n_y \times n_y} + \mathbf{C}_c(k)\mathbf{T}(k))\mathbf{F}(k)(\mathbf{C}_c(k) \\ &+ \mathbf{C}_\delta(k))\mathbf{x}(k) + \mathbf{C}_c(k)\mathbf{z}(k), \end{aligned} \quad (27)$$

where $\mathbf{r}_o(k)$ represents the residual of the fault-free system, and $\mathbf{T}(k)$, $\mathbf{z}(k)$ have the following forms:

$$\mathbf{T}(k)\mathbf{E}_f(k) = \mathbf{L}(k-1)\mathbf{T}(k-1)\mathbf{E}_f(k-1) + \mathbf{M}(k-1), \quad (28)$$

where $\mathbf{P}_x(k|k-1)$ is defined in (16).

Proof. The proof of the theorem is straightforward, thus omitted here. \square

Most available robust filters usually depend heavily on the structural information of the parameter uncertainties and thus can only be designed at the criterion of norm index. Unlike these, as can be seen from Theorems 1–3, the advantages of our designed robust filter are that it is only based on the statistical information and thus can be designed at the criterion of minimum estimation error covariance. Although our design method may be a little bit complex, it takes not time for the designer to learn it.

The residual evaluation function is as follows:

$$T_D(k) = \mathbf{r}(k)^T \mathbf{r}(k). \quad (42)$$

And the corresponding detection threshold is set as:

$$J_D(k) = \lambda \text{tr}(\Sigma_{\mathbf{r}(k)}), \quad (43)$$

where λ is the user-determined parameter chosen by test and requirement. Then the occurrence and disappearance of the fault can be determined by comparing $T_D(k)$ and $J_D(k)$ according to the following detection logic:

$$\begin{cases} T_D(k) > J_D(k) \Rightarrow \text{with fault} \Rightarrow \text{fault alarm,} \\ T_D(k) \leq J_D(k) \Rightarrow \text{no fault} \Rightarrow \text{fault release.} \end{cases} \quad (44)$$

Summarizing the above discussion, the schematic diagram of the intermittent multiplicative sensor fault detection strategy is given in Figure 1.

4 | SIMULATION EXAMPLE

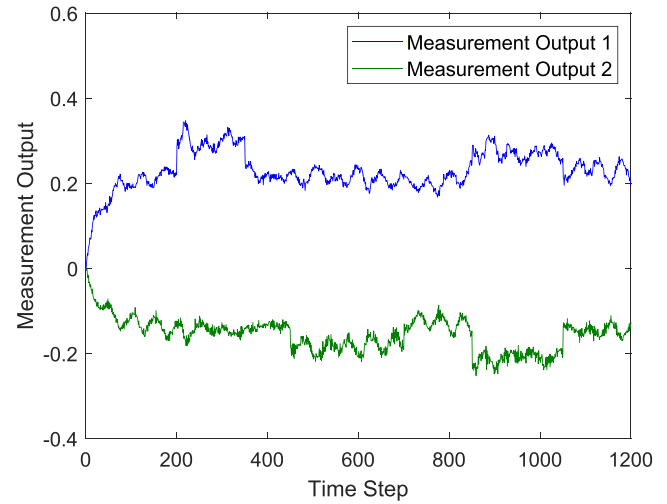
In this section, we apply the fault detection strategy to an example system to illustrate the effectiveness of our obtained results. Consider a stochastic uncertain LTV system with the following parameters:

$$\mathbf{A}_c(k) = \begin{bmatrix} 0.95 & 0.25 + 0.04 \cos(0.15k) \\ 0.14 + 0.06 \sin(0.13k) & -0.72 \end{bmatrix},$$

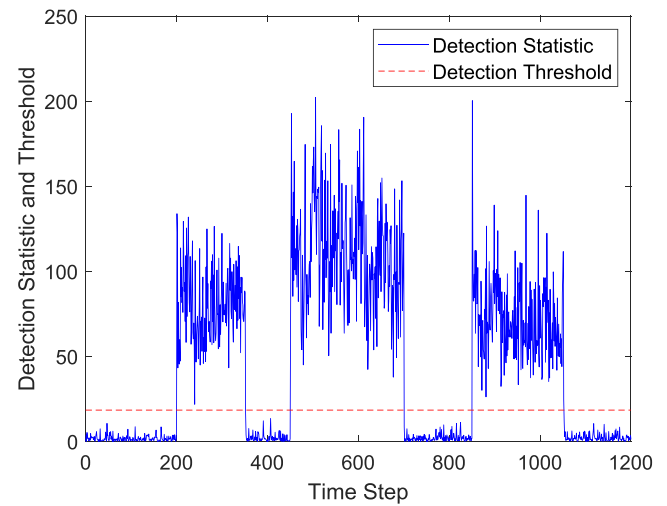
$$\mathbf{B}_c(k) = \begin{bmatrix} 0.62 + 0.08 \sin(0.12k) \\ 0.4 + 0.05 \cos(0.14k) \end{bmatrix},$$

$$\mathbf{C}_c(k) = \begin{bmatrix} 0.92 & 0.46 + 0.02 \sin(0.16k) \\ -0.68 + 0.03 \cos(0.12k) & 0.85 \end{bmatrix}.$$

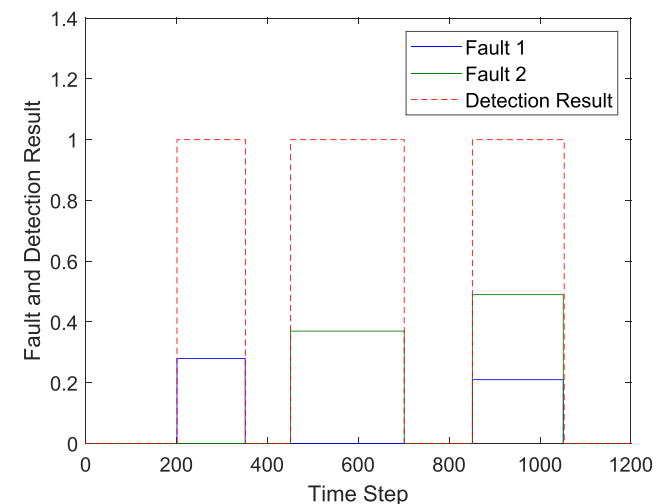
The initial state, process noise, and measurement noise are mutually independent zero-mean Gaussian white processes with covariances $\Sigma_{\mathbf{x}(0)} = 1.1\mathbf{I}_{n_x \times n_x} \times 10^{-5}$, $\Sigma_{\mathbf{w}(k)} = 1.3\mathbf{I}_{n_x \times n_x} \times 10^{-5}$, and $\Sigma_{\mathbf{v}(k)} = 1.2\mathbf{I}_{n_y \times n_y} \times 10^{-5}$, respectively. The uncertain parameter matrices are mutually independent zero-mean white processes with covariances:



(A) Measurement output

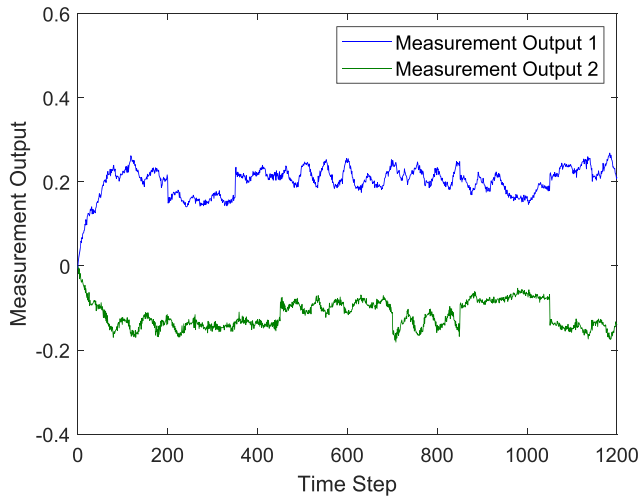


(B) Detection statistic and threshold

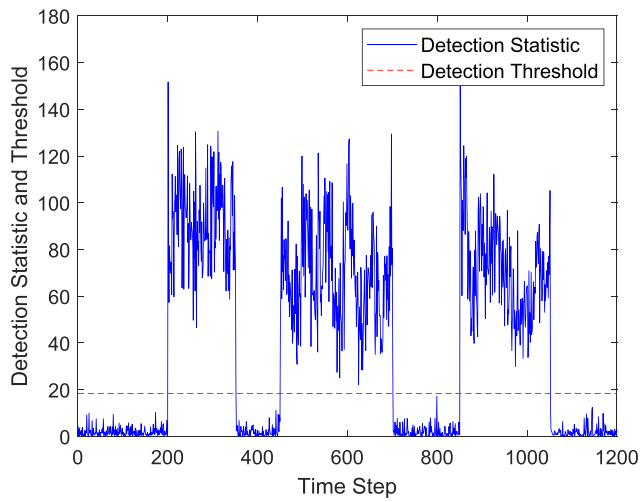


(C) Fault and detection result

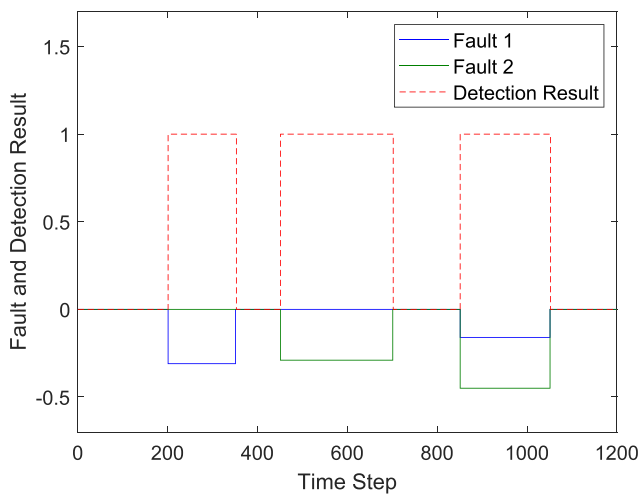
FIGURE 2 Detection result of the fault in Case 1 [Color figure can be viewed at wileyonlinelibrary.com]



(A) Measurement output

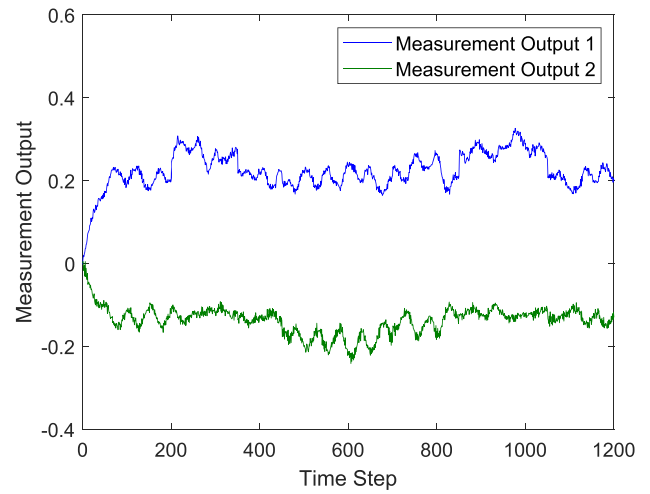


(B) Detection statistic and threshold

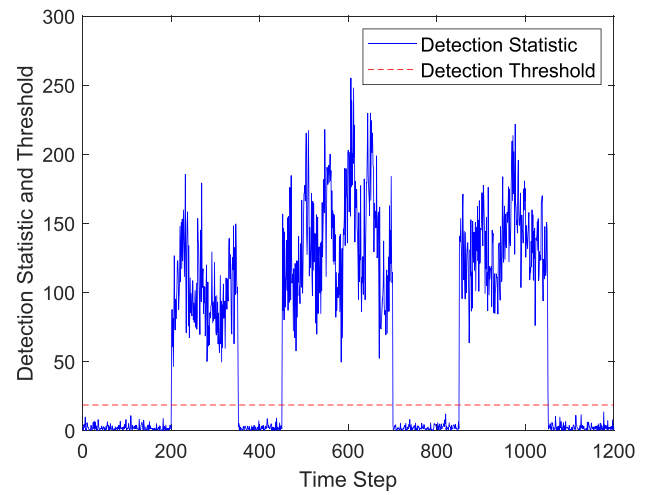


(C) Fault and detection result

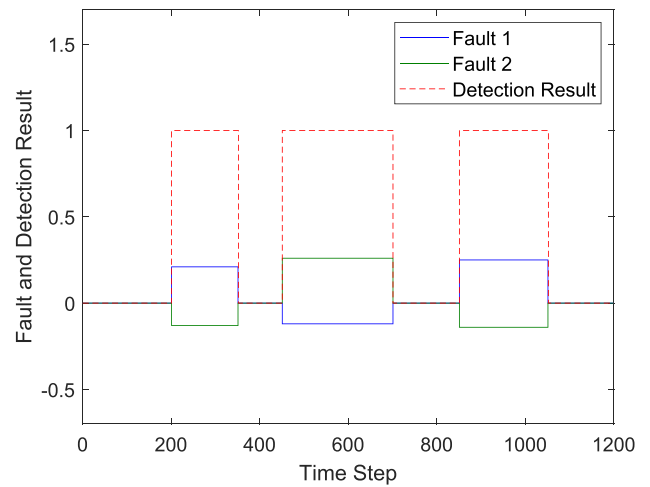
FIGURE 3 Detection result of the fault in Case 2
[Color figure can be viewed at wileyonlinelibrary.com]



(A) Measurement output



(B) Detection statistic and threshold



(C) Fault and detection result

FIGURE 4 Detection result of the fault in Case 3
[Color figure can be viewed at wileyonlinelibrary.com]

$$\Sigma_{\mathbf{A}_\delta(k)} = \begin{bmatrix} 1.2 & & \\ & 1 + 0.1 \sin(0.12k) & \\ & & 1.4 \\ & & & 1.3 + 0.2 \cos(0.14k) \end{bmatrix} \times 10^{-6},$$

$$\Sigma_{\mathbf{B}_\delta(k)} = \begin{bmatrix} 1.5 + 0.4 \sin(0.13k) & \\ & 1.2 + 0.3 \cos(0.12k) \end{bmatrix} \times 10^{-6},$$

$$\Sigma_{\mathbf{C}_\delta(k)} = \begin{bmatrix} 1.3 + 0.2 \cos(0.16k) & & \\ & 1.6 & \\ & & 1 + 0.5 \sin(0.15k) \\ & & & 1.2 \end{bmatrix} \times 10^{-6}.$$

To show the effectiveness of our proposed strategy, various fault cases are all taken into serious consideration as follows.

Case 1: Only positive fault occurs:

$$\mathbf{F}(k) = \begin{cases} \begin{bmatrix} 0.28 & \\ & 0 \end{bmatrix}, k \in [201, 350], \\ \begin{bmatrix} 0 & \\ & 0.37 \end{bmatrix}, k \in [451, 700], \\ \begin{bmatrix} 0.21 & \\ & 0.49 \end{bmatrix}, k \in [851, 1050], \\ \mathbf{0}_{2 \times 2}, \text{ otherwise.} \end{cases}$$

Case 2: Only negative fault occurs:

$$\mathbf{F}(k) = \begin{cases} \begin{bmatrix} -0.31 & \\ & 0 \end{bmatrix}, k \in [201, 350], \\ \begin{bmatrix} 0 & \\ & -0.29 \end{bmatrix}, k \in [451, 700], \\ \begin{bmatrix} -0.16 & \\ & -0.45 \end{bmatrix}, k \in [851, 1050], \\ \mathbf{0}_{2 \times 2}, \text{ otherwise.} \end{cases}$$

Case 3: Positive and negative fault simultaneously occur:

$$\mathbf{F}(k) = \begin{cases} \begin{bmatrix} 0.21 & \\ & -0.13 \end{bmatrix}, k \in [201, 350], \\ \begin{bmatrix} -0.12 & \\ & 0.26 \end{bmatrix}, k \in [451, 700], \\ \begin{bmatrix} 0.25 & \\ & -0.14 \end{bmatrix}, k \in [851, 1050], \\ \mathbf{0}_{2 \times 2}, \text{ otherwise.} \end{cases}$$

Simulation results are shown in Figures 2, 3, and 4. The measurement output, detection statistic and threshold, and corresponding detection result are depicted for all cases. We observe that our proposed fault detection strategy can detect both the occurrence and disappearance of the fault regardless of the fault type (positive/negative) or faulty sensors (sensor 1/sensor 2). The effectiveness of our proposed strategy is thus demonstrated successfully.

5 | CONCLUSION

This paper has addressed the intermittent multiplicative sensor fault detection problem for linear time-varying systems with stochastic noise and parameter uncertainties. By utilizing a quadratic optimization technique, we have designed a novel filter to ensure that the estimation error covariance is minimized iteratively. Based on the above, we have constructed a residual generator and analyzed the quantitative effect of the fault on it. Then we have designed a corresponding evaluation function and detection threshold to detect both the occurrence and disappearance of the fault. It should be noted that our proposed strategy takes model uncertainties into full consideration but does not depend on their structural information. Finally, we have also carried out a simulation study to demonstrate the effectiveness and applicability of the proposed strategy.

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