Real-Time Preventive Sensor Maintenance Using Robust Moving Horizon Estimation and Economic Model Predictive Control

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Conducting preventive maintenance of measurement sensors in real-time during process operation under feedback control while ensuring the reliability and improving the economic performance of a process is a central problem of the research area focusing on closed-loop preventive maintenance of sensors and actuators. To address this problem, a robust moving horizon estimation (RMHE) scheme and an economic model predictive control system are combined to simultaneously achieve preventive sensor maintenance and optimal process economic performance with closed-loop stability. Specifically, given a preventive sensor maintenance schedule, a RMHE scheme is developed that accommodates varying numbers of sensors to continuously supply accurate state estimates to a Lyapunov-based economic model predictive control (LEMPC) system. Closed-loop stability for this control approach can be proven under fairly general observability and stabilizability assumptions to be made precise in the manuscript. Subsequently, a chemical process example incorporating this RMHE-based LEMPC scheme demonstrates its ability to maintain process stability and achieve optimal process economic performance as scheduled preventive maintenance is performed on the sensors.
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Introduction

In the context of process operation, sensor and actuator maintenance programs and policies can greatly impact the process reliability, manufacturing safety and process economic performance. Among modern maintenance programs for process manufacturing, preventive maintenance of sensors and actuators in real-time can significantly mitigate the damage from production losses, process upsets, and downtime based on specific routine regulations.[1] Specifically, sensor preventive maintenance programs are important in the chemical processing industry.[2] For example, sensors working under severe conditions need to be frequently replaced as they may not withstand their working environment and consequently result in inaccurate readings.[3] In terms of when to conduct preventive maintenance on a sensor, the predicted time of failure or the replacement frequency can usually be obtained from the past sensor readings and probability distribution data of sensor failure.[4,5] Furthermore, continuous use of old sensors even without any obvious faults occurring may still result in unacceptably high manufacturing and maintenance costs in the long term.[1]

From a maintenance logistics point of view, sensors present a challenge because they are widely dispersed throughout a manufacturing plant. During a maintenance procedure when no redundant sensors are available, sensor replacement will directly result in sensor data losses which may cause significant process performance degradation under continuous operation. For example, when conducting the maintenance of some important sensors, vital machinery may be affected and bring process operation to a halt which makes large production losses unavoidable. Consequently, control system designs accounting for sensor data losses have received a lot of attention lately.[6–9] More specifically, in Ref. 7, a sensor reconfiguration-based method was proposed through characterizing stability regions for different control configurations utilizing different numbers of sensors to ensure stability after a sensor is taken off-line. In Ref. 9, a Lyapunov-based model predictive control system was designed to explicitly account for sensor data losses and implement the last computed optimal input trajectory (when sensor measurements are available for the last time) when sensor data is not available. However, in the context of smart manufacturing, integrating preventive sensor maintenance and process

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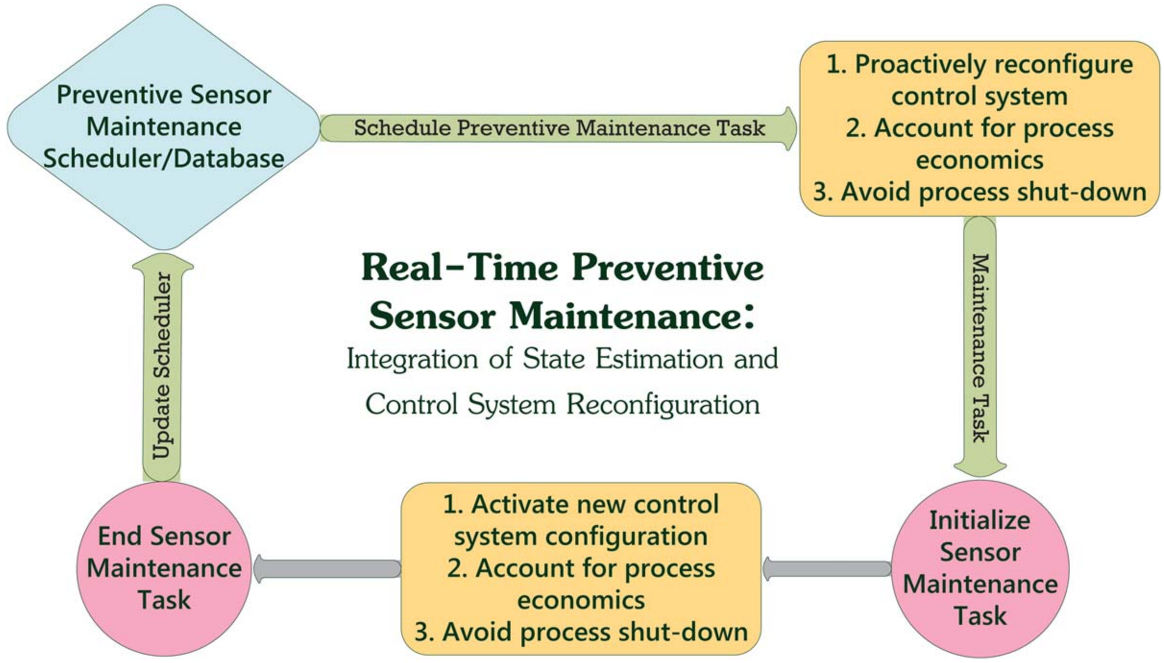

Figure 1. Logic sequence for real-time preventive sensor maintenance, incorporating maintenance events, economic optimization, and process control.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

economic optimization becomes an important problem to avoid unnecessary plant shutdown.[10,11]

To address the process performance degradation brought by sensor data losses, state estimation methodology can be adopted to provide accurate estimated state values when a sensor is taken off-line. In terms of state estimation for nonlinear systems, deterministic observer design approaches that explicitly account for nonlinearities have been developed, such as high-gain observers (HGOs) (including Luenberger-type observers)[12–15]; stochastic approaches or optimization-based methods are other choices for state estimation for nonlinear systems, such as the moving horizon estimation (MHE) method.[16,17] To achieve better robustness of the high-gain observer to measurement noise, robust moving horizon estimation (RMHE)[18] can be utilized to provide more robust state estimates than HGOs. In RMHE, the optimal state estimates are calculated within a confidence region depending on the measurement noise characteristics.

A recent theoretical development within the process control community is the establishment of a theoretical basis for economic model predictive control (EMPC), an optimization-based control methodology with a structure based on that of traditional tracking model predictive control (MPC) but with a cost function that represents the process economics. Thus, the control actions computed by EMPC will, ideally, force the process operation to remain economically optimal throughout time. Examples of recent developments for EMPC are asymptotic average performance bounds for EMPC using a self-tuning terminal cost and generalized terminal region constraint,[19] a performance and stability analysis for EMPC without terminal costs or constraints,[20] the use of event-triggering to initiate EMPC evaluations and reduce computation requirements,[21] an output feedback EMPC scheme with RMHE,[22] and a fast EMPC scheme by employing nonlinear programming sensitivities[23] (see, also, the review[24] for more recent results on EMPC).

In terms of control system design accounting for preventive actuator maintenance, our previous works[25,26] proposed integrating preventive maintenance on actuators and control system reconfiguration for tracking MPC[25] and economic MPC.[26] To complete the preventive maintenance tasks on sensors in real-time, it is necessary to design a control system that can simultaneously compute economically optimal control actions for a process and maintain process closed-loop stability even as the number of sensors is varied. While the preventive maintenance on actuators in Ref. 26 requires that the control system be able to adjust to a change in the number of manipulated inputs that it controls, preventive maintenance on sensors as described in the present work requires the control system to be able to adjust to changes in the number of measurements that it receives for feedback.

Considering all of the above requirements and objectives, EMPC is one natural approach to accomplish these tasks. In this work, a state estimation methodology is used with the EMPC system to provide state estimates when some state measurements become unavailable; such a state estimation methodology is not required in the case of preventive maintenance of actuators. Also, from the viewpoint of closed-loop stability, the way to handle sensor maintenance is different from that of handling actuator maintenance tasks. Once the actuator number changes in the actuator maintenance program, the available control energy changes which may lead to a change in the region of stable operation (i.e., EMPC may need to force the closed-loop state to another operating region to maintain closed-loop stability). For the sensor maintenance task, the available control energy stays the same during the maintenance, but the number of measurements changes. While the change in the number of measurements may not require a change in the region of operation, the state estimation problem changes as sensors are taken offline/brought back online, which poses unique theoretical and implementation challenges.

In this work, handling scheduled preventive sensor maintenance via the EMPC system design is considered. A RMHE scheme is developed that accommodates a varying number of sensors to continuously supply accurate state estimates to an EMPC system. Figure 1 illustrates the overall scheme integrating the real-time preventive sensor maintenance, process economics optimization, and control system reconfiguration. The Lyapunov-based EMPC (LEMPC) combined with RMHE is proved to be stabilizing under certain observability and stabilizability assumptions. Then, a chemical process example utilizing the proposed RMHE-based LEMPC is presented for which the controller maintains the process stability, accomplishes control system reconfiguration under a changing number of online sensors, and achieves minimal economic performance degradation by adjusting the optimization problem as the number of online sensors changes.

Preliminaries

Notation

The Euclidean norm of a vector is denoted by $|\cdot|$, and the notation $|\cdot|_Q$ denotes the weighted Euclidean norm with weighting matrix Q. A scalar-valued function $V:\mathbb{R}^n\rightarrow\mathbb{R}$ is positive definite if it evaluates to a positive scalar for all vectors $x\in\mathbb{R}^n$ in its domain except that $V(0)=0$. The level set of a positive definite scalar-valued function is denoted $\Omega_r:=\{x\ :\ V(x)\leq r\}$. A continuous, strictly increasing function $\alpha:[0,a)\rightarrow[0,\infty)$ with $\alpha(0)=0$ is said to be a class $\mathcal{K}$ function. The relative complement of A with respect to B is denoted as $B\setminus A:=\{x\in B\ :\ x\notin A\}$. The symbol $\text{diag}(v)$ signifies a diagonal matrix with the elements of the vector v on the diagonal. The sequence $\{t_{k\geq 0}\}$ denotes a synchronous partitioning of $\mathbb{R}_+$ where $t_k:=k\Delta$ for $k=0,\ 1,\ \ldots$ and $\Delta>0$ is the sampling period.

Class of nonlinear process systems

The control methodology that accounts for preventive sensor maintenance, as presented in this work, is developed for nonlinear systems of the following form

$$\begin{aligned}\dot{x}(t)&=f(x(t))+g(x(t))u(t)+l(x(t))w(t)\\ y(t)&=h(x(t))+v(t)\end{aligned}\tag{1}$$

where the vector of states is $x\in\mathbb{R}^n$, the vector of manipulated inputs is $u\in\mathbb{R}^m$, the vector of disturbances is $w\in\mathbb{R}^l$, the vector of measured outputs is $y\in\mathbb{R}^q$, the vector of measurement noise is $v\in\mathbb{R}^q$, and f, g, l, and h are sufficiently smooth vector or matrix functions of their arguments. Without loss of generality, we assume that $f(0)=0$ (the origin is assumed to be the equilibrium of the unforced system) and the initial time $t_0=0$. It is assumed that the bounds on the available control energy restrict its allowable values to a convex set

$$\mathbb{U}=\left\{u\in\mathbb{R}^m\ :\ u_i^{\min}\leq u_i\leq u_i^{\max},\ i=1,\ \ldots,\ m\right\}\tag{2}$$

The disturbance and measurement noise vectors are also assumed to be bounded

$$W=\left\{w\in\mathbb{R}^l\ :\ |w|\leq\theta_w\right\}\tag{3}$$

$$V=\{v\in\mathbb{R}^q\ :\ |v|\leq\theta_v\}\tag{4}$$

where θ_w and θ_v are positive constants that bound the disturbance and measurement noise vectors, respectively. The output measurement vector y of the system is assumed to be continuously available. It is assumed that the instantaneous value of the real-time process economics of the system of Eq. 1 can be modeled with a time-invariant, scalar-valued cost function $L_e(x,u)$.

The real-time preventive sensor maintenance schedule is defined as the change of the sensor group used in the EMPC system in real-time from the ith sensor group with q_i number of sensors functioning well to the jth sensor group with q_j number of sensors functioning well. This change in the number of online sensors occurs at the time $t=t_m$ determined by a scheduler or decision-maker, at which time one or more sensors are taken offline for preventive maintenance. As a result, the measurement vector changes from $y_i(t)=h_i(x(t))+v_i(t)\in\mathbb{R}^{q_i}$ to $y_j(t)=h_j(x(t))+v_j(t)\in\mathbb{R}^{q_j}$ (q_i and q_j are less than or equal to q). The state estimation structure will change to the jth group of sensors ($y_j(t)$) from the original ith group of sensors ($y_i(t)$).

Stabilizability assumption under state feedback control

A state feedback controller $u=k(x)$ that can asymptotically (and locally exponentially) stabilize the origin of the system of Eq. 1 when no disturbances are applied (the nominal closed-loop system, with $w(t)\equiv 0$) is assumed to exist. It is further assumed that this controller $k(x)$ meets all input constraints ($k(x)\in\mathbb{U}$) for all the initial states inside a given compact set containing the origin. These assumptions ensure the existence of class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i=1,2,3,4$ and of a continuously differentiable Lyapunov function $V(x)$ for the closed-loop nominal system under the feedback control law $k(x)$ that satisfy[27–29]

$$\begin{aligned}\alpha_1(|x|)\leq V(x)&\leq\alpha_2(|x|)\\ \frac{\partial V(x)}{\partial x}(f(x)+g(x)k(x))&\leq-\alpha_3(|x|)\\ \left|\frac{\partial V(x)}{\partial x}\right|&\leq\alpha_4(|x|)\end{aligned}\tag{5}$$

These inequalities hold for all $x\in\mathbb{D}\subseteq\mathbb{R}^n$ where $\mathbb{D}$ is an open neighborhood of the origin. The region of attraction of the nominal closed-loop system under the controller $k(x)$ is estimated as a level set of a Lyapunov function $V(x)$ for this closed-loop system ($\Omega_\rho\subseteq\mathbb{D}$) and is termed the "stability region."

Lyapunov-based economic model predictive control

This section describes the mathematical formulation of Lyapunov-based economic model predictive control (LEMPC),[30] a control strategy that will be used in this work to develop the structure and stability properties of a controller that can account for real-time preventive sensor maintenance. LEMPC is an optimization-based control methodology that calculates an input trajectory based on the following optimization problem

$$\max_{u\in S(\Delta)}\int_{t_k}^{t_{k+N}}L_e(\tilde{x}(\tau),u(\tau))\,d\tau\tag{6a}$$

$$\text{s.t.}\ \dot{\tilde{x}}(t)=f(\tilde{x}(t))+g(\tilde{x}(t))u(t)\tag{6b}$$

$$\tilde{x}(t_k)=x(t_k)\tag{6c}$$

$$u(t)\in\mathbb{U},\ \forall\,t\in[t_k,t_{k+N})\tag{6d}$$

$$\begin{aligned}V(\tilde{x}(t))\leq\rho_e,\ \forall\,t\in[t_k,t_{k+N})\\ \text{if}\ V(x(t_k))\leq\rho_e\end{aligned}\tag{6e}$$

$$\frac{\partial V}{\partial x}(f(x(t_k))+g(x(t_k))u(t_k)) \leq \frac{\partial V}{\partial x}(f(x(t_k))+g(x(t_k))k(x(t_k)))$$
$$\text{if } V(x(t_k)) > \rho_e \tag{6f}$$

The LEMPC maximizes the economics-based objective function in Eq. 6a to find the optimal value of the input u within the set $S(\Delta)$, which denotes the set of all piecewise constant input trajectories with period Δ. The input u must be maintained in the set of allowable inputs $\mathbb{U}$ per Eq. 6d, and also must cause the predicted state trajectory $\tilde{x}$ (from Eq. 6b) to satisfy the stability constraints of Eqs. 6e and 6f. Equations 6e and 6f define two modes of operation of the LEMPC, depending on the measured value $x(t_k)$ (Eq. 6c) of the actual process system state at time t_k, the time at the beginning of a sampling period Δ. The first mode (Mode 1) maintains the state within the level set Ω_{ρ_e} of the Lyapunov function V, where $\Omega_{\rho_e} \subset \Omega_\rho$. The set Ω_{ρ_e} is a region within which, if the process state at the beginning of a sampling period is within Ω_{ρ_e}, the process state at the end of the sampling period will still be within Ω_ρ even if bounded process noise is present. If the process state is not in Ω_{ρ_e} at the beginning of a sampling period, the constraint of Eq. 6f, which represents Mode 2, is activated. This constraint forces the time derivative of the Lyapunov function V along the trajectories of the nominal closed-loop system under LEMPC at the time t_k to decrease by at least as much as the time derivative of the Lyapunov function along the trajectories of the closed-loop system under the controller $k(x)$ at time t_k. The constraint of Eq. 6f is guaranteed to drive the closed-loop system state under LEMPC from $\Omega_\rho \backslash \Omega_{\rho_e}$ into Ω_{ρ_e} in finite time.

LEMPC is a receding-horizon strategy, which means that it receives a measurement of the process state x at the current time t_k, solves the optimization problem of Eq. 6 for the trajectory of u throughout the (finite) N sampling periods in the prediction horizon, and then only implements $u^*(t_k)$, the value of u for the first sampling period (sample-and-hold implementation). At the beginning of the next sampling period, the LEMPC receives a new state measurement and is resolved. The two-mode LEMPC design maintains stability of the closed-loop system in the sense that the state trajectories are always within Ω_ρ for any initial condition within this stability region.[30]

Observability assumption

For both the ith and jth state estimation structures, it is assumed that there exists a deterministic observer that takes the following general form (to facilitate the presentation, the subscript i/j is used to denote that an observer, which is required to satisfy specific assumptions given below, is defined for both the ith and the jth groups of sensors)

$$\dot{z}_{i/j}=F_{i/j}(\epsilon_{i/j}, z_{i/j}, y_{i/j}) \tag{7}$$

where $z_{i/j}$ are the observer states which are estimates of the actual system states, $y_{i/j}$ is the output measurement vector, and $\epsilon_{i/j}$ are positive parameters. When the state feedback controller $u=k(x)$ uses state estimates from the observers, it becomes an output feedback controller: $\dot{z}_{i/j}=F(\epsilon_{i/j}, z_{i/j}, y_{i/j})$, $u=k(z_{i/j})$. The following assumptions are made:

ASSUMPTION 1 (c.f. Ref. 22).

1. *there exist positive constants* θ_w^*, $\theta_{vi/j}^*$ *such that for each pair* $\{\theta_w, \theta_{vi/j}\}$ *with* $\theta_w \leq \theta_w^*$, $\theta_{vi/j} \leq \theta_{vi/j}^*$, *there exist* $0 < \rho_{1i/j} < \rho$, $e_{m0i/j} > 0$, $\epsilon_{Li/j}^* > 0$, $\epsilon_{Ui/j}^* > 0$ *such that if* $x(0) \in \Omega_{\rho_{1i/j}}$, $|z_{i/j}(0)-x(0)| \leq e_{m0i/j}$ *and* $\epsilon_{i/j} \in (\epsilon_{Li/j}^*, \epsilon_{Ui/j}^*)$, *the trajectories of the closed-loop system are bounded in* Ω_ρ *for all* $t \geq 0$;

2. *there exists* $e_{mi/j}^* > 0$ *such that for each* $e_{mi/j} \geq e_{mi/j}^*$, *there exists* $t_{bi/j}(\epsilon_{i/j})$ *such that* $|z_{i/j}(t)-x(t)| \leq e_{mi/j}$ *for all* $t \geq t_{bi/j}(\epsilon_{i/j})$.

An example of an observer for which these assumptions hold is a high-gain observer.[15] To increase the speed of estimation error convergence, the observer parameter $\epsilon_{i/j}$ should be chosen as small as possible; however, when the parameter $\epsilon_{i/j}$ is too small (i.e., the observer gain is too large), it will make the observer state estimate very sensitive to measurement noise. Thus, the observer parameter $\epsilon_{i/j}$ must be picked to be small enough so that the estimation error is reduced as quickly as possible, but not so small that the state estimates are corrupted by the noise. In the remainder of this work, the estimate given by the observer $F_{i/j}$ will be denoted as $z_{i/j}$.

REMARK 1. *Assumption 1 defines a state-space region (i.e.,* $\Omega_{\rho_{1i/j}}$*) from which the closed-loop system will be bounded in the stability region* Ω_ρ *for all initial conditions in* $\Omega_{\rho_{1i/j}}$ *and a required time length (i.e.,* $t_{bi/j}$*) which the observer needs for the state estimates to converge to a neighborhood of the actual process state values.*

State Estimation with Varying Number of Sensors

In this section, to compute the state estimates for both the ith and jth state estimation structures, the RMHE method is adopted with a deterministic observer used to calculate a confidence region. One of the deterministic observer designs is the high-gain observer formulation for multiple-input multiple-output systems.[14] For the sake of brevity, only the RMHE formulation is provided below.

Robust moving horizon estimation

To achieve considerable convergence speed of the state observer while significantly reducing its sensitivity to measurement noise, a RMHE scheme[18] is adopted with the following formulation

$$\min_{\tilde{X}(t_k)} \sum_{i=k-N_e}^{k-1} |w(t_i)|_{Q^{-1}}^2 + \sum_{i=k-N_e}^{k} |v(t_i)|_{R^{-1}}^2 + V_T(t_{k-N_e}) \tag{8a}$$

$$\text{s.t. } \dot{\tilde{x}}(t)=f(\tilde{x}(t))+g(\tilde{x}(t))u(t)+l(\tilde{x}(t))w(t) \tag{8b}$$

$$v(t)=y(t)-h(\tilde{x}(t)), \ \forall\, t \in [t_{k-N_e}, t_k] \tag{8c}$$

$$w(t) \in \mathbb{W},\ v(t) \in \mathbb{V},\ \tilde{x}(t) \in \Omega_\rho \tag{8d}$$

$$\dot{z}(t)=F(\epsilon, z(t), y(t)) \tag{8e}$$

$$z(t_{k-1})=\hat{x}^*_{\text{RMHE}}(t_{k-1}) \tag{8f}$$

$$|\tilde{x}(t_k)-z(t_k)| \leq \kappa|y(t_k)-h(z(t_k))| \tag{8g}$$

$$\tilde{X}(t_k) := \tilde{x}(t_{k-N_e}), \ldots, \tilde{x}(t_k) \tag{8h}$$

where N_e is the estimation horizon, κ is a positive adjustable parameter, Q is the covariance matrix for w, and R is the covariance matrix for v. The function $V_T(t_{k-N_e})$ is an arrival cost, which is a function that contains information on past process states, $\tilde{x}$ is the estimate of the state x which follows the dynamic model in Eq. 8b, $y(t_k)$ is the vector of measured outputs at time t_k, and $z(t_k)$ is the estimate of the process states from the deterministic state observer of Eq. 8e at time t_k based on continuous measurement of the output vector $y(t)$. The value of the observer state at t_{k-1} is initialized with the optimal estimate of the state $\hat{x}^*_{\text{RMHE}}(t_{k-1})$ from the RMHE at time t_{k-1} (Eq. 8f). This RMHE scheme is implemented with a finite horizon by approximating $w(t)$ and $v(t)$ as piecewise constant functions with sampling period Δ.

The RMHE is evaluated at time instants $\{t_{k\geq 0}\}$. It explicitly uses distribution/boundedness information on w, v, and x by considering that the most recent N_e measurements and past measurements are accounted for in the arrival cost $V_T(t_{k-N_e})$. The RMHE scheme optimizes the state estimate within a confidence region by maintaining the difference between the predicted and observer states within the region specified by Eq. 8g. In this way, the RMHE inherits the robustness properties of the deterministic observer and gives estimates with bounded errors. The solution to the optimization problem gives the optimal estimate of the current state which is denoted by

$$\hat{x}^*_{\text{RMHE}}(t_k)=\tilde{x}^*(t_k) \tag{9}$$

Preventive Sensor Maintenance via RMHE-Based LEMPC

We assume that the process operates under the RMHE-based LEMPC system with q_i sensors online (ith state estimation structure); at t_m, one or more sensors are taken offline for preventive maintenance and subsequently, the RMHE-based LEMPC system with q_j sensors (jth state estimation structure) is used. We note that the case that the sensors are brought back online could also be handled within this framework. In this section, the details are provided for the RMHE-based LEMPC design that facilitates preventive sensor maintenance, and in addition, stability of the process of Eq. 1 in closed-loop with such a controller is proved under the assumptions to be given below.

RMHE-based LEMPC design

As a result of the considered bounded process noise and uncertainties in the state estimation, subsets of the stability region Ω_ρ will be used to bound the process states in the design of the RMHE-based LEMPC. Specifically, the sets are defined as follows: $\Omega_{\rho_{ei}}$ is the subset of Ω_ρ for the state estimation based on the ith group of sensors and $\Omega_{\rho_{ej}}$ is the subset of Ω_ρ under the state estimation based on the jth group of sensors. To deal with the process uncertainty and measurement noise when the sensor state estimation structure changes from the ith group of sensors to the jth group of sensors, we need to ensure that the process state is driven into the new operation region $\Omega_{\rho_{ej}}$ by the time the available sensor group changes at $t=t_m$. Specifically, the RMHE-based LEMPC drives the process state into the suitable operation region $\Omega_{\rho_{ej}}$ by the time the q_j sensors are used to measure y_j. Once the jth group of sensors is active, the RMHE method based on the new observer denoted as F_j using the jth group of sensors and the corresponding new measurement vector, y_j, is used to provide the optimal state estimate $\hat{x}^*_{\text{RMHE}}(t_k)$ to the RMHE-based LEMPC. To facilitate practical implementation, we assume that $t_{bi/j}$ is an integer multiple of the sampling time, Δ, and $t_m \geq t_{bi/j}$ so that the process state estimation can converge before the available number of sensors changes.

The proposed RMHE-based LEMPC scheme can incorporate these issues that arise from real-time preventive sensor maintenance by solving the following

$$\max_{u\in S(\Delta)} \int_{t_k}^{t_{k+N}} L_e(\tilde{x}(\tau),u(\tau))\,d\tau \tag{10a}$$

$$\text{s.t.}\ \dot{\tilde{x}}(t)=f(\tilde{x}(t))+g(\tilde{x}(t))u(t) \tag{10b}$$

$$\tilde{x}(t_k)=\begin{cases} z_i(t_k), & \text{if } t_k < t_{bi} \\ \hat{x}^*_{\text{RMHE}i}(t_k), & \text{if } t_{bi}\leq t_k < t_m \end{cases} \tag{10c}$$

$$u(t)\in \mathbb{U},\ \forall\, t\in[t_k,t_{k+N}) \tag{10d}$$

$$\begin{aligned} &V(\tilde{x}(t))\leq \rho_{ei},\ \forall\, t\in[t_k,t_{k+N}) \\ &\text{if } V(\hat{x}(t_k))\leq \rho_{ei} \text{ and } t_k < t_m-t_s \end{aligned} \tag{10e}$$

$$\begin{aligned} &\frac{\partial V}{\partial x}(f(\hat{x}(t_k))+g(\hat{x}(t_k))u(t_k)) \leq \frac{\partial V}{\partial x}(f(\hat{x}(t_k))+g(\hat{x}(t_k))k(\hat{x}(t_k))) \\ &\text{if } V(\hat{x}(t_k)) > \rho_{ei} \text{ or } t_k \geq t_m-t_s \end{aligned} \tag{10f}$$

where the notation follows that in Eq. 6. When $t < t_{bi}$, state estimates, $z_i(t)$, are provided by an observer denoted as F_i using the ith group of sensors. When $t_{bi}\leq t < t_m$, the RMHE based on the observer F_i is utilized to provide the state estimate, which is denoted $\hat{x}^*_{\text{RMHE}i}(t)$, to the RMHE-based LEMPC. After t_m, the RMHE-based LEMPC problem is defined similarly to Eq. 10 and is based on the observer F_j (i.e., for $t\in[t_m,t_m+t_{bj})$, the state estimate is provided by the observer F_j and for $t\geq t_m+t_{bj}$, the state estimate is provided by the RMHE based on the observer F_j).

The Mode 1 and Mode 2 Lyapunov-based constraints of Eq. 6 are modified for the RMHE-based LEMPC strategy to account for the changing number of sensors. The Mode 2 constraint of the RMHE-based LEMPC is triggered not only if the state is outside $\Omega_{\rho_{ei}}$, but also when the time is between t_m-t_s and t_m. This adjustment to the constraints is made for stability reasons which are explained further in the "Stability Analysis" subsection below (i.e., $t_s > 0$ is selected such that the RMHE-based LEMPC operates in Mode 2 for sufficiently long to ensure closed-loop stability after t_m).

Remark 2. *The objective of this work is to propose a control framework that integrates process economic optimization, process control, and preventive sensor maintenance. Because EMPC is a control scheme that integrates economic optimization within the context of feedback control, it has been chosen for use as the controller for this work. Also, it is important to point out that EMPC may be considered a more general form of MPC than tracking MPC, which uses a positive definite cost function with respect to a prespecified reference trajectory or set-point. While other MPC schemes may be considered in place of EMPC for preventive sensor maintenance, the specific EMPC scheme considered (Lyapunov-based EMPC) may dictate a time-varying operating policy along with allowing for closed-loop stability guarantees. Furthermore, as the available number of sensors that are online changes, the state estimation problem must change to account for the varying number of measurements, which is not an issue of whether EMPC or MPC is used, but rather, an issue of the estimation scheme design to account for a varying number of measurements.*

Remark 3. *While the integration of scheduling and control has become a popular research topic in the process/systems engineering community, it is not immediately clear what, if any, benefit would be achieved by integrating* *scheduling of sensor maintenance tasks into the proposed control framework because it is likely that the use of a scheduler or another decision-maker would be used in determining which sensor needs to be taken offline for maintenance to minimize the impact of process performance degradation. The scheduler may be based on the life-cycle data of sensors, the observability of the process which has one or more sensors under maintenance, and the priority of different sensors. Nevertheless, closed-loop simulations like the type performed in the example section may help to develop a plan/schedule for sensor maintenance based on stability, reliability and economics concerns.*

Implementation strategy

The implementation strategy of the preventive sensor maintenance method is illustrated by Figure 2. Specifically, the control system initially uses the ith group of sensors where q_i ($q_i < n$) outputs are continuously measured to compose the measurement vector, y_i. A state estimation method is utilized to obtain the state estimate based on these q_i sensor measurements by either observer F_i or RMHE. Initially, the RMHE is not utilized until the state estimation error converges to a small value based on the observability assumption (i.e., up to $t=t_{bi}$ only the observer is utilized to provide state estimates to the RMHE-based LEMPC for the optimal input trajectory calculation). Then, starting from $t=t_{bi}$, the RMHE-based LEMPC takes advantage of the state estimate from the RMHE to compute the optimal input trajectories. At the same time, as the optimal state estimate from RMHE is expected to be more accurate after $t=t_{bi}$, the current state of the observer F_i is reset to be the optimal state estimate from RMHE at every sampling period. A similar RMHE implementation holds using F_j after the sensor group changes at t_m.

LEMPC is the proposed control framework for this preventive sensor maintenance problem because of its ability to economically optimize the process even while switching constraints as the online sensor groups are varied. The RMHE-based LEMPC is a receding horizon control strategy like the LEMPC of Eq. 6a. The following algorithm describes the logic for the implementation of the RMHE-based LEMPC, including the change in the state estimation structure:

1. Initialize the observer F_i with $z_i(0)$ and implement the observer F_i continuously based on the continuous output measurement $y_i(t)$.
2. At the current sampling instance t_k, if $t_k < t_{bi}$ or $t_k \in [t_m, t_m+t_{bj})$, go to Step 2.1; else if $t_k \in [t_{bi}, t_m)$ or $t_k \geq t_m+t_{bj}$, go to Step 2.2.
   2.1 The RMHE-based LEMPC receives the state estimate from the deterministic observer of Eq. 7; then go to Step 3.
   2.2 Based on the state estimate provided by the state observer of Eq. 7 and the output measurements at the current and previous sampling instants, the RMHE of Eq. 8 calculates the optimal state estimate $\hat{x}^*_{\mathrm{RMHE}}(t_k)$ which is sent to the RMHE-based LEMPC; then go to Step 3.
3. If $x(t_k) \notin \Omega_{\rho_{ei}}$ and $t_k \in [0, t_m-t_s)$, or if $x(t_k) \notin \Omega_{\rho_{ej}}$ and $t_k \geq t_m$, or if $t_k \in [t_m-t_s, t_m)$, go to Step 3.1. Else, go to Step 3.2.
   3.1 The RMHE-based LEMPC operates in Mode 2. Go to Step 4.
   3.2 The RMHE-based LEMPC operates in Mode 1. Go to Step 4.

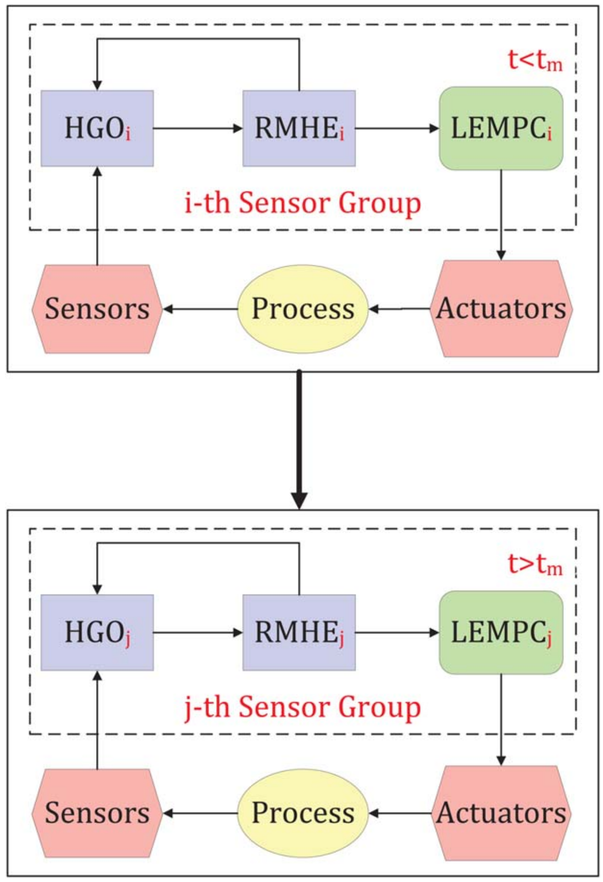

Figure 2. RMHE-based LEMPC system reconfiguration diagram for real-time preventive sensor maintenance (LEMPC$_i$ denotes the LEMPC scheme with the *i*th sensor group based on the state estimates from the RMHE denoted as RMHE$_i$ and the HGO denoted as HGO$_i$; the same holds for LEMPC$_j$ but with *i* replaced by *j*).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

4. The RMHE-based LEMPC calculates N vectors of control actions, one for each sampling period in the prediction horizon $t \in [t_k, t_{k+N})$, and sends the control action $u^*(t_k)$ to the actuators for sample-and-hold implementation for one sampling period ($t \in [t_k, t_{k+1})$).
5. Go to Step 2 ($k \leftarrow k+1$).

Remark 4. *Variations of the maintenance schedule outside of the schedule considered in the article can be handled by the proposed RMHE-based LEMPC scheme and are a conceptually straightforward extension of the present work. For example, a sensor maintenance schedule where some sensors are taken offline at time t_m, but only a few are brought back online at a later time can be handled. For simplicity of presentation and notation, we have only presented the most (conceptually) challenging case where sensors are taken offline at a time for preventive sensor maintenance. Nevertheless, all of the arguments could be repeated to handle the case where (some) sensors are being brought online through redefining the notation (e.g., t_m now becomes the time at which the sensors are being brought back online). From a stability and observability perspective, the stabilizability and* *observability assumptions must be satisfied for the new sensor group, which may require the design of an additional deterministic observer and RMHE scheme for this sensor group.*

Remark 5. *From a practical perspective, when sensors are being brought back online or taken offline and the estimation problem changes, the estimation error may be sufficiently small such that one may use the RMHE to provide state estimates to the LEMPC starting at t_m (i.e., applying the deterministic observer from t_m to t_m+t_{bj} may not be required). This is the case in the chemical process example presented in this work.*

Remark 6. *For output feedback-based control of nonlinear systems, typically what is required to prove closed-loop stability is that the estimation error converges sufficiently fast relative to the time-scale of the process dynamics (i.e., there is a time-scale separation between the estimation error dynamics and the process dynamics). This means that it takes very little time for the estimation error to converge. Therefore, there is little, if not no, limitation to conducting sensor maintenance after the state estimation problem has converged. For the case that a sensor is taken offline before the estimation error has converged, one may simply disregard the measurements of the sensor(s) that will be taken offline and formulate the estimation problem at $t=0$ utilizing the measurements of sensors that will remain online. Moreover, for the proposed RMHE-based LEMPC scheme to work in either case (whether the sensor is taken offline before the estimation error converges or the sensor is taken offline after the state estimator has converged), the same assumptions need to be satisfied. We note that when the state estimation problem converges, it converges to a neighborhood of the actual state value (i.e., the estimated state never becomes identically equal to the actual state).*

Stability analysis

This section addresses the stability of the system of Eq. 1 when controlled by the RMHE-based LEMPC of Eq. 10a. We first present several propositions and then summarize the main results in a theorem. Proposition 1 characterizes the continuity property of the Lyapunov function V. Proposition 2 characterizes the effects of bounded state estimation error and process noise. The proofs of these statements can be found in the sources referenced for each.

Proposition 1 (c.f. Refs. 9 and 31). *Consider the Lyapunov function $V(\cdot)$. There exists a quadratic function $f_V(\cdot)$ such that*

$$V(x) \le V(\hat{x})+f_V(|x-\hat{x}|) \tag{11}$$

for all $x, \hat{x} \in \Omega_\rho$ with

$$f_V(s)=\alpha_4(\alpha_1^{-1}(\rho))s+M_v s^2 \tag{12}$$

where M_v is a positive constant.

Proposition 2 (c.f. Ref. 22). *Consider the systems*

$$\begin{aligned}\dot{x}_a(t)&=f(x_a)+g(x_a)u(t)+l(x_a)w(t)\\ \dot{x}_b(t)&=f(x_b)+g(x_b)u(t)\end{aligned} \tag{13}$$

with initial states $|x_a(0)-x_b(0)| \le \delta_{xi/j}$. There exists a function $f_W(\cdot,\cdot)$ such that

$$|x_a(t)-x_b(t)| \le f_W(\delta_{xi/j}, t) \tag{14}$$

for all $x_a(t), x_b(t) \in \Omega_\rho$ and $u \in \mathbb{U}$, $w \in \mathbb{W}$ with

$$f_W(s,\tau) := \left(s+\frac{M_l\theta_w}{L_f+L_g u^{\max}}\right)e^{(L_f+L_g u^{\max})\tau}-\frac{M_l\theta_w}{L_f+L_g u^{\max}} \tag{15}$$

where L_f, L_g, M_l are positive constants associated with functions f, g, l.

If the system is initialized using either the ith sensor group or jth sensor group and the sensor group does not change online throughout the length of operation, then closed-loop stability follows if certain conditions hold.[22] The following result provides sufficient conditions such that the closed-loop state under the RMHE-based LEMPC of Eq. 10 will be bounded in Ω_ρ for the case that the available sensors do not change online.

Proposition 3 (Ref. 22). *Consider the system of* Eq. 1 *in closed-loop under the RMHE-based LEMPC of* Eq. 10 *based on an observer satisfying Assumption 1 (formulated for either the ith sensor group or the jth sensor group) and a controller that renders the origin of the closed-loop system asymptotically (and locally exponentially) stable under state feedback and continuous implementation. Let $\theta_w \le \theta_w^*$, $\theta_{vi/j} \le \theta_{vi/j}^*$, $\epsilon_{i/j} \in (\epsilon_{Li/j}^*, \epsilon_{Ui/j}^*)$, and $|z_{i/j}(0)-x(0)| \le e_{m0i/j}$. Also, let $\epsilon_w > 0$, $\Delta > 0$ and $\rho > \rho_{1i/j} > \rho_{ei/j} > \rho_{\min,i/j} > \rho_{si/j} > 0$ and $\kappa_{i/j} \ge 0$ satisfy the following conditions*

$$\rho_{ei/j} \le \rho-\max\{f_V(f_W(\delta_{xi/j},\Delta))+f_V(\delta_{xi/j}), Mt_{bi/j}\alpha_4(\alpha_1^{-1}(\rho))\} \tag{16}$$

$$\begin{aligned}&-\alpha_3(\alpha_2^{-1}(\rho_{si/j}))+\left(L_V^f+L_V^g u^{\max}\right)(M\Delta+\delta_{xi/j})+M_V^l\theta_w\\ &\quad\le -\epsilon_w/\Delta\end{aligned} \tag{17}$$

where $\delta_{xi/j}=(\kappa_{i/j}L_{hi/j}+1)e_{mi/j}+\kappa_{i/j}\theta_{vi/j}$, $L_{hi/j}$, L_V^f, L_V^g are Lipschitz constants associated with $h_{i/j}$, $\frac{\partial V}{\partial x}f$, and $\frac{\partial V}{\partial x}g$, respectively, M is a constant that bounds the time derivative of x (i.e., $|\dot{x}| \le M$) and M_V^l is a constant that bounds $|\frac{\partial V}{\partial x}l|$ for $x \in \Omega_\rho$. Then, if $x(0) \in \Omega_{\rho_{ei/j}}$, then $x(t) \in \Omega_\rho$ for all $t \ge 0$. Moreover, if after some time, the RMHE-based LEMPC operates in Mode 2 only then the state is ultimately bounded in $\Omega_{\rho_{\min,i/j}}$.

To cope with the changing number of sensors at t_m, the state needs to be forced to a compact set containing the origin (i.e., $\Omega_{\rho_{ej}}$) in preparation for the switch from sensor group i to sensor group j. This is addressed by enforcing the Mode 2 constraint in the RMHE-based LEMPC of Eq. 10 from t_m-t_s to t_m. If $x(t_m) \in \Omega_{\rho_{ej}}$, we can apply the results of Proposition 3 and thus, guarantee that the closed-loop state is bounded in Ω_ρ. This result is stated in the following theorem.

Theorem 1. *Let $\rho_{\min,i} \le \rho_{ej}$ ($\Omega_{\rho_{\min,i}} \subseteq \Omega_{\rho_{ej}}$), $e_{mi} \le e_{m0j}$, and the assumptions of* Proposition 3 *be satisfied for the ith and jth sensor groups. If $t_s > 0$ is sufficiently large and $x(0) \in \Omega_{\rho_{ei}}$, then the closed-loop state under the RMHE-based LEMPC of* Eq. 10 *is bounded in Ω_ρ for $t \ge 0$.*

Proof. It is necessary to show that the estimation error is less than e_{m0j} at t_m and that the RMHE-based LEMPC drives the state of the closed-loop system into $\Omega_{\rho_{ej}}$ by t_m. If both of these can be shown, Proposition 3 states that the closed-loop trajectory for $t > t_m$ will remain bounded in Ω_ρ.

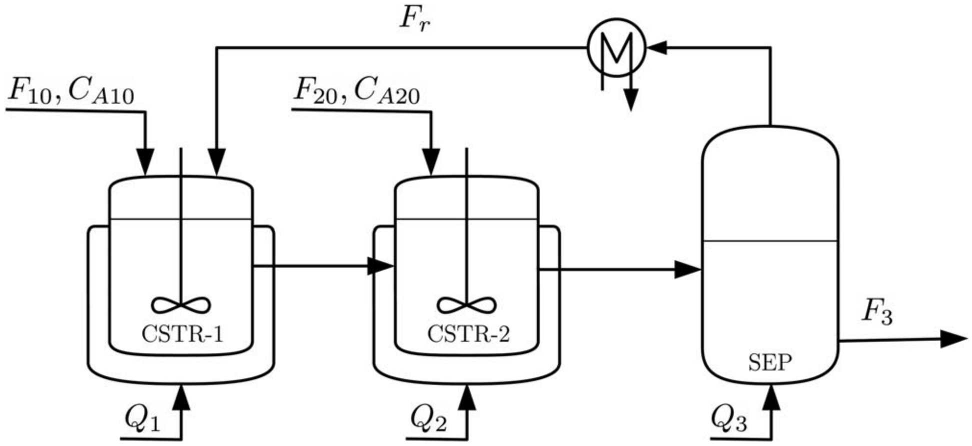

Figure 3. Process flow diagram of the CSTR-CSTR-Separator process network from Ref. 32.

After t_{bi} (i.e., after the deterministic observer F_i has converged), the estimation error is bounded by e_{mi} owing to the properties of the deterministic observer (Assumption 1). If the deterministic observer F_j is initialized at t_m with the state estimate computed by the RMHE for the ith sensor group at t_m, then the initial error when F_j takes over at t_m is bounded by $|z(t_m)-x(t_m)| \le e_{mi} \le e_{m0j}$ if $e_{mi} \le e_{m0j}$. Thus, the estimation error is less than e_{m0j} at t_m.

It is possible for the RMHE-based LEMPC with Mode 2 operation only, in a finite but sufficiently long time interval, to drive the state from any initial condition in Ω_ρ into $\Omega_{\rho_{\min,i}}$ and maintain the state within that set (i.e., $\Omega_{\rho_{\min,i}}$ is forward invariant under the RMHE-based LEMPC). This follows from the fact that the state is ultimately bounded in $\Omega_{\rho_{\min,i}}$ (Proposition 3) if the RMHE-based LEMPC operates exclusively in Mode 2 and no changes are made to the sensors throughout the length of operation (see, also, the proof of [Ref. 22, Theorem 1] for a complete proof of this fact). If $\rho_{\min,i} \le \rho_{ej}$, then there exists $N^* \in \mathbb{I}_+$ such that under the RMHE-based LEMPC of Eq. 10 operating under Mode 2 only $x(N^*\Delta) \in \Omega_{\rho_{\min,i}} \subseteq \Omega_{\rho_{ej}}$ for all $x(0) \in \Omega_{\rho_{ei}}$. If $t_s \ge N^*\Delta$, then $x(t_m) \in \Omega_{\rho_{\min,i}} \subseteq \Omega_{\rho_{ej}}$ under the RMHE-based LEMPC of Eq. 10. From Proposition 3, boundedness of the closed-loop state in Ω_ρ under the RMHE-based LEMPC when the available sensors change at t_m follows. ■

Remark 7. *In terms of the relationship between $t_{bi/j}$ and $N_e\Delta$, we note here that when $t \ge t_{bi/j}$, the RMHE is activated and its horizon length is chosen as $\min\{\frac{t_{bi/j}}{\Delta}, N_e\}$ considering the assumption that $t_{bi/j}$ is an integer multiple of Δ. Based on this choice, after the RMHE is activated (i.e., $t \ge t_{bi/j}$) it is possible for the process to have a time-varying horizon length for several sampling times if $t_{bi/j} \le N_e\Delta$.*

Application to a Chemical Process Network

Description of the chemical process network

The chemical process network from Ref. 32 is used to illustrate the design of the proposed RMHE-based LEMPC and its usefulness when sensor preventive maintenance is scheduled. The process consists of three vessels: two continuously stirred tank reactors (CSTRs) in series, and a flash tank that receives the effluent from the second CSTR. The process is depicted by the process flow diagram of Figure 3. A second-order reaction with rate r occurs in the CSTRs

$$A \xrightarrow{r} B$$

Both CSTRs (CSTR-1 and CSTR-2) receive fresh feeds of the reactant A in an inert solvent C at flow rates F_{10} and F_{20} with concentrations C_{A10} and C_{A20}, respectively. In addition, to recover unreacted A, CSTR-1 receives recycled condensed vapor at flow rate F_r from the flash tank separator (SEP). The desired product B is obtained from the liquid that exits the flash tank. The temperature in the vessels is adjusted by controlling the heating/cooling rate Q_j, $j=1, 2, 3$ to each of the vessels. It is assumed that the heat of reaction and heat capacity are constant in the temperature range considered and that the liquid has constant density such that all three vessels have static holdup. In the separator, the reaction rate of the reaction $A \xrightarrow{r} B$ is negligible. Material and energy balances were used to derive the following dynamic equations for the system

$$\frac{dT_1}{dt}=\frac{F_{10}}{V_1}(T_{10}-T_1)+\frac{F_r}{V_1}(T_3-T_1)+\frac{-\Delta H}{\rho C_p}ke^{\frac{-E}{RT_1}}C_{A1}^2+\frac{Q_1}{\rho C_p V_1} \tag{18a}$$

$$\frac{dC_{A1}}{dt}=\frac{F_{10}}{V_1}(C_{A10}-C_{A1})+\frac{F_r}{V_1}(C_{Ar}-C_{A1})-ke^{\frac{-E}{RT_1}}C_{A1}^2 \tag{18b}$$

$$\frac{dC_{B1}}{dt}=\frac{-F_{10}}{V_1}C_{B1}+\frac{F_r}{V_1}(C_{Br}-C_{B1})+ke^{\frac{-E}{RT_1}}C_{A1}^2 \tag{18c}$$

$$\frac{dT_2}{dt}=\frac{F_1}{V_2}(T_1-T_2)+\frac{F_{20}}{V_2}(T_{20}-T_2)+\frac{-\Delta H}{\rho C_p}ke^{\frac{-E}{RT_2}}C_{A2}^2+\frac{Q_2}{\rho C_p V_2} \tag{18d}$$

$$\frac{dC_{A2}}{dt}=\frac{F_1}{V_2}(C_{A1}-C_{A2})+\frac{F_{20}}{V_2}(C_{A20}-C_{A2})-ke^{\frac{-E}{RT_2}}C_{A2}^2 \tag{18e}$$

$$\frac{dC_{B2}}{dt}=\frac{F_1}{V_2}(C_{B1}-C_{B2})-\frac{F_{20}}{V_2}C_{B2}+ke^{\frac{-E}{RT_2}}C_{A2}^2 \tag{18f}$$

$$\frac{dT_3}{dt}=\frac{F_2}{V_3}(T_2-T_3)-\frac{\Delta H_{\mathrm{vap}}F_{rm}}{\rho C_p V_3}+\frac{Q_3}{\rho C_p V_3} \tag{18g}$$

$$\frac{dC_{A3}}{dt}=\frac{F_2}{V_3}(C_{A2}-C_{A3})-\frac{F_r}{V_3}(C_{Ar}-C_{A3}) \tag{18h}$$

$$\frac{dC_{B3}}{dt}=\frac{F_2}{V_3}(C_{B2}-C_{B3})-\frac{F_r}{V_3}(C_{Br}-C_{B3}) \tag{18i}$$

where the temperatures of vessels $j=1, 2, 3$, corresponding to CSTR-1, CSTR-2, and SEP, respectively, are denoted as T_j, C_{Aj}, and C_{Bj} are the concentrations of species A and B in vessel

Table 1. Description of Process Parameters and Their Values for the Reactor and Separator Process Network[32]

Parameter/Value	Description
$F_{10}=5.0\ \mathrm{m^3/h}$	Flow rate of CSTR-1 inlet
$F_{20}=5.0\ \mathrm{m^3/h}$	Flow rate of CSTR-2 inlet
$F_r=2.0\ \mathrm{m^3/h}$	Recycle flow rate of SEP
$T_{10}=300\ \mathrm{K}$	Temperature of F_{10}
$T_{20}=300\ \mathrm{K}$	Temperature of F_{20}
$V_1=5.0\ \mathrm{m^3}$	Volume of CSTR-1
$V_2=5.0\ \mathrm{m^3}$	Volume of CSTR-2
$V_3=3.0\ \mathrm{m^3}$	Volume of SEP
$k=1.9\times10^9\ \mathrm{m^3/(kmol\ h)}$	Pre-exponential factor
$E=7.1\times10^4\ \mathrm{kJ/kmol}$	Activation energy
$\Delta H=-7.8\times10^3\ \mathrm{kJ/kmol}$	Heat of reaction
$\Delta H_{\mathrm{vap}}=4.02\times10^4\ \mathrm{kJ/kmol}$	Heat of vaporization
$C_p=0.231\ \mathrm{kJ/(kg\ K)}$	Heat capacity
$R=8.314\ \mathrm{kJ/(kmol\ K)}$	Gas constant
$\rho=1000\ \mathrm{kg/m^3}$	Liquid solution density
$\alpha_A=3.0$	Relative volatility of A
$\alpha_B=0.8$	Relative volatility of B
$\alpha_C=1.0$	Relative volatility of C
$M_A=18\ \mathrm{kg/kmol}$	Molecular weight of A
$M_B=18\ \mathrm{kg/kmol}$	Molecular weight of B
$M_C=40\ \mathrm{kg/kmol}$	Molecular weight of C

j, and the molar flow rate of the recycle stream is F_r. The parameter notation and values are given in Table 1 where 'h' is the unit for hours. The flow rates F_1, F_2, and F_3 are the outlet flow rates from CSTR-1, CSTR-2, and SEP, respectively, and their values can be obtained by setting the sums of the flow rates into each vessel equal to the sums of the flow rates out of each vessel because of the constant fluid density assumption used with the mass balances.

It is assumed that approximating the relative volatilities of A, B, and C as constants results in a sufficiently accurate model for this process. Under that assumption, the flash separator overhead composition can be described using algebraic equations

$$C_{C3}=(\rho-C_{A3}M_A-C_{B3}M_B)/M_C \tag{19a}$$

$$C_{ir}=\frac{\alpha_i\rho C_{i3}}{\alpha_A C_{A3}M_A+\alpha_B C_{B3}M_B+\alpha_C C_{C3}M_C},\quad i=A,\,B,\,C \tag{19b}$$

$$F_{rm}=F_r(C_{Ar}+C_{Br}+C_{Cr}) \tag{19c}$$

where α_i and C_{ir} are the relative volatility and the overhead vapor concentration of each species, respectively, and F_{rm} is the molar flow rate of the recycle stream.

Control objectives and process economics

The five manipulated inputs for the process are the heat rates Q_j to vessels $j=1,\,2,\,3$ and the concentrations C_{A10} and C_{A20} of A in the fresh feeds to CSTR-1 and CSTR-2, respectively. These inputs are bounded as follows

$$|Q_j|\ \le\ 1.0\times10^5\ \mathrm{kJ/h},\quad j=1,\,2 \tag{20a}$$

$$2.2\times10^6\le Q_3\le 2.6\times10^6\,\mathrm{kJ/h} \tag{20b}$$

$$0.5\le C_{Aj0}\le 7.5\ \mathrm{kmol/m^3},\quad j=1,\,2 \tag{20c}$$

where the input vector has the form $u=[Q_1\,Q_2\,Q_3\,C_{A10}\,C_{A20}]^T=[u_1\,u_2\,u_3\,u_4\,u_5]^T$. In addition to the input constraints, state constraints are imposed on the operating temperature of the two CSTRs and of the separator of the form

$$370\le T_j\le 395\ \ \mathrm{K},\quad j=1,\,2 \tag{21a}$$

$$380\le T_3\le 400\ \ \mathrm{K} \tag{21b}$$

The control objective is to maximize the revenue per unit cost under the proposed RMHE-based LEMPC while accounting for preventive sensor maintenance. The process economic objective function chosen to accomplish this goal is

$$L_e(x,u)=\frac{P_pC_{B3}}{P_h(|Q_1|+|Q_2|+|Q_3|)+P_m(F_{10}C_{A10}+F_{20}C_{A20})} \tag{22}$$

where the unit price of product is $P_p=10$, the unit price of reactant material is $P_m=2$, and the unit price of heat supply/removal is $P_h=1.0\times10^{-4}$. The same pricing is taken to apply for both heating and cooling, but this assumption does not limit the essence of the results of this study and can be readily relaxed.

The economically optimal steady state is determined so that its stability region can be used in the RMHE-based LEMPC. The economically optimal steady state is determined by solving

$$\max_{x_s,u_s}\quad L_e(x_s,u_s) \tag{23a}$$

$$\text{s.t.}\quad f(x_s)+g(x_s)u_s=0 \tag{23b}$$

$$u_s\in\mathbb{U} \tag{23c}$$

$$x_s\in\mathbb{X} \tag{23d}$$

where $L(x_s,u_s)$ is the revenue per unit cost that will be used as the RMHE-based LEMPC objective (Eq. 22). The algebraic equation in Eq. 23b provides the steady-state solutions to the nominal input-affine dynamic system of Eq. 18, and the constraints in Eqs. 23c and 23d correspond to the input constraints of Eq. 20 and the state constraints of Eq. 21, respectively. The solution to this steady-state optimization problem is x_s^*, which satisfies Eq. 23b when paired with u_s^*

$$\begin{aligned}x_s^*&=[T_{1s}^*\quad C_{A1s}^*\quad C_{B1s}^*\quad T_{2s}^*\quad C_{A2s}^*\quad C_{B2s}^*\quad T_{3s}^*\quad C_{A3s}^*\quad C_{B3s}^*\,]^T\\&=[395\quad 2.32\quad 2.17\quad 395\quad 2.75\quad 2.15\quad 380\quad 1.88\quad 2.31]^T\\u_s^*&=[Q_{1s}^*\quad Q_{2s}^*\quad Q_{3s}^*\quad C_{A10s}^*\quad C_{A20s}^*\,]^T\\&=[-3.25\times10^3\quad -1.08\times10^3\quad 2.2\times10^6\quad 0.5\quad 0.5]^T\end{aligned} \tag{24}$$

The units for each variable in the optimal steady-state vector x_s^* and optimal steady-state input vector u_s^* are the same as for the corresponding variables in Table 1. The steady state of Eq. 24 is open-loop unstable.

It is assumed that the sensors in this chemical process example undergo routine sensor preventive maintenance to prevent the potentially large economic losses that could occur due to the consequences of sensor failure, such as product contamination or plant shut-down. Thus, we assume that the given process can be safely operated with the remaining sensors when the maintenance procedures considered in this example are performed (the process can continue to be safely operated even as certain sensors are taken offline).

The process is outfitted with sensors that provide measurements of C_{A3}, C_{B1}, C_{B2}, C_{B3}, T_1, T_2, and T_3. A state estimation method is applied to the process to compute state estimates of C_{A1} and C_{A2}. There is a scheduled maintenance task on the sensor that measures C_{A3} at $t_m=0.3\ h$ (i.e., for $t\ge t_m$, the control system will not receive any measurements from the sensor of C_{A3} any longer). The sensors that provide the measurements of C_{B1}, C_{B2}, C_{B3}, T_1, T_2, and T_3 will continue to be available to the control system and the states C_{A1}, C_{A2}, and C_{A3} need to be estimated. Thus, the first sensor group consists of all the available sensors and the second sensor group consists of all the available sensors except the sensor that measures C_{A3}.

Deterministic observer and RMHE design

Here, we use a HGO as the deterministic observer in the RMHE design. We note that the HGO changes after the preventive sensor maintenance is conducted at t_m. We will use HGO-1 and HGO-2 to denote the HGOs formulated for the first and second sensor groups, respectively. The design of HGO-2 (see Ref. 14) is provided here, and the design of HGO-1 follows from the design of HGO-2. The measurement vector after the C_{A3} sensor is taken offline at t_m is defined as $y=[h_1(x)\ h_2(x)\ h_3(x)\ h_4(x)\ h_5(x)\ h_6(x)]=[T_1\ C_{B1}\ T_2\ C_{B2}\ T_3\ C_{B3}]$. To obtain the state estimates, a high-gain observer is formulated as follows

$$\frac{d\bar{z}_1}{dt}=\frac{\alpha_1}{\epsilon}(y_1-\bar{z}_1) \tag{25a}$$

$$\frac{d\bar{z}_2}{dt}=\frac{\alpha_2}{\epsilon}(y_2-\bar{z}_2)+\bar{z}_3 \tag{25b}$$

$$\frac{d\bar{z}_3}{dt}=\frac{\alpha_3}{\epsilon^2}(y_2-\bar{z}_2) \tag{25c}$$

$$\frac{d\bar{z}_4}{dt}=\frac{\alpha_4}{\epsilon}(y_3-\bar{z}_4) \tag{25d}$$

$$\frac{d\bar{z}_5}{dt}=\frac{\alpha_5}{\epsilon}(y_4-\bar{z}_5)+\bar{z}_6 \tag{25e}$$

$$\frac{d\bar{z}_6}{dt}=\frac{\alpha_6}{\epsilon^2}(y_4-\bar{z}_5) \tag{25f}$$

$$\frac{d\bar{z}_7}{dt}=\frac{\alpha_7}{\epsilon}(y_5-\bar{z}_7) \tag{25g}$$

$$\frac{d\bar{z}_8}{dt}=\frac{\alpha_8}{\epsilon}(y_5-\bar{z}_8)+\bar{z}_9 \tag{25h}$$

$$\frac{d\bar{z}_9}{dt}=\frac{\alpha_9}{\epsilon^2}(y_6-\bar{z}_8) \tag{25i}$$

where the observer states are defined as

$$\bar{z}=T(x)=[T_1\ C_{B1}\ \dot{C}_{B1}\ T_2\ C_{B2}\ \dot{C}_{B2}\ T_3\ C_{B3}\ \dot{C}_{B3}]^T$$

and the mapping $T:\mathbb{R}^n\rightarrow\mathbb{R}^n$ is an appropriately chosen invertible coordinate change. The design parameters of HGO-2 are ϵ, which is a small positive design parameter, and $\alpha=[\alpha_1\ \cdots\ \alpha_9]^T$. Based on the mapping $\bar{z}=T(x)$ of Eq. 25, the estimated state, $z=[z_1\ \cdots\ z_9]^T$, is derived as $z=T^{-1}(\text{sat}(\bar{z}))$ where $\text{sat}(\cdot)$ is a saturation function of the form

$$\text{sat}(\bar{z}_i):=\begin{cases}\bar{z}_{i,m}, & \bar{z}_i\geq\bar{z}_{i,m}\\ \bar{z}_i, & -\bar{z}_{i,m}\leq\bar{z}_i\leq\bar{z}_{i,m}\\ -\bar{z}_{i,m}, & \bar{z}_i\leq-\bar{z}_{i,m}\end{cases} \tag{26}$$

where $\bar{z}_{i,m}$ $(i=1,\ldots,9)$ is the saturation limit of the transformed state $\bar{z}_i$. The saturation function is used to prevent the peaking phenomenon.

The following design parameters for the HGOs and for the RMHE scheme of Eq. 8 were used in all case studies below. It was verified through extensive closed-loop simulations that these parameters achieved good estimation performance. The parameters of HGO-1 were chosen to be

$$[\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5\ \alpha_6]=[2\ \ 15\ 10^{-2}\ \ 2\ \ 15\ \ 10^{-2}] \tag{27}$$

and $\epsilon=0.01$, and for HGO-2

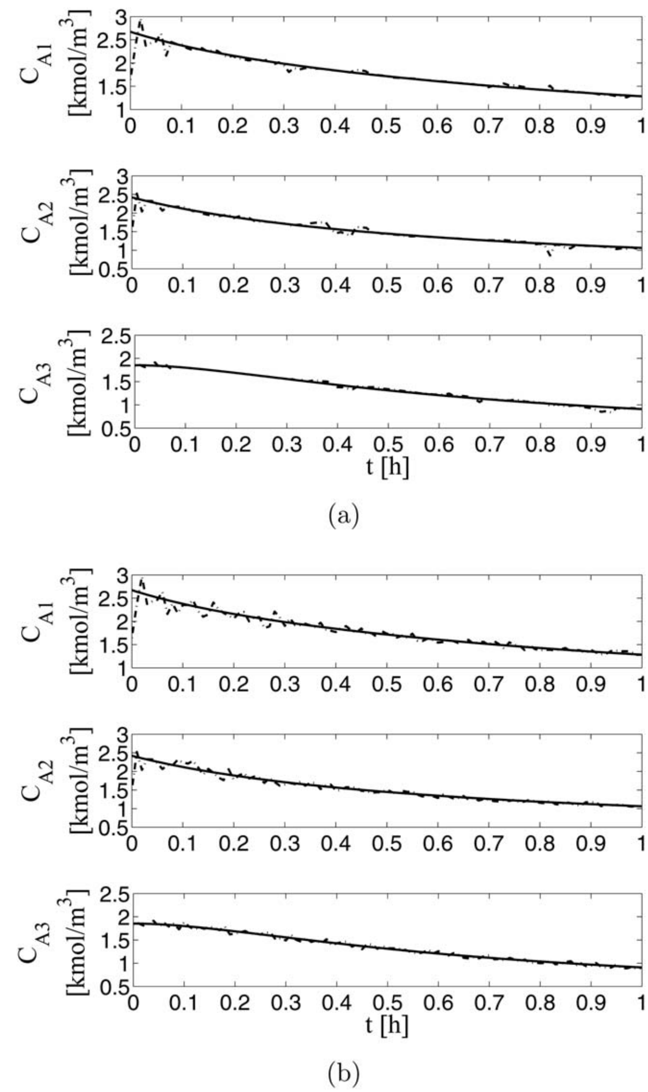

Figure 4. The estimated reactant concentration profiles (dashed lines) compared with the closed-loop reactant concentration profiles (solid lines) of the process network of Eq. 18 under (a) the RMHE-based output feedback LEMPC and (b) the HGO-based output feedback LEMPC for Case I (no sensor maintenance).

$$\begin{aligned}&[\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5\ \alpha_6\ \alpha_7\ \alpha_8\ \alpha_9]\\&=[1\ \ 10\ \ 10^{-2}\ \ 1\ \ 10\ \ 10^{-2}\ \ 1\ \ 10\ \ 10^{-2}]\end{aligned} \tag{28}$$

and $\epsilon=0.01$. The design parameters of the RMHE of Eq. 8 were chosen as $\kappa=0.4$ and $N_e=8$. The estimation horizon was chosen so that acceptable estimation performance was achieved without the use of an arrival cost. In the subsequent sections, we use RMHE-1 to denote the RMHE designed based on HGO-1 and RMHE-2 to denote the RMHE designed based on HGO-2.

RMHE-based LEMPC design handling real-time preventive sensor maintenance

A Lyapunov-based controller is designed for the process that can asymptotically stabilize the economically optimal steady state. It will be used to define the Lyapunov-based constraints of the RMHE-based LEMPC. The heat rate inputs Q_1, Q_2, and Q_3 have a larger impact on closed-loop stability of

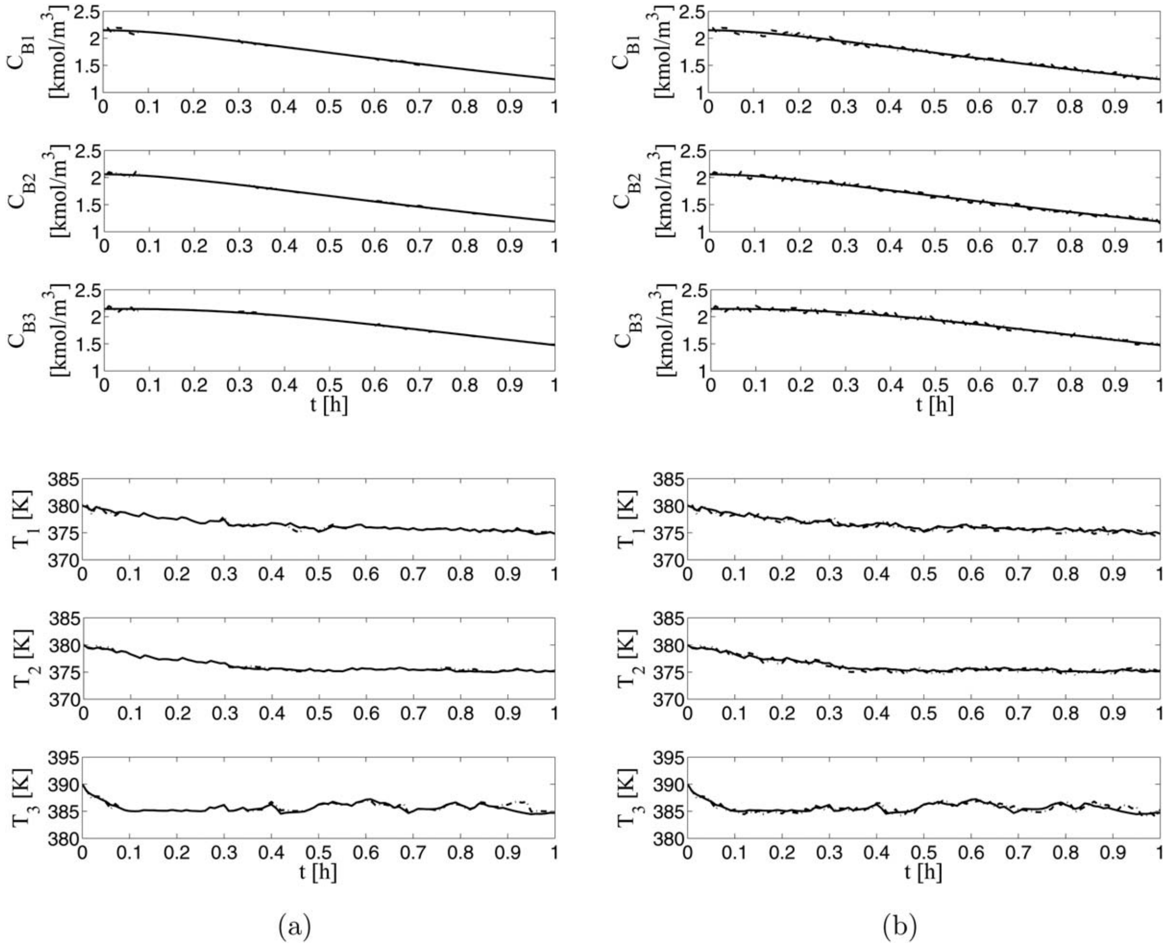

Figure 5. The estimated product concentration and temperature profiles (dashed lines) compared with the closed-loop product concentration and temperature profiles (solid lines) of the process network of Eq. 18 under (a) the RMHE-based output feedback LEMPC and (b) the HGO-based output feedback LEMPC for Case I (no sensor maintenance).

this process system than the concentration inputs C_{A10} and C_{A20}. For this reason, the inputs in vector u_q ($u_q=[Q_1\, Q_2\, Q_3]^T$) are used to stabilize the closed-loop system, while the inputs in vector u_c ($u_c=[C_{A10}\, C_{A20}]^T$) are primarily used to attain better economic performance of the closed-loop process and $u=[u_q^T\, u_c^T]^T$. The Lyapunov-based controller will be designed using different control laws for the heat rates and for the inlet concentrations. The heat rate inputs are controlled per the following feedback control law[33]

$$k_{qi}(x)=\begin{cases} -\dfrac{L_fV+\sqrt{(L_fV)^2+(L_{g_i}V)^4}}{(L_{g_i}V)^2}L_{g_i}V & \text{if } L_{g_i}V\neq 0 \\ 0 & \text{if } L_{g_i}V=0 \end{cases} \tag{29}$$

for $i=1,\ 2,\ 3$ where g_i denotes the ith column of g and $L_fV=(\partial V/\partial x)f(x)$ and $L_{g_i}V=(\partial V/\partial x)g_i(x)$ denote the Lie derivatives of V with respect to the vector fields f and g_i, respectively. The elements of u_c per the Lyapunov-based strategy are $k_c(x)=[0.5\ \ 0.5]^T$. The full Lyapunov-based control law for all five inputs is $k(x)=[k_q^T(x)\, k_c^T(x)]^T$.

A quadratic function (i.e., $V(x)=(x-x_s^*)^TP(x-x_s^*)$) was chosen for the Lyapunov function of the process under the controller $k(x)$, and extensive closed-loop simulations under this controller facilitated the choice of the matrix P and an estimate of the stability region of the process under the $k(x)$ control law. The stability region estimate was made by choosing the largest level set of V within which $\dot{V}<0$ along the trajectories of the closed-loop system. The positive definite matrix P is given by

$$P=\text{diag}([10^{-3}\ \ 1.5\ \ 0.5\ \ 10^{-3}\ \ 1.5\ \ 0.5\ \ 10^{-3}\ \ 1.5\ \ 0.5]) \tag{30}$$

The stability region is estimated to be the level set Ω_ρ with $\rho=12.4$. The subsets of the stability region Ω_ρ are estimated to be level sets: $\Omega_{\rho_{e1}}$ with $\rho_{e1}=10$ and $\Omega_{\rho_{e2}}$ with $\rho_{e2}=8$.

To deal with the scheduled preventive maintenance on the sensor of C_{A3} at $t_m=0.3\,h$, the proposed RMHE-based LEMPC of Eq. 10 for this chemical process network has the following form (for $N=8$)

$$\max_{u\in S(\Delta)}\int_{t_k}^{t_{k+8}} L_e(\tilde{x}(\tau),u(\tau))\,d\tau \tag{31a}$$

$$\text{s.t. } \dot{\tilde{x}}(t)=f(\tilde{x}(t))+g(\tilde{x}(t))u(t) \tag{31b}$$

$$\tilde{x}(t_k)=\begin{cases}\hat{x}_{\text{HGO}-1}(t_k), & \text{if } t_k<t_{b1}=0.08\\ \hat{x}^*_{\text{RMHE}-1}(t_k), & \text{if } 0.08\leq t_k<0.30\end{cases} \tag{31c}$$

$$u(t)\in\mathbb{U},\quad \forall\, t\in[t_k,t_{k+8}) \tag{31d}$$

$$\begin{aligned}\tilde{x}(t)\in\mathbb{X},&\quad \forall\, t\in[t_k,t_{k+8})\\ V(\tilde{x}(t))\leq 10,&\quad \forall\, t\in[t_k,t_{k+8})\end{aligned} \tag{31e}$$

$$\text{if } V(\hat{x}(t_k)) \le 10 \quad \text{and} \quad t_m \notin [t_k, t_{k+8}) \tag{31f}$$

$$\frac{\partial V}{\partial x}(f(\hat{x}(t_k))+g(\hat{x}(t_k))u(t_k)) \le \frac{\partial V}{\partial x}(f(\hat{x}(t_k))+g(\hat{x}(t_k))k(\hat{x}(t_k)))$$
$$\text{if } (V(\hat{x}(t_k)) > 10 \quad \text{and} \quad t_m \notin [t_k, t_{k+8}))$$
$$\text{or}(t_m \in [t_k, t_{k+8})) \tag{31g}$$

which is used for $t \in [0, t_m)$ and

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_{k+8}} L_e(\tilde{x}(\tau), u(\tau))\, d\tau \tag{32a}$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t)) + g(\tilde{x}(t))u(t) \tag{32b}$$

$$\tilde{x}(t_k) = \hat{x}^*_{\text{RMHE}-2}(t_k), \text{ if } t_k \ge 0.30 \tag{32c}$$

$$u(t) \in \mathbb{U}, \forall\, t \in [t_k, t_{k+8}) \tag{32d}$$

$$\tilde{x}(t) \in \mathbb{X}, \forall\, t \in [t_k, t_{k+8}) \tag{32e}$$

$$V(\tilde{x}(t)) \le 8, \forall\, t \in [t_k, t_{k+8}) \quad \text{if } V(\hat{x}(t_k)) \le 8 \tag{32f}$$

$$\frac{\partial V}{\partial x}(f(\hat{x}(t_k))+g(\hat{x}(t_k))u(t_k)) \le \frac{\partial V}{\partial x}(f(\hat{x}(t_k))+g(\hat{x}(t_k))k(\hat{x}(t_k))),$$
$$\text{if } V(\hat{x}(t_k)) > 8 \tag{32g}$$

which is used for $t \ge t_m$. For this example, t_{b1} is estimated to be 0.08 h. Considering that this means $t_{b1} = N_e \Delta$ for this specific case, HGO-1 is utilized to provide the state estimates of C_{A1} and C_{A2} during the first N_e sampling times. For $t \ge 0.08$ h, RMHE-1 is activated to provide the estimates of C_{A1} and C_{A2} until $t_m = 0.3$ h. After $t_m = 0.3$ h, RMHE-2 is utilized to provide state estimates of not only C_{A1} and C_{A2} but also C_{A3} as well. RMHE-2 is immediately used at t_m to provide the state estimate to the LEMPC of Eq. 32 because the state estimation error is small at $t = t_m$, which was verified through simulations (i.e., applying HGO-2 from t_m to $t_m + t_{b2}$ to ensure the convergence of the estimation error to a small value was not needed for this particular example).

The sampling time used for the RMHE-based LEMPC is $\Delta = 0.01$ h and the prediction horizon used is $N = 8$. The optimization software Ipopt[34] was used to find a local solution to the RMHE-based LEMPC optimization problem. The simulations were carried out using the Java programming language in a Intel® i7 3.40 GHz processor running a Windows 7 Professional system.

Case studies

As we discussed in the Introduction of this work, routine sensor replacement is necessary to avoid potential sensor failure which may bring production loss to the process. We now compare four case studies which a process operation manager may evaluate when weighing the risks, costs, and benefits of conducting a sensor maintenance procedure

I. Process operation without preventive sensor maintenance on the sensor of C_{A3}.

II. Process operation with preventive sensor maintenance on the sensor of C_{A3} at $t_m = 0.3$ h.

III. Process operation subject to faulty sensor readings of C_{A3} after $t_f = 0.5$ h.

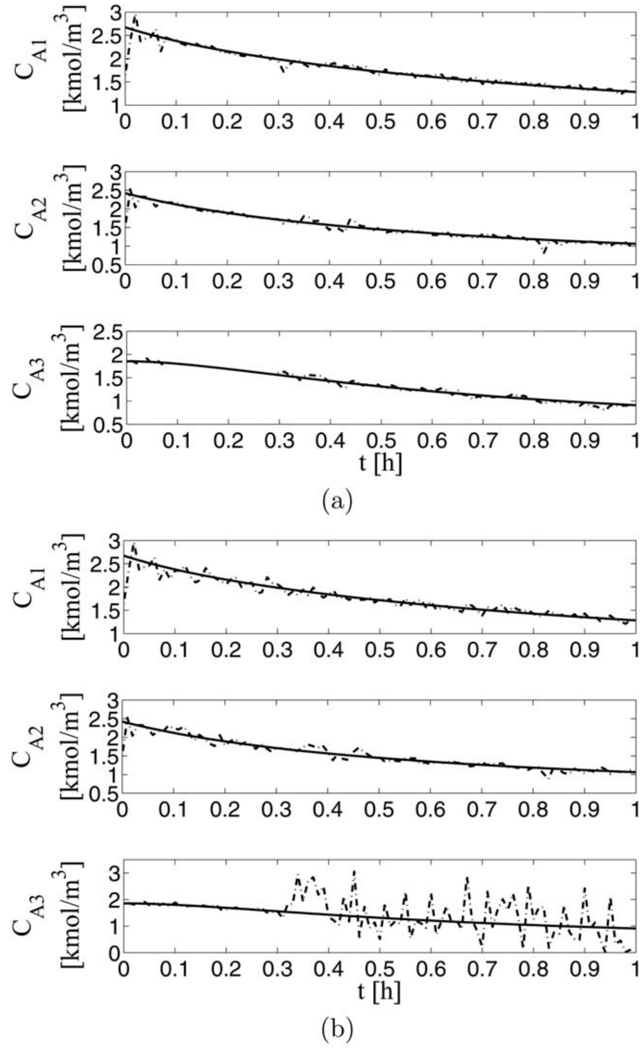

Figure 6. The estimated reactant concentration profiles (dashed lines) compared with the closed-loop reactant concentration profiles (solid lines) of the process network of Eq. 18 under (a) the RMHE-based output feedback LEMPC and (b) the HGO-based output feedback LEMPC for Case II where preventive maintenance is conducted on the C_{A3} sensor making it unavailable at 0.3 h.

IV. Process operation with preventive maintenance of the sensor of C_{A3} at $t_m = 0.3$ h followed by preventive maintenance of the actuator of C_{A20} at $t_a = 0.6$ h.

In these four case studies, the RMHE-based LEMPC formulation of Eqs. 31 and 32 is applied to the process of Eq. 18. The state estimation accuracy between the high-gain observer and the RMHE method is compared, and the RMHE-based LEMPC's ability to handle the preventive sensor maintenance task is demonstrated in all four cases; the process economic performance degradation is also evaluated for different cases under the preventive sensor maintenance.

To model the process and measurement noise, bounded Gaussian white noise is added to the process state and measurement values, respectively, with a zero mean and standard deviation $\sigma_{w,C_A} = \sigma_{w,C_B} = 0.05$, $\sigma_{w,T} = 5.0$, $\sigma_{v,C_A} = \sigma_{v,C_B} = 0.03$, and $\sigma_{v,T} = 5.0$, and subject to the bounds $\theta_{w,C_A} = \theta_{w,C_B} = 0.1$ kmol/m^3, $\theta_{w,T} = 10.0$ K, $\theta_{v,C_A} = \theta_{v,C_B} = 0.05$ kmol/m^3 and

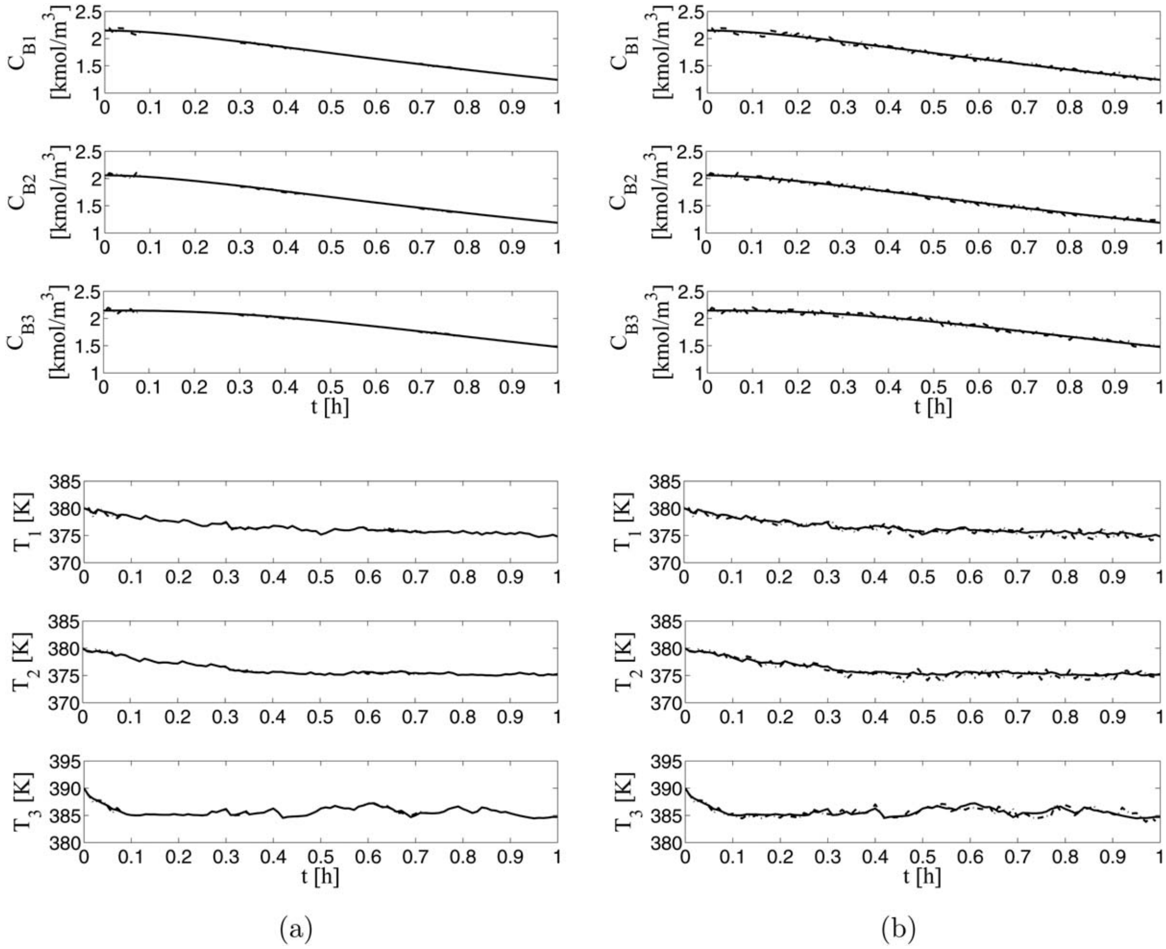

Figure 7. The estimated product concentration and temperature profiles (dashed lines) compared with the closed-loop product concentration and temperature profiles (solid lines) of the process network of Eq. 18 under (a) the RMHE-based output feedback LEMPC and (b) the HGO-based output feedback LEMPC for Case II where preventive maintenance is conducted on the C_{A3} sensor making it unavailable at 0.3 *h*.

$\theta_{v,T}=5.0$ K. For each case study, the same realization of the noise was used to compare the various control strategies.

Case I: RMHE vs. HGO—No Preventive Sensor Maintenance. In the first case, we compare the state estimation performance between the RMHE method (Eq. 8) and the HGO method (HGO-1) for the process network of Eq. 18. For this case, no preventive sensor maintenance is completed. The RMHE method consists of the output feedback RMHE-based LEMPC of Eq. 31 with $t_m \rightarrow \infty$, while the HGO method consists of the output feedback RMHE-based LEMPC of Eq. 31 with $t_m \rightarrow \infty$ and $t_{b1} \rightarrow \infty$ (i.e., HGO-1 provides the state estimate to the LEMPC for all time).

HGO-1 for both the RMHE and HGO methods is initialized such that initially there is nonzero estimation error (i.e., it is initialized with an initial condition not equal to the actual state value). The estimated state and closed-loop state profiles under the HGO and RMHE methods are shown in Figures 4 and 5. From Figure 4, the HGO initially computes a state estimate close to the actual values of C_{A1} and C_{A2}. After RMHE-1 is activated at 0.08 h in the RMHE method (Figure 4a), it computes state estimates of C_{A1} and C_{A2} very close to the actual state values throughout the length of operation. When comparing the estimated state profiles of C_{A3}, C_B, and T, the HGO is more sensitive to the measurement noise (e.g., compare Figure 5a with Figure 5b). Thus, the RMHE method provides better state estimation performance and robustness to measurement noise when compared with the HGO method. In addition, the closed-loop economic performance under the RMHE method is 14.8% greater than that under steady-state operation for the 1 h operation period.

Case II: RMHE vs. HGO—Preventive Sensor Maintenance. In this case study, we consider that preventive sensor maintenance on the sensor of C_{A3} will be conducted as scheduled at $t_m=0.3$ h. Again, we compare the RMHE method with the HGO method, but here the RMHE method is applied according to the output feedback RMHE-based LEMPC strategy of Eqs. 31 and 32, and the HGO method is applied using HGO-1 and HGO-2 to provide state estimates to the output feedback-based LEMPC schemes of Eqs. 31 and 32 before and after t_m, respectively.

We compare the estimated state profiles from the RMHE and HGO methods with the actual closed-loop state profiles in Figures 6 and 7. From Figure 6a, we can see that the estimated C_{A3} trajectory from the RMHE method is nearly overlapping with the actual state trajectories after the C_{A3} sensor is taken offline at $t_m=0.3$ h. However, the state estimate for C_{A3} from the HGO method significantly deviates from the actual state profile as shown by Figure 6b. From Figure 6b, the state

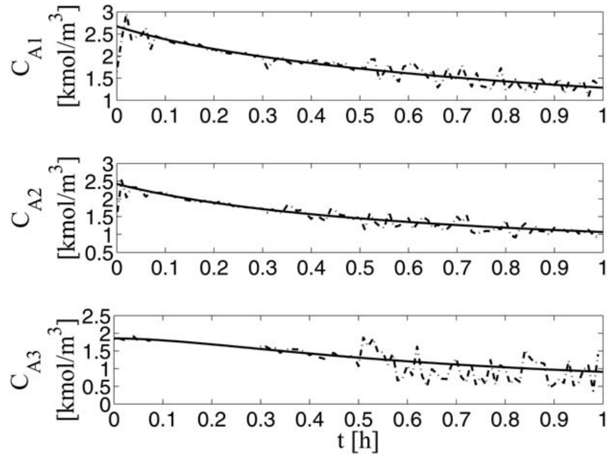

Figure 8. The estimated reactant concentration profiles (dashed lines) compared with the closed-loop reactant concentration profiles (solid lines) of the process network of Eq. 18 under the RMHE-based output feedback LEMPC for Case III where no preventive maintenance is completed on the C_{A3} sensor and the sensor provides faulty readings at 0.5 *h*.

estimates of C_{A1} and C_{A2} from the HGO are also affected by its sensitivity to the measurement noise and the removal of the measurement of C_{A3}. Figure 7 displays the T and C_B profiles of the process under the RMHE and HGO methods. The inaccurate estimation of C_{A3} from the HGO decreases the estimation accuracy of the estimated profiles of C_B and T as shown in Figures 7a, b. These results demonstrate the advantage of the RMHE method for state estimation when the measurements are corrupted by noise as it provides accurate state estimates within a small neighborhood of the actual process states. They also show that the proposed RMHE-based LEMPC is able to smoothly deal with the preventive sensor maintenance task without leading to poor performance and process shut-down.

Case III: RMHE vs. Faulty Sensor Readings. The state profiles of the third case are shown in Figures 8 and 9. The third case with the faulty sensor illustrates the consequences of not performing sensor maintenance and then having a sensor fault. It is developed using the output feedback RMHE-based LEMPC of Eq. 31, except that this LEMPC is used for all times and it does not account for the faulty sensor readings. To model the faulty sensor, random noise is added to the process measurements starting at time t_f.

From Figure 8, the faulty reading from the sensor of C_{A3} causes large deviations of the estimated states from the actual closed-loop states. The different estimated state values, which are provided to the RMHE-based LEMPC system of Eqs. 31 and 32, result in a different computed input trajectory for Case III than for Cases I and II as shown in Figure 9. Specifically, the input profiles from the process with preventive maintenance on the sensor of C_{A3} are close to those from the process without preventive sensor maintenance on the sensor of C_{A3} due to the accurate state estimate of C_{A3}, while for the process with a faulty sensor reading of C_{A3}, the controller requires increased energy consumption than that actually required due to the inaccurate state estimates.

In terms of the economic cost of Eq. 22, the average revenue per unit cost over the 1 h operation period for the process with faulty sensor readings decreases 5.18% when compared with the process using state feedback without sensor maintenance and with fully functional sensors. However, the process economic performance degradation for the process conducting the preventive sensor maintenance work based on the proposed output feedback RMHE-based LEMPC is only 1.27% due to the accurate state estimates provided by the RMHE

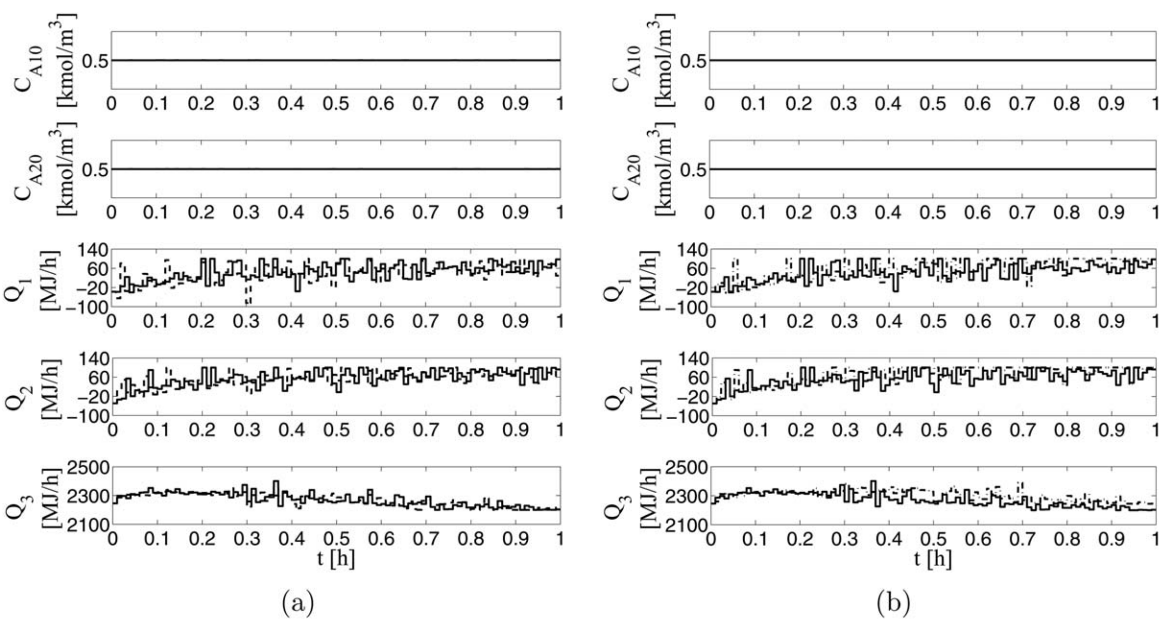

Figure 9. Manipulated input profiles from the RMHE-based LEMPC of Eqs. 31 and 32 when sensor maintenance is completed on the C_{A3} sensor at 0.3 *h* [dashed lines in (a)], when the C_{A3} sensor provides faulty readings of C_{A3} after 0.5 *h* [dashed lines in (b)].

For a comparison, the manipulated input profiles from the RMHE-based LEMPC when the sensor of C_{A3} is available and functioning well for all times are given as the solid lines.

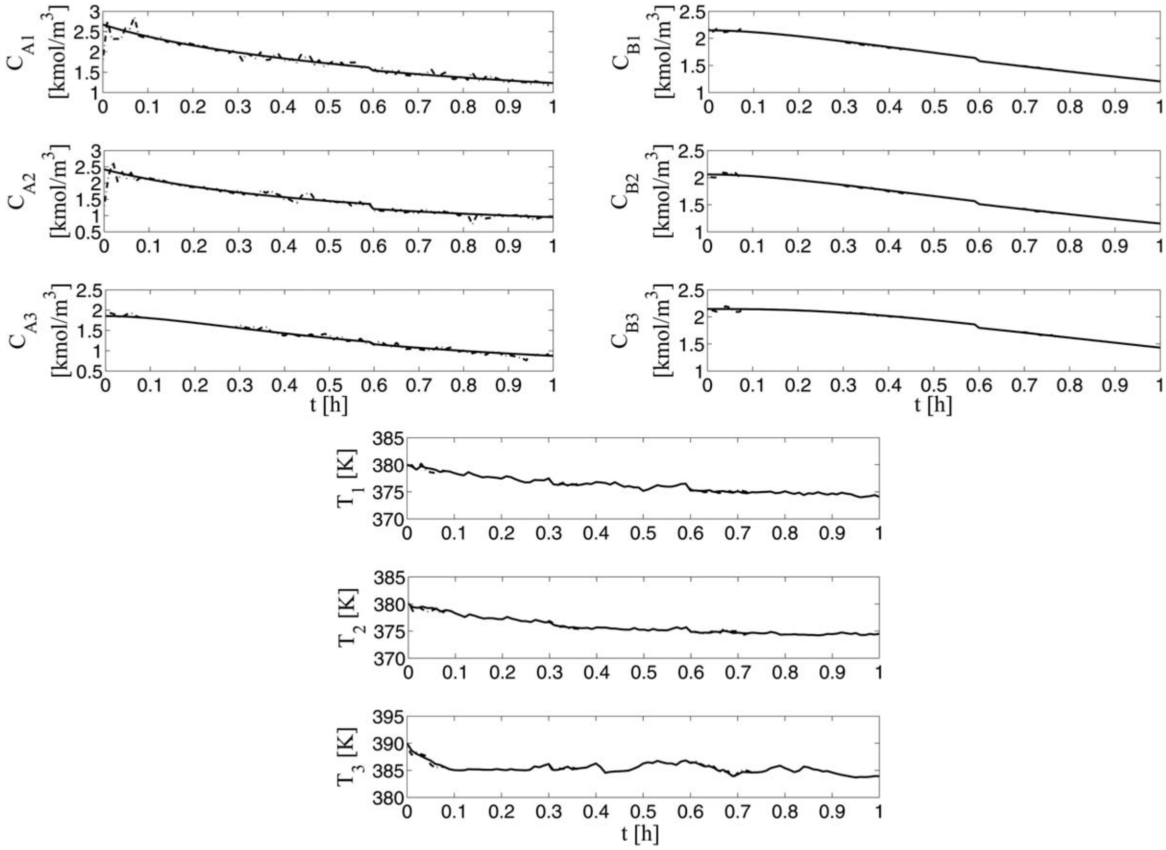

Figure 10. The closed-loop state (solid lines) and estimated state (dashed) profiles for the process network of Eq. 18 under the RMHE-based LEMPC for handling sensor and actuator maintenance where the sensor of C_{A3} is unavailable after $t_m=0.3$ h and actuator of C_{A20} is unavailable after $t_a=0.6$ h.

method and the control system reconfiguration of the LEMPC scheme of Eqs. 31 and 32 when t approaches t_m. In particular, the RMHE-based LEMPC takes full advantage of the estimation performance from the RMHE method.

Remark 8. *Sensor faults can come in a variety of forms. For the purposes of the Case III comparison made in this article, the sensor fault type simulated demonstrates that sensor faults can be more costly than removing a sensor for preventive maintenance using the proposed control strategy. If the faulty sensors have more severe problems, such as constant or drifting signals, the process economic performance degradation will likely be much larger than that demonstrated in Case III.*

Case IV: Integrating Actuator and Sensor Maintenance. The chemical processing industry is concerned not only with maintenance of sensors, but also with preventive maintenance of actuators. It is possible to consider the situation that the maintenance work for the sensor and actuator is scheduled in sequence. To address this problem, we apply a LEMPC scheme handling preventive maintenance on both sensors and actuators to the chemical process network based on the LEMPC design handling preventive actuator maintenance[26] and the LEMPC handling the preventive sensor maintenance of Eqs. 31 and 32.

In this case, the preventive sensor maintenance on C_{A3} is scheduled at $t_m=0.3$ h, and we assume the actuator of C_{A20} is taken offline at $t_a=0.6$ h when it is no longer available to the LEMPC. In Figure 10, closed-loop state and estimated state profiles under the RMHE-based LEMPC are compared. From Figure 10, we can see that the proposed RMHE-based LEMPC still provides accurate state estimates even after the actuator of

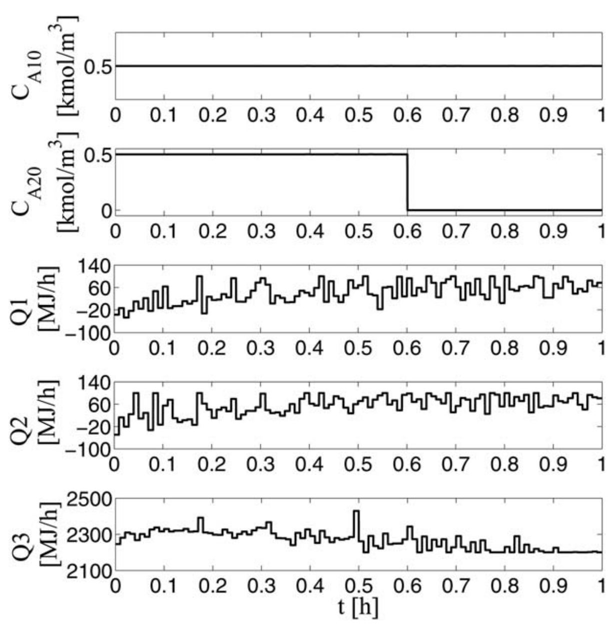

Figure 11. Manipulated input profiles with the sensor of C_{A3} unavailable after $t_m=0.3$ h and the actuator of C_{A20} unavailable after $t_a=0.6$ h under the RMHE-based LEMPC for handling both sensor and actuator preventive maintenance.

C_{A20} is taken offline (i.e., control system reconfiguration caused by the loss of the actuator of C_{A20}). Based on the manipulated input profiles shown in Figure 11, the manipulated input value of C_{A20} is set to be 0 when its actuator is taken offline for maintenance at $t_a=0.6\ h$. The process economic performance following Eq. 22 decreases 2.63% compared with the situation shown in Case II where the actuator maintenance is not conducted over this 1 h operation but sensor maintenance is performed. This Case IV simulation run demonstrates that the integration of the method proposed in the present manuscript and the results of Ref. 26 produces a control scheme capable of handling both sensor and actuator maintenance in a single framework.

Conclusions

This article establishes a novel RMHE scheme that accommodates a varying number of sensors to continuously supply accurate state estimates to a LEMPC system. It was shown that the proposed RMHE-based LEMPC scheme can maintain process closed-loop stability under standard observability and stabilizability assumptions. Then, the proposed RMHE-based LEMPC was applied to a chemical process; the simulation results exhibited its ability to accomplish control system reconfiguration under a changing number of online sensors and to achieve minimal economic performance degradation by operating the process in an economically optimal fashion, while preserving closed-loop stability.

Acknowledgment

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