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A tutorial review of economic model predictive control methods

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ABSTRACT

An overview of the recent results on economic model predictive control (EMPC) is presented and discussed addressing both closed-loop stability and performance for nonlinear systems. A chemical process example is used to provide a demonstration of a few of the various approaches. The paper concludes with a brief discussion of the current status of EMPC and future research directions to promote and stimulate further research potential in this area.

1. Introduction

Optimal operation and control of dynamic systems and processes has been a subject of significant research for many years. Important early results on optimal control of dynamic systems include optimal control based on the Hamilton–Jacobi–Bellman equation [16], Pontryagin's maximum principle [135], and the linear quadratic regulator [84]. Within the context of the chemical process industries, room for improvement in process operations will always exist given that it is unlikely for any process to operate at the true or theoretically global optimal operating conditions for any substantial length of time. One methodology for improving process performance is to employ the solution of optimal control problems (OCPs) on-line. In other words, control actions for the manipulated inputs of a process are computed by formulating and solving a dynamic optimization problem on-line that takes advantage of a dynamic process model while accounting for process constraints. With the available computing power of modern computers, solving complex dynamic optimization problems (e.g., large-scale, nonlinear, and non-convex optimization problems) on-line is becoming an increasingly viable option to use as a control scheme to improve the steady-state and dynamic performance of process operations.

The process performance of a chemical process refers to the process economics of process operations and encapsulates many objectives: profitability, efficiency, variability, capacity, sustainability, etc. As a result of continuously changing process economics (e.g., variable feedstock, changing energy prices, etc.), process operation objectives and strategies need to be frequently updated to account for these changes. Traditionally, economic optimization and control of chemical processes has been addressed in a multi-layer hierarchical architecture (e.g., [106]) which is depicted in Fig. 1. In the upper-layer called real-time optimization (RTO), a metric usually defining the operating profit or operating cost is optimized with respect to up-to-date, steady-state process models to compute optimal process set-points (or steady-states). The set-points are used by the lower-layer feedback process control systems (i.e., supervisory control and regulatory control layers) to steer the process to operate at these set-points using the manipulated inputs to the process (e.g., control valves, heating jackets, etc.). In addition to the previously stated objective, process control also must work to reject disturbances and ideally, guide the trajectory of the process dynamics along an optimal path.

The supervisory control layer of Fig. 1 consists of advanced control algorithms that are used to account for process constraints, coupling of process variables, and processing units. In the supervisory control layer, model predictive control (MPC) (e.g., [116,109,140]), a control strategy based on optimal control concepts, has been widely implemented in the chemical process industry. MPC uses a dynamic model of the process in the optimization problem to predict the future evolution of the process over a finite-time horizon to determine the optimal input trajectory with respect to a specified

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performance index. Furthermore, MPC can account for the process constraints and multi-variable interactions in the optimization problem. Thus, it has the ability to optimally control constrained multiple-input multiple-output nonlinear systems. The conventional formulations of MPC use a quadratic performance index, which is essentially a measure of the predicted deviation of the error of the states and inputs from their corresponding steady-state values, to force the process to the (economically) optimal steady-state. The regulatory control layer includes mostly single-input single-output control loops like proportional-integral-derivative (PID) control loops that work to implement the computed control actions by the supervisory control layer.

The overall control architecture of Fig. 1 invokes intuitive time-scale separation arguments between the various layers. For instance, RTO is executed at a rate of hours-days, while the regulatory control layer computes control actions for the process at a rate of seconds-minutes (e.g., [11,147]). Though this paradigm has been successful, we are witnessing the growing need for dynamic market-driven operations which include more efficient and nimble process operation [7,81,150,36]. To enable next-generation operations, novel control methodologies capable of handling dynamic optimization of process operations must be proposed and investigated. In other words, there is a need to develop theory, algorithms, and implementation strategies to tightly integrate the layers of Fig. 1. The benefits of such work may be transformative to process operations and usher in a new era of dynamic (off steady-state) process operations.

To this end, it is important to point out that while steady-state operation is typically adopted in chemical process industries, steady-state operation may not necessarily be the economically best operation strategy. The chemical process control literature is rich with both experimental and simulated chemical processes that demonstrate performance improvement with dynamic process operation (see [41,94,13,151,149,158,159,131,133,132,153,23,97,126,24,105,152], and the numerous references therein for results in this direction). In particular, periodic operation of chemical reactors has been perhaps the most commonly studied example (e.g., [151]). Periodic control strategies have also been developed for several applications (for instance, [97,126,23,149,133]). Several techniques have been proposed to help identify systems where performance improvement is achieved through periodic operation which mostly include frequency response techniques and the application of the maximum principle [41,9,21,8,66,158].

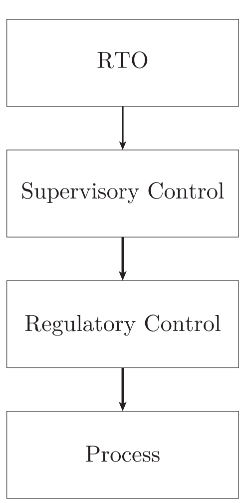

Fig. 1. The traditional paradigm employed in the chemical process industries for process optimization and control.

In an attempt to integrate economic process optimization and process control as well as realize the possible process performance improvement achieved by consistently dynamic, transient, or time-varying operation (i.e., not forcing the process to operate at a pre-specified steady-state), economic MPC (EMPC) has been proposed which incorporates a general cost function or performance index (i.e., objective function) in its formulation [72,56,141]. The cost function may be a direct or indirect reflection of the process economics. However, a by-product of this modification is that EMPC may operate a system in a possibly time-varying fashion to optimize the process economics (i.e., may not operate the system at a specified steady-state or target). The rigorous design of EMPC systems that operate large-scale processes in a dynamically optimal fashion while maintaining stability (safe operation) of the closed-loop process system is challenging as traditional notions of stability (e.g., asymptotic stability of a steady-state) may not apply to the closed-loop system under EMPC. It is important to point out that the use of OCPs with an economic cost function is not a new concept. In fact, MPC with an economic cost is not new either (e.g., one such EMPC framework was presented in [72]). However, closed-loop stability and performance under EMPC has only recently been considered and proved for various EMPC formulations.

This article attempts to organize the recent theoretical developments on EMPC. Further explanation of the theory is given where possible in an attempt to make the theory tractable and accessible to even a beginning graduate student working in the area of process control. The remainder of the paper is organized as follows. In the next section, the preliminaries are presented which include the notation used throughout this work, the class of nonlinear process systems considered, as well as a more thorough description of real-time optimization and model predictive control. The subsections on RTO and MPC are not meant to be comprehensive, but rather, are presented to provide some historical background on the challenges addressed in this area. The third section examines closed-loop stability under EMPC and outlines the various types of constraints and modifications to the objective function that have been presented to guarantee some notion of closed-loop stability. The fourth section discusses closed-loop performance under EMPC. Various EMPC formulations are subsequently applied to a chemical process example in the fifth section. An overall discussion and analysis is provided in the sixth section which attempts to provide our perspective on the current status of EMPC. Lastly, the review concludes with a discussion of future research directions.

2. Preliminaries

2.1. Notation

The operator $|\cdot|$ is used to denote the Euclidean norm of a vector, while the operator $|\cdot|_Q^2$ is used to denote a square of a weighted Euclidean norm of a vector where Q is a positive definite matrix (i.e., $|x|_Q^2 = x^TQx$). The symbol $S(\Delta)$ denotes the family of piecewise constant functions with period Δ. A continuous function $\alpha : [0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$ and belongs to class $\mathcal{K}_\infty$ if $a = \infty$ and α is radially unbounded. A continuous, scalar-valued function, $\beta : \mathbb{R}^{n_x} \to \mathbb{R}$ is positive definite with respect to x_s if $\beta(x_s) = 0$ and $\beta(x) > 0$ for all $x \in \mathbb{R}^{n_x} \setminus \{x_s\}$. The symbol Ω_ρ denotes a level set of a scalar function $V(\cdot)$ (i.e., $\Omega_\rho = \{x \in \mathbb{R}^{n_x} | V(x) \leq \rho\}$). The set operators $\oplus$ and $\ominus$ denote the following set operations:

$$\mathbb{A} \oplus \mathbb{B} = \{c = a + b | a \in \mathbb{A}, b \in \mathbb{B}\}$$
$$\mathbb{A} \ominus \mathbb{B} = \{c | \{c\} \oplus \mathbb{B} \subseteq \mathbb{A}\}$$

or in other words, $\mathbb{A} \oplus \mathbb{B}$ is a set with elements constructed from the addition of any element of the set $\mathbb{A}$ with any element of the set $\mathbb{B}$ and $\mathbb{A} \ominus \mathbb{B}$ is a set where the addition of any element of the set $\mathbb{A} \ominus \mathbb{B}$ with any element of the set $\mathbb{B}$ forms a set that is a subset of or is equal to the set $\mathbb{A}$.

2.2. Classes of process systems

Throughout this tutorial review, unless otherwise noted, the class of process systems typically encountered within the chemical process industries is considered which are continuous-time systems. Owing to the complex reaction mechanisms and thermodynamic relationships that govern the underlying physics of chemical processes, most process systems are inherently nonlinear. Mathematically stated, the class of continuous-time, time-invariant nonlinear systems described by the following state-space form is considered:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \quad (1)$$

where $x \in \mathbb{X} \subseteq \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{U} \subset \mathbb{R}^{n_u}$ is the manipulated input vector, $w \in \mathbb{W} \subset \mathbb{R}^{n_w}$ is the disturbance vector and the notation $\dot{x}$ denotes the time derivative of the state. The set $\mathbb{X}$ denotes the set of admissible states. The input vector is bounded in the set of the available control energy $\mathbb{U}$ where $\mathbb{U} = \{u \in \mathbb{R}^{n_u} | u_{\min,i} \leq u_i \leq u_{\max,i}, i = 1, 2, \ldots, n_u\}$. The disturbance vector includes unknown external forcing of the system, modeling errors, and other forms of uncertainty and is bounded in the following set: $\mathbb{W} = \{w \in \mathbb{R}^{n_u} | |w| \leq \theta, \theta > 0\}$.

In addition to continuous-time nonlinear systems, other models have been considered for the design of EMPC systems. Specifically, many EMPC schemes have been developed for systems described by a discrete-time nonlinear model, possibly obtained from the discretization of a nonlinear continuous-time model of the form of Eq. (1). The discrete-time system analogous to the continuous-time system of Eq. (1) is given by the nonlinear time-invariant difference equation:

$$x(k + 1) = f_d(x(k), u(k), w(k)) \quad (2)$$

with $x \in \mathbb{X} \subseteq \mathbb{R}^{n_x}$, $u \in \mathbb{U} \subset \mathbb{R}^{n_u}$, and $w \in \mathbb{W} \subset \mathbb{R}^{n_w}$ where k is used to denote the current time step and the notation $f_d(\cdot)$ is used to distinguish the discrete-time nonlinear state-transition map and the continuous-time nonlinear vector field denoted by $f(\cdot)$. In other cases, linear systems are considered. A linear process model may arise from the linearization of Eq. (1) around an operating steady-state or when a linear model can provide sufficient accuracy describing the evolution of the process system. The continuous-time linear (time-invariant) model is given by

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3)$$

where A and B are $n_x \times n_x$ and $n_x \times n_u$ matrices, respectively.

A state measurement of the process state is assumed to be available at synchronous time instants given by the sequence $\{\tau_{k \geq 0}\}$ where $\tau_k = \tau_0 + k\Delta$ and Δ is the sampling period. The kth time step of the discrete-time model (Eq. (2)) corresponds to the sampling time instance τ_k of the continuous-time model. To distinguish between continuous time and the discrete sampling time, the notation t is used for continuous time and the symbols τ_k and k are used for the discrete sampling time instances for the continuous-time model and the discrete-time model, respectively. Output feedback and asynchronous sampling are discussed in Sections 7.1 and 7.3. For the remainder of the manuscript, the predictive controllers described below will take advantage of the (open-loop) solution to the nominal model ($w(t) \equiv 0$) of Eq. (1) or (2) for a given piecewise constant input trajectory $u(t)$. This state trajectory or solution to the initial value problem of Eq. (1) with $w(t) \equiv 0$ for a given initial condition and input trajectory is defined as the open-loop predicted state trajectory which is denoted as $\tilde{x}(t)$ and can be obtained by recursively solving the process model. If the vector field $f(\cdot)$ is a continuously differentiable function of its arguments, the existence and uniqueness of this trajectory is guaranteed for all times when this trajectory is proved to remain within a compact set (e.g., [87]).

For the process systems of interest, a continuous function of the form $l_e : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}$ is used as a measure of the instantaneous process operating cost (or profit). As the function $l_e(x, u)$ is a direct or indirect reflection of the (instantaneous) process economics it is typically referred to as the economic cost function, economic cost functional, or economic stage cost (here, $l_e(\cdot)$ will be referred to as the economic cost function in subsequent sections). A wide range of economic costs have been considered such as the net instantaneous operating profit (i.e., the instantaneous profit minus the instantaneous cost) as well as more traditional chemical engineering performance metrics like production rates of desired products, desired product selectivity, and product yield. Given the generality of the classes of the systems encompassed by Eqs. (1) and (2), further assumptions are placed on the class of systems and are stated in the subsequent sections as the topics that require these assumptions are introduced.

The (economically) optimal steady-state is defined to be the minimizer of the following optimization problem:

$$\underset{x_s, u_s}{\text{minimize}} \quad l_e(x_s, u_s) \quad (4a)$$

$$\text{subject to} \quad f(x_s, u_s, 0) = 0 \quad (4b)$$

$$g(x_s, u_s) \leq 0 \quad (4c)$$

$$g_e(x_s, u_s) \leq 0 \quad (4d)$$

where $g : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_p}$ denotes the process constraints which may include input and state constraints as well as mixed input and state constraints and $g_e : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_e}$ denotes economic constraints like constraints to achieve desired production rates to meet customer demand, product specifications and quality, and feedstock availability to name a few. The optimal solution of Eq. (4) is denoted x_s^* and u_s^*. Without loss of generality, the optimal steady-state is assumed to be unique and the origin of Eq. (1) (i.e., $f(x_s^*, u_s^*, 0) = f(0, 0, 0) = 0$) and similarly, $f_d(x_s^*, u_s^*, 0) = f_d(0, 0, 0) = 0$ for the system of Eq. (2).

Remark 1. The state x is assumed to be in the set $\mathbb{X} \subseteq \mathbb{R}^{n_x}$ (i.e., $\mathbb{X}$ may be a subset of or equal to $\mathbb{R}^{n_x}$). The case that $\mathbb{X}$ is the entire $\mathbb{R}^{n_x}$ corresponds to the case when no state constraints are considered. However, the input u is assumed to belong to a set $\mathbb{U} \subset \mathbb{R}^{n_u}$. The assumption that $\mathbb{U}$ is only a subset of $\mathbb{R}^{n_u}$ is because of the physical limitations of control actuators. Lastly, the disturbance is assumed to be bounded in a subset of $\mathbb{R}^{n_w}$ (i.e., $\mathbb{W} \subset \mathbb{R}^{n_w}$) owing to the fact that closed-loop stability under a particular control structure in the presence of disturbances is typically proved for a sufficiently small bounded disturbance. In general, it is difficult to prove closed-loop stability of the closed-loop system of Eq. (1) in the presence of possibly unbounded disturbances.

2.3. Real-time optimization

The traditional method for optimization of chemical processes is real-time optimization (RTO) (e.g., [59,106,148,56,35]). Typically, RTO is executed with a much larger sampling period than the supervisory control layer (e.g., MPC layer); that is, RTO may be computed on the order of hours-days and the supervisory control layer may be computed on the order of minutes-hours [106,56,150]. Although RTO is responsible for process optimization as its name suggests, it covers more responsibilities than just optimization in industrial applications. These responsibilities can be summarized in a four-step algorithm. First, the RTO system analyzes process data to detect if the system has reached steady-state. When steady-state has been detected, data validation and reconciliation is completed followed by model parameter estimation and model updating using various techniques to update the steady-state process model. After the model has been updated, optimization, of the form of Eq. (4), is completed. Lastly, a decision maker decides whether to implement the new operating conditions (i.e., send the computed steady-state to the process control layer which forces the process operation to the newly computed steady-state). While RTO has become an important information system in chemical process industries, RTO has three main drawbacks. A complete discussion of the issues arising in the context of RTO is not within the scope of the present paper, but rather a brief summary of these issues is provided below.

Since optimizing over an accurate process model is important for RTO to yield good performance, RTO has traditionally used more complex nonlinear steady-state models than the supervisory control layer [60,61]. In the lower feedback control layers, linear models are often used which may be derived from a number of techniques like a linearization of a nonlinear first-principles model around the desired operating steady-state or via model identification techniques (e.g., [20]). The discrepancies between the two models may result in a computed operating point by the RTO layer that is unreachable by the feedback control layer often leading to an offset between the actual operating steady-state and the desired operating steady-state. Also, optimization (or re-optimization) is completed after steady-state of the process is detected. Since the process is inherently dynamic and possibly under the influence of time-varying disturbances, waiting for the process to reach steady-state may delay the computation of the new optimal operating condition. Thus, re-optimization may be completed only infrequently, thereby adversely affecting the process performance. One solution to this problem is to solve the optimization problem more frequently (e.g., [148]), but this may lead to stability issues of the closed-loop system [56].

At a more fundamental level, many have questioned whether steady-state operation is the best operating strategy owing to time-varying process economics and inherent characteristics of nonlinear process systems [7,81]. As such, researchers have explored using dynamic models instead of steady-state process models in the optimization step of RTO, and the resulting system is typically referred to as dynamic RTO (D-RTO) (e.g., [72,83,107,82,161,176,81,167,169,168]. Dynamic RTO has a similar structure to that of EMPC, in that both optimization problems that characterize these systems are typically dynamic optimization problems that work to minimize an economic objective subject to a dynamic process model. The main differences between D-RTO and EMPC are D-RTO is not typically used directly for feedback control, but rather it is used in the RTO layer of the hierarchical structure of Fig. 1 above with the process control layers (i.e., supervisory control and regulatory control layers). Furthermore, only limited work has been done on a theoretical treatment of closed-loop stability with D-RTO. On the other hand, EMPC is typically implemented for feedback control, its dynamic model is typically implicitly or explicitly assumed to be consistent with the model of the optimization layer (e.g., RTO) and its formulation is tailored to account for closed-loop stability (see below).

2.4. Model predictive control

Model predictive control (MPC), also referred to as receding horizon control, is an on-line optimization-based control technique that optimizes a performance index or cost function over a prediction (control) horizon by taking advantage of a dynamic nominal process model (i.e., Eq. (1) with $w(t) \equiv 0$) while accounting for process constraints (e.g., [62,116,109,140,137,26]). The main objective of conventional or tracking MPC is to steer the system to and maintain operation thereafter at the economically optimal steady-state or the economically optimal trajectory computed in an upper-layer optimization problem like the optimization problem of Eq. (4) (e.g., RTO or D-RTO). To manage the trade-off between the speed of response of the closed-loop system and the amount of control energy required to generate the response, MPC is typically formulated with a quadratic objective function which penalizes the deviations of the state and inputs from their corresponding optimal steady-state values over the prediction horizon. Specifically, MPC is given by the following dynamic optimization problem (recall the assumption that the origin of the system of Eq. (1) is the economically optimal steady-state):

$$\underset{u \in S(\Delta)}{\text{minimize}} \quad \int_0^{\tau_N} \left(|\tilde{x}(t)|^2_{Q_c} + |u(t)|^2_{R_c} \right) dt \tag{5a}$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \tag{5b}$$

$$\tilde{x}(0) = x(\tau_k) \tag{5c}$$

$$g(\tilde{x}(t), u(t)) \le 0, \quad \forall t \in [0, \tau_N) \tag{5d}$$

where the positive definite matrices $Q_c > 0$ and $R_c > 0$ are tuning matrices that manage the trade-off between the speed of response and the cost of control action. The state trajectory $\tilde{x}(t)$ is the predicted evolution of the state using the nominal dynamic model ($w(t) \equiv 0$) of Eq. (1) under the piecewise constant input profile computed by the MPC. The initial conditions on the dynamic model are given in Eq. (5c) which are obtained at each sampling period through a measurement of the current state. The constraints of Eq. (5d) are the process constraints imposed on the computed input profile (e.g., input and state constraints) which are typically pointwise constraints, so the constraints of Eq. (5d) are usually written as:

$$g(\tilde{x}(\tau_j), u(\tau_j)) \le 0 \tag{6}$$

for $j = 0, 1, \ldots, N$. When the prediction horizon N is finite, it is well-known that the MPC scheme of Eq. (5) may not be stabilizing (e.g., [109]). Various constraints and variations to the cost function may be made to guarantee stability of the closed-loop system when N is finite (see, for example, [109], and the references therein).

To address the drawbacks of the two-layer RTO and MPC hierarchical control structure, much of recent research has focused on a tighter integration of RTO and MPC. Specifically, to handle unreachable set-points, an intermediate layer called the (steady-state) target optimization layer may be introduced that converts the optimal steady-state computed in the RTO layer to a reachable set-point for the feedback control layer (e.g., [117,22,172,125,137,171,93,160]). This concept is also referred to as two-stage MPC because of its components. Specifically, a quadratic program (QP) or linear program (LP) is used to convert the unreachable desired steady-state into a reachable target and then, an MPC of the form of Eq. (5) forces the closed-loop state to the reachable target. Target optimization or the first stage of the two-stage MPC also allows for more frequent optimization since it is typically executed at the same rate as the MPC. Within this context, most of the research on this topic has focused on MPC with a linear model (i.e., using the model of Eq. (3) for the constraint of Eq.(5b)).

Another option is to completely integrate economic optimization of process operations and MPC into the same algorithm. Early research (and still on-going) on this topic has focused on combining steady-state economic optimization and linear MPC (i.e., MPC formulated with the linear model of Eq. (3)) into one optimization problem. Specifically, MPC schemes that integrate steady-state optimization use a cost function of the form:

$$L_{\text{MPC/RTO}}(x(t), u(t)) = \int_0^{\tau_N} \left(|x(t)|^2_{Q_c} + |u(t)|^2_{R_c} \right) dt + l_e(x(\tau_N), u_s) \tag{7}$$

which has both a quadratic (tracking) component and an economic cost component in the cost function. These MPC schemes have the following general formulation:

$$\underset{u \in S(\Delta), u_s}{\text{minimize}} \quad L_{\text{MPC/RTO}}(\tilde{x}(t), u(t)) \tag{8a}$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \tag{8b}$$

$$\tilde{x}(0) = x(\tau_k) \tag{8c}$$

$$f(\tilde{x}(\tau_N), u_s) = 0, u_s \in \mathbb{U} \tag{8d}$$

$$g(\tilde{x}(t), u(t)) \leq 0, \quad \forall t \in [0, \tau_N) \tag{8e}$$

where the decision variables of the optimization problem include both the input trajectory over the prediction horizon and the steady-state input u_s. The constraint of Eq. (8d) enforces that the predicted state trajectory $\tilde{x}(t)$ converges to an admissible steady-state. The remaining constraints and notation are similar to the MPC of Eq. (5). Work in this direction has primarily been application driven (e.g., [127,37,173]) with more general frameworks presented in [15,157,3].

3. Economic model predictive control schemes

The quadratic cost of conventional MPC (Eq. (5a)) allows for tunable closed-loop response. However, it may not be an adequate representation of managing real-time process operation with respect to the process economic performance. A positive deviation from the target may represent a profit, while a negative deviation from the target may represent a loss (or vice versa) [150]. For example, consider an input that supplies heat energy to a reactor (e.g., a steam jacket). Supplying more steam to the jacket than the target is more costly in terms of the energy consumption of the reactor, while supplying less steam consumes less energy. Owing to this drawback of using a quadratic cost function in the MPC, the main three drawbacks of RTO, and the calls to unify process economic optimization and process control, the idea of using the economic cost function $l_e(\cdot)$ directly in an MPC scheme was proposed (e.g., [56,141]). The resulting MPC scheme is called economic MPC (EMPC). Since EMPC accounts directly for process economics which is aligned with the core ideas of next-generation manufacturing (e.g., Smart Manufacturing [31,36], market-driven manufacturing [7], and real-time energy management [150]), its popularity amongst researchers has significantly increased within the last few years.

Broadly, economic model predictive control can be characterized by the following optimization problem:

$$\underset{u \in S(\Delta)}{\text{minimize}} \quad \int_0^{\tau_N} l_e(\tilde{x}(t), u(t))\, dt \tag{9a}$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \tag{9b}$$

$$\tilde{x}(0) = x(\tau_k) \tag{9c}$$

$$g(\tilde{x}(t), u(t)) \leq 0, \quad \forall\, t \in [0, \tau_N) \tag{9d}$$

where the decision variable to the optimization problem is the input trajectory over the prediction horizon. The objective function of Eq. (9a) is the process economic cost function (e.g., operating cost) that the EMPC optimizes through dynamic operation of the process. A dynamic model, typically the nominal process model, is used as a constraint (Eq. (9b)) and is initialized through a state measurement obtained at a sampling instance (Eq. (9c)). The constraint of Eq. (9d) represents process constraints (e.g., input and state constraints) which are implemented as in Eq. (6). In addition to the constraints of Eqs. (9b)–(9d), economics-based constraints (e.g., the raw material that may be fed to a process over a period of operation may be fixed) are often added. The general formulation is given by:

$$g_e(\tilde{x}(t), u(t)) \leq 0 \tag{10}$$

for all $t \in [0, \tau_N)$. With slight abuse of notation, the constraints of Eq. (10) are not necessarily equivalent to the economics-based constraints of Eq. (4d), in that they may also incorporate integral, summation, and average constraints. The constraints of Eq. (10) may play an important role in the solution of the optimization problem of Eq. (9) especially when no upper-layer optimization is used to account for these constraints and when the optimal operating strategy dictated by Eq. (9) leads to dynamic (transient) operation (i.e., not steady-state). In either case, the enforcement of these constraints in the EMPC is needed to ensure that these constraints are satisfied over the entire length of process operation.

The implementation strategy of the EMPC of Eq. (9) is identical to the conventional MPC of Eq. (5). Specifically, EMPC is solved in a receding horizon fashion. At a sampling instance τ_k, the EMPC receives a state measurement of the current process state which is used to initialize the EMPC. An optimal piecewise input trajectory, according to the optimization problem of Eq. (9), is computed over the prediction horizon corresponding to the time $t \in [\tau_k, \tau_{k+N})$ in real-time. The optimal input trajectory computed at a given sampling instance is denoted as $u^*(t|\tau_k)$. The first control action, denoted as $u^*(0|\tau_k)$ is sent to the control actuators to be implemented over the sampling period from τ_k to τ_{k+1}. At the next sampling period, the EMPC is re-solved. The resulting input profile computed by the EMPC that is applied to the system of Eq. (1) is denoted as $u^*(t)$ and is given by

$$u^*(t) = u^*(0|\tau_k), \quad \text{for } t \in [\tau_k, \tau_{k+1}), k = 0, 1, \ldots. \tag{11}$$

In the general context (i.e., for the general system of Eq. (1) or (2)), there are three main issues to consider and address with respect to the optimal control problem of the EMPC. First is the issue of feasibility of the optimization problem. Specifically, one must carefully consider the conditions that guarantee that the EMPC is both initially feasible for a given initial condition $x(\tau_0)$ and recursively feasible at each subsequent sampling period. Assuming that one can show recursive feasibly, it is important to consider the stability or type of stability the closed-loop system will exhibit under the EMPC. Recall, no explicit assumption is placed on the economic cost to be positive definite with respect to a steady-state and thus, the EMPC may dictate a time-varying operating policy. Finally, one should consider the closed-loop performance under the EMPC. Even though the EMPC optimizes the process economics, it does so over a finite-time prediction or control horizon. Thus, over long periods of operation, no guarantees, in general, can be made on closed-loop performance under EMPC. For provable results on feasibility, closed-loop stability, and closed-loop performance under EMPC, additional assumptions must be placed on the closed-loop system and typically, the addition of stability and/or performance constraints are added to the formulation of the EMPC. These areas are discussed in depth in the subsequent sections.

Several application-oriented formulations of the EMPC of Eq. (9) have been presented in the literature where an appropriate cost function and constraints (not of the explicit form discussed below) have been formulated after an in-depth knowledge of the application has been gained. The additional elements added to the EMPC formulation are tailored for the particular application to allow for desirable stability, operation, and performance properties. These properties are typically demonstrated and evaluated through simulation [56,74,76,75,80,104,1,73]. For theoretical works that consider EMPC of the form of Eq. (9) (without stability constraints), only a limited amount of work has been completed including [64]. The main advantage of these types of formulations is that no additional constraints must be added or precomputed. As we will see below, the stability constraints are typically obtained from a steady-state optimization problem of the form of Eq. (4). Therefore, the EMPC of Eq. (9) will have less constraints than the ones discussed below and no steady-state optimization problem is required to be solved. The main disadvantage of the EMPC of Eq. (9) is that, at present, stability and performance cannot be guaranteed in general unless a sufficiently long prediction horizon is used and other controllability assumptions and turnpike conditions are satisfied [64].

In the remaining subsections, EMPC formulations with provable closed-loop stability properties are discussed. Closed-loop performance under the various EMPC formulations will be discussed in the subsequent sections. The EMPC formulations are given using the continuous-time nominal model ($w(t) \equiv 0$) of Eq. (1) except for the EMPC with a terminal constraint. It is straightforward to cast these formulations with the discrete-time nominal model of Eq. (2).

Remark 2. It is important to point out that in addition to the EMPC schemes discussed below a few EMPC schemes have been designed for explicitly handling noise and uncertainty. In [2,67] adaptive EMPC schemes were proposed for handling uncertainties and in [39], an EMPC was presented utilizing stochastic optimization techniques.

3.1. Infinite-horizon economic model predictive control

To address closed-loop stability, one may consider employing an infinite horizon in the EMPC of Eq. (9). In other words, let N tend to infinity (similarly, let τ_N tend to infinity) in the objective function of Eq. (9a). This is perhaps a more appropriate prediction horizon because chemical processes are continuously operated over long periods of time (practically infinite time). At least intuitively, one may be able to guarantee, if a solution is returned, that the state of the system is maintained in the set of admissible states $\mathbb{X}$. Then, stability in the sense of boundedness of the closed-loop state may be guaranteed. Theoretically, one could guarantee that the operating policy dictated by the infinite-horizon EMPC is the economically optimal one by the principle of optimality. However, it is difficult to solve a general optimization problem with an infinite number of decision variables. Since optimal control problems such as Eq. (9) with $N \to \infty$ often arise in the context of economics, it is important to point out that many ideas for solving various classes of these problems have been proposed, and schemes for obtaining an approximate solution to the optimization problem have been devised especially when the open-loop predicted state trajectory displays a turnpike property [28,27,89] which corresponds to the case in process operations when steady-state operation is likely the optimal operating strategy (see Section 4 for an illustration of the turnpike property).

Several works on infinite-horizon EMPC have been presented [169,79,38,77,112,113,130,170]. In [169,170], a few methods were given for solving the infinite-horizon EMPC with an economic cost that maximizes a discounted profit function. In other words, the EMPC is formulated with an objective function:

$$L_e(x(t), u(t)) = -\int_0^{\infty} e^{-\rho t} l_e(x(t), u(t))\, dt \tag{12}$$

where $\rho > 0$ is the discount factor used to account for the present value of money. Specifically, a time transformation was introduced to convert the infinite-time interval to a finite-time interval. The time transformation introduces a singularity which is handled by imposing a boundary condition at the final time. An adaptive temporal discretization scheme was then employed to solve the optimization problem. In [79], a discount factor similar to Eq. (12) was used in the economic cost. Nominal stability of the economically-optimal cyclic steady-state was proved when certain assumptions on the economic cost function were satisfied. This methodology was extended to robust stability of the cyclic steady-state (i.e., input-to-state stability with respect to a bounded disturbance) [77] where no discount factor was used in the economic cost function. In [77], this approach was demonstrated by approximating the infinite horizon with a long finite-time horizon which is a typical approach to implement an infinite-horizon optimal control formulation. In [38], an auxiliary control law was used to formulate the infinite-horizon problem with a finite number of decision variables corresponding to a finite-time horizon and approximating the infinite-horizon tail through the auxiliary control law. It was shown that the resulting EMPC asymptotically stabilizes the economically optimal steady-state when a strong duality assumption is satisfied [38]. Along the same lines as [38], the infinite-horizon EMPC problem was divided into a finite-time horizon and an infinite-horizon tail in [112,113,130]. Of particular interest to the various applications studied in these works was time-varying economic prices. To deal with the infinite-horizon time, an unconstrained infinite-time tail was analytically solved for. An economic linear optimal control policy was proposed which statistically constrained the unconstrained problem and its value was added to the finite-time horizon EMPC problem as a terminal cost [130].

3.2. Economic model predictive control with terminal constraints

Given the difficulty of solving an infinite-horizon EMPC for general cost functions of the form $l_e(x, u)$ and for a general nonlinear system, a finite-time prediction horizon approach is typically adopted. The objective function that the EMPC minimizes is

$$L_e(x(t), u(t)) = \int_0^{\tau_N} l_e(x(t), u(t))\, dt \tag{13}$$

where $\tau_N = N\Delta$ and $N < \infty$ is the finite-time prediction horizon. To better approximate the infinite-horizon solution and to ensure robustness of the control solution to disturbances and instabilities, the finite-horizon EMPC is implemented with a receding horizon; that is, the EMPC optimization problem is solved at every sampling instance τ_k to compute a control action to be applied in a sample-and-hold fashion (i.e., zeroth-order hold) over the sampling period from τ_k to τ_{k+1}. At the next sampling instance τ_{k+1}, the (finite-horizon) EMPC is computed by rolling the horizon one sampling period forward.

Much of the recent theoretical work on EMPC investigates the extension of conventional or tracking MPC (Eq. (5)) stabilizing elements to EMPC such as adding a terminal constraint and/or terminal cost (e.g., see, for instance, [109] for more details on the use of terminal constraint and cost). Numerous EMPC formulations and theoretical developments which include a terminal constraint and/or terminal cost have been proposed and studied [143,141,58,4,38,79,95,6,119,42,96,142,5,57,63,73,120–122,12, 166,174]. This class of EMPC schemes has the following general formulation which is given with a discrete-time model as most of the work on this type of EMPC has been done for discrete-time systems:

$$\underset{u(0),u(1),\cdots,u(N-1)}{\text{minimize}} \quad \sum_{j=0}^{N-1} l_e(\tilde{x}(j), u(j)) + V_f(\tilde{x}(N)) \tag{14a}$$

$$\text{subject to} \quad \tilde{x}(j+1) = f_d(\tilde{x}(j), u(j), 0) \tag{14b}$$

$$\tilde{x}(0) = x(k) \tag{14c}$$

$$\tilde{x}(N) \in \mathbb{X}_f \tag{14d}$$

$$(\tilde{x}(j), u(j)) \in \mathbb{Z}, \quad \forall j \in \mathbb{I}_{0:N-1} \tag{14e}$$

where $\mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U}$ is a compact, time-invariant set that includes the process constraints like input and state constraints and $\mathbb{I}_{0:N-1}$ is the set of integers ranging from 0 to $N-1$. At a sampling instance k corresponding to the time τ_k in continuous-time, the EMPC of Eq. (14) receives a measurement of the current state (Eq. (14c)) and optimizes the economic cost of Eq. (14a) with respect to the process dynamics (Eq. (14b)) and process constraints (Eq. (14e)). For stability and performance (the latter will be discussed in Section 4 below), a terminal constraint is added (Eq. (14d)). If the terminal constraint is a point-wise constraint $\tilde{x}(\tau_N) \in \mathbb{X}_f = \{x_s^*\}$, the terminal cost, denoted as $V_f(\tilde{x}(\tau_N))$, is often dropped as it is not required for stability and performance guarantees (refer to Section 4 for details of the latter point). When a terminal region constraint is used, that is, $\mathbb{X}_f$ is some compact set containing x_s^* in its interior, the terminal cost is often used.

With respect to provable closed-loop stability under the EMPC of Eq. (14), an assumption must be placed on the controllability or stabilizability properties of the system of Eq. (2) (or similarly, Eq. (1)). Before the assumption can be stated, a few definitions are required. First, a feasible input solution and the optimal input solution to the EMPC of Eq. (14) at time step k are denoted as $u(0|k), u(1|k), \ldots, u(N-1|k)$ and $u^*(0|k), u^*(1|k), \ldots, u^*(N-1|k)$, respectively. The set of admissible initial states and inputs is the set

$$\begin{aligned}\mathbb{Z}_N = \{&(x(0), u(0), u(1), \ldots, u(N-1)) | \tilde{x}(j+1) = f_d(\tilde{x}(j), u(j), 0), \tilde{x}(0) \\ &= x(0), \tilde{x}(N) \in \mathbb{X}_f, (\tilde{x}(j), u(j)) \in \mathbb{Z}, \forall j \in \mathbb{I}_{0:N-1}\} \qquad (15)\end{aligned}$$

where $u(j) = u(j|0)$. The set $\mathbb{Z}_N$ clearly depends on the prediction horizon length for both a point-wise terminal constraint and a terminal region constraint. The set of admissible initial states, denoted as $\mathbb{X}_N$, is the projection of $\mathbb{Z}_N$ onto $\mathbb{X}$. It is important to note that it is difficult to explicitly characterize the sets $\mathbb{Z}_N$ and $\mathbb{X}_N$ in general. The following assumption is placed on the type of discrete systems considered which bounds the amount of control energy required to force an initial state in $\mathbb{X}_N$ to x_s^*. The assumption of weak controllability ensures a non-empty feasible set for a sufficiently long prediction horizon.

Assumption 1. [Weak controllability] For the system of Eq. (2), there exists a feasible input trajectory $u(0), u(1), \ldots, u(N-1)$ for each $x(0) \in \mathbb{X}_N$ and there exists a $\mathcal{K}_\infty$ function $\gamma(\cdot)$ such that

$$\sum_{j=0}^{N-1} |u(j) - u_s^*| \le \gamma(|x(0) - x_s^*|). \qquad (16)$$

Recursive feasibility of the EMPC of Eq. (14) is guaranteed for the nominally operated system (Eq. (2) with $w(k) \equiv 0$ and when Assumption 1 is satisfied) for any initial state $x(0) \in \mathbb{X}_N$. The closed-loop state trajectory under the EMPC of Eq. (14) will remain bounded under nominal operation ($w(t) \equiv 0$) if the economic cost $l_e(\cdot)$ and the state transition mapping $f_d(\cdot)$ are continuous on $\mathbb{Z}$ (recall that $\mathbb{Z}$ is a compact set), x_s^* is contained in the interior of $\mathbb{X}_N$, and Assumption 1 holds. In other words, $x \in \mathbb{X}_N$ for all $k \ge 0$ when $x(0) \in \mathbb{X}_N$ ($\mathbb{X}_N$ is a forward invariant set). This form of stability is much different than the forms of stability typically shown for the closed-loop system under conventional MPC (e.g., nominal asymptotic stability when applying the discrete control sequence to the discrete-time system of Eq. (2) or practical stability when applying the discrete control sequence to the continuous-time system of Eq. (1)). In other words, the EMPC of Eq. (14) will lead to dynamic, transient, or time-varying operation in general which is depicted in Fig. 2. This type of stability property has been demonstrated in numerous applications to be an important property of EMPC leading to closed-loop economic performance improvement over traditional control methodologies (e.g., tracking MPC).

Still, it is important to understand under what conditions the EMPC of Eq. (14) will render the economically optimal steady-state

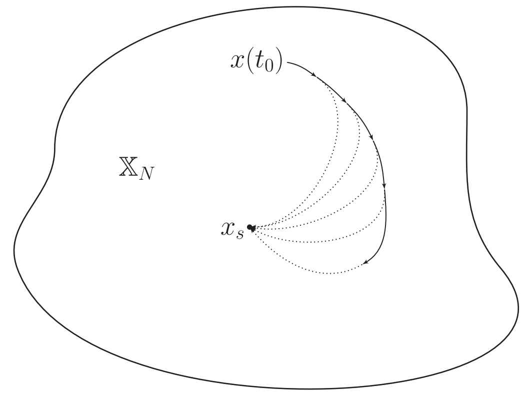

Fig. 2. A state-space illustration of the state trajectory under the EMPC of Eq. (14) with a point-wise terminal constraint over several sampling periods. The solid line is the closed-loop state trajectory and the dotted line is the open-loop predicted trajectory $\tilde{x}(t)$ computed at each sampling period.

x^* asymptotically stable for the closed-loop system of Eq. (2) [6,4]. One method is to make additional assumptions regarding the nonlinear system which extends the notion of dissipativity [6] (first presented for continuous-time systems [165] and extended to discrete-time systems [25]).

Definition 1. [Dissipativity [25,6]] A closed-loop system is dissipative with respect to a supply rate $s : \mathbb{X} \times \mathbb{U} \to \mathbb{R}$ if there exists a function $\lambda : \mathbb{X} \to \mathbb{R}$ such that

$$\lambda(f_d(x, u)) - \lambda(x) \le s(x, u) \qquad (17)$$

for all $(x, u) \in \mathbb{Z} \subseteq \mathbb{X} \times \mathbb{U}$. If there exists a positive definite function $\beta : \mathbb{X} \to \mathbb{R}_{\ge 0}$ such that

$$\lambda(f_d(x, u)) - \lambda(x) \le -\beta(x) + s(x, u) \qquad (18)$$

then the system is strictly dissipative.

Assumption 2. [Dissipativity of the Closed-loop System under EMPC [6]] The closed-loop system of Eq. (2) under the EMPC of Eq. (14) is strictly dissipative with a supply rate given by:

$$s(x, u) = l_e(x, u) - l_e(x_s^*, u_s^*). \qquad (19)$$

When Assumptions 1 and 2 are satisfied for the closed-loop system under the EMPC of Eq. (14), the steady-state x_s^* is asymptotically stable for any initial condition $x(0) \in \mathbb{X}_N$. A stronger assumption than dissipativity that has been used to derive a Lyapunov function for the closed-loop system under the EMPC of Eq. (14) is strong duality (i.e., strong duality implies dissipativity [6]).

Assumption 3. [Strong Duality of the Steady-State Problem [38]] There exists a λ_s so that (x_s^*, u_s^*) is the unique minimizer of

$$\begin{aligned}&\underset{x,u}{\text{minimize}}\; l_e(x, u) + [x - f_d(x, u)]^T \lambda_s \\ &\text{subject to}\; (x, u) \in \mathbb{Z}\end{aligned} \qquad (20)$$

and there exists a function $\hat{\beta}$ of class $\mathcal{K}_\infty$ such that the rotated economic cost $L(x, u)$ satisfies:

$$L(x, u) \ge \hat{\beta}(|x - x_s^*|) \qquad (21)$$

where the rotated cost is defined as

$$L(x, u) := l_e(x, u) + [x - f_d(x, u)]^T \lambda_s - l_e(x_s^*, u_s^*). \qquad (22)$$

It was shown in [38] that the rotated cost of Eq. (22) is a Lyapunov function for the closed-loop system under the EMPC of Eq. (14) formulated with a point-wise terminal constraint when the closed-loop system satisfies Assumptions 1 and 3.

Within the context of economics-based constraints, one class of constraints that are of interest within the context of EMPC are average constraints. Reflecting back on the traditional paradigm for economic optimization of chemical processes with RTO, the computed operating conditions satisfy the economic constraints (Eq. (4d)) asymptotically since the closed-loop system is asymptotically forced to the operating conditions. Within the context of EMPC which may not force the system to operate at the economically optimal steady-state, it may be important to enforce economics-based constraints directly in the EMPC. For example, constrain the EMPC solution such that the time-averaged raw material amount is fixed. One notion of economics-based average constraints is to construct constraints that asymptotically satisfy an average, as is the case in the traditional operation paradigm.

One method for handling asymptotic average constraints [6] is to define an auxiliary variable as follows (the method is summarized below since it is applied in Section 5; the interested reader is referred to [6] for a complete discussion of this method):

$$y(k) = h(x(k), u(k)) \tag{23}$$

where $h : \mathbb{Z} \to \mathbb{R}^{n_y}$ is continuous on $\mathbb{Z}$ and y contains all the average constraints that should be asymptotically satisfied. The average of y is

$$\frac{\sum_{i=0}^{n} y(k)}{n+1} \tag{24}$$

Owing to the fact that h is continuous on the compact set $\mathbb{Z}$, the average as n tends to infinity is finite. However, taking the limit of Eq. (24) as n tends to infinity may be meaningless because it may not be properly defined (i.e., have a unique value). Instead, the following is used for the definition of the asymptotic average in [6]:

$$\mathrm{Av}[y] = \left\{ \bar{y} \in \mathbb{R}^{n_y} \,|\, \exists t_n \to \infty,\ \lim_{n\to\infty} \frac{\sum_{k=0}^{t_n} y(k)}{t_n + 1} = \bar{y} \right\} \tag{25}$$

which deals with the fact that the asymptotic average might be a set of numbers. Also, the asymptotic average of y is a non-empty set. The (set) constraint on the asymptotic average of y is denoted as $\mathbb{Y}$ which is assumed to be a closed and convex set. Furthermore, it is assumed that

$$h(x_s, u_s) \in \mathbb{Y}. \tag{26}$$

At every sampling period, the following constraint is imposed in the optimization problem of Eq. (14):

$$\sum_{j=0}^{N-1} h(\tilde{x}(j), u(j)) \in \mathbb{Y}_k \tag{27}$$

where

$$\mathbb{Y}_k = \mathbb{Y}_{00} \oplus (k+N)\mathbb{Y} \ominus \sum_{j=0}^{N-1} h(\tilde{x}(j), u^*(j)) \tag{28}$$

and $\mathbb{Y}_{00}$ is an arbitrary compact set containing $h(x_s, u_s)$ in its interior. The constraint of Eq. (27) ensures that the asymptotic average constraint is satisfied for the nominal closed-loop system of Eq. (2) under the EMPC of Eq. (14) formulated with a point-wise terminal constraint (i.e., $\mathbb{X}_f = \{x_s^*\}$ and $V_f(\cdot) \equiv 0$). This was shown in [6]. Asymptotic average constraints were extended to EMPC with a terminal region and terminal cost in [123]. Furthermore, a general method for enforcing transient average constraints by adding $(N+T-1)n_y$ additional constraints to the EMPC where T is the number of sampling periods that the average constraint must be satisfied was presented in [121].

Other works on EMPC of the form of Eq. (14) address various issues. In [143,141], the use of unreachable set-points in the cost function of a tracking MPC (i.e., with a quadratic cost) was discussed, and a demonstration of the approach was provided. The demonstration showed better closed-loop performance of a tracking MPC formulated with unreachable set-points compared to using a tracking MPC formulated with reachable targets generated from (steady-state) target optimization (see above for a discussion of target optimization). In [58], an EMPC was formulated for changing economic criterion using a terminal cost and a point-wise terminal constraint of the form: $\tilde{x}(N) = \tilde{x}(N+1)$. This type of terminal constraint essentially forces the open-loop predicted state trajectory $\tilde{x}$ to converge to an equilibrium manifold instead of a single equilibrium point. EMPC with a terminal region constraint and terminal cost was introduced and analyzed in [4]. A Lyapunov stability analysis was given in [79] for EMPC with a terminal constraint based on optimal cyclic steady-states. In [6], asymptotic average performance, the optimality of steady-state operation, periodic terminal constraints, and asymptotic average constraints were presented and analyzed. In [119], it was shown that the dissipativity property is robust to small changes in the constraint set. In [5], direct methods were employed to formulate the EMPC problem as a large-scale nonlinear program (NLP) and solve it with an interior point nonlinear solver [163,19] with automatic differentiation [164]. The idea of enforcing a generalized terminal constraint in EMPC (i.e., enforce the predicted state trajectory to converge to an equilibrium manifold) was explored further in [57,120]. In [57], a MPC (or EMPC) scheme was proposed and it was shown that with the proposed MPC (or EMPC) algorithm the control solution converges to that of an MPC (or EMPC) algorithm with a terminal constraint chosen to be the economically optimal steady-state. This idea was further extended with a self-tuning terminal cost which may lead to improved closed-loop performance compared to a fixed terminal weight [120]. In [122], a Lyapunov stability analysis of asymptotically average constrained EMPC was given and the necessity of dissipativity for optimal steady-state operation was discussed. A Lyapunov function was derived for EMPC formulated with a periodic terminal constraint in [174]. In [166], a two-layer control scheme was proposed that featured an EMPC in the upper layer and a fast hybrid neighboring-extremal controller in the lower layer for nonlinear hybrid systems.

3.3. *Economic model predictive control with Lyapunov-based constraints*

Another method for designing an EMPC with provable stability properties is to formulate Lyapunov-based constraints by taking advantage of an explicit stabilizing controller (i.e., the explicit controller is used as an auxiliary controller). The resulting EMPC is the so-called Lyapunov-based EMPC (LEMPC) [68,29,69,54,70,71,50,49,162,51–53,55,91,90,92]. Before the formulation of LEMPC is given, the main assumptions are stated. The vector field f of the nonlinear system of Eq. (1) is assumed to be a locally Lipschitz vector function on $\mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_w}$. Like the EMPC with a terminal constraint, a notion of controllability and/or stabilizability of the system of Eq. (1) must be imposed. The following assumption is essentially a stabilizability assumption for the system of Eq. (1) and is comparable to assuming the (A, B) pair is stabilizable for the linear system of Eq. (3) (i.e., the (A, B) pair is stabilizable if all its uncontrollable modes are stable or in other words, the eigenvalues of the uncontrollable modes are in the left-half of the complex plane).

Assumption 4. [Existence of a Lyapunov-based Controller] There exists a Lyapunov-based controller $k(x)$ which renders the origin of the nominal closed-loop system of Eq. (1) under continuous implementation of $k(x)$ asymptotically stable with $u = k(x) \in U$ for all $x \in D \subseteq \mathbb{R}^{n_x}$ where D is an open neighborhood of the origin.

Using converse theorems [108,99,87], Assumption 4 implies the existence of a continuously differentiable Lyapunov function $V(x)$ for the nominal closed-loop system of Eq. (1) under the continuous implementation of the controller $k(x)$ that satisfies the following inequalities:

$$\alpha_1(|x|) \le V(x) \le \alpha_2(|x|) \quad (29a)$$

$$\frac{\partial V(x)}{\partial x} f(x, k(x)) \le -\alpha_3(|x|) \quad (29b)$$

$$\left| \frac{\partial V(x)}{\partial x} \right| \le \alpha_4(|x|) \quad (29c)$$

for all $x \in D$ where $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ are class $\mathcal{K}$ functions. The region $\Omega_\rho \subseteq D$ is the (estimated) stability region of the closed-loop system under the Lyapunov-based controller and is taken to be a level set of the Lyapunov function where the time-derivative of the Lyapunov function is negative along the closed-loop state trajectory. Several control laws that satisfy Assumption 4 have been developed for various classes of nonlinear systems including control laws that provide explicit characterization of the region of attraction for the closed-loop system under the controller $k(x)$ which accounts for input constraints (see, for example, [32,48,88,100,156] and the references therein for results in this direction). Applying the controller $k(x)$ in a sample-and-hold fashion with a sufficiently small sampling period will ensure practical stability of the origin of the closed-loop system (i.e., $k(x)$ is applied as an emulation controller); see, for instance, [128,65,118,85,129] and the references therein for results and analysis of sampled-data systems. Practical stability means convergence to a small neighborhood of the origin for sufficiently large time.

Much like the design methodology of EMPC with terminal constraint/cost, LEMPC takes advantage of design techniques originally developed for Lyapunov-based MPC (LMPC) which is an MPC technique which uses a quadratic cost (e.g., [114,115,118,33]). Utilizing the stability region Ω_ρ under the explicit (auxiliary) controller $k(x)$, LEMPC is a two-mode control strategy and its formulation is given by the following optimization problem:

$$\underset{u \in S(\Delta)}{\text{minimize}} \quad L_e(\tilde{x}(t), u(t)) \quad (30a)$$

$$\text{subject to} \quad \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \quad (30b)$$

$$\tilde{x}(0) = x(\tau_k) \quad (30c)$$

$$u(t) \in U, \ \forall\, t \in [0, \tau_N) \quad (30d)$$

$$V(\tilde{x}(t)) \le \rho_e, \quad \forall\, t \in [0, \tau_N) \quad \text{if } V(x(\tau_k)) < \rho_e \ \text{ and } \ t < t_s \quad (30e)$$

$$\frac{\partial V}{\partial x} f(x(\tau_k), u(\tau_k), 0) \le \frac{\partial V}{\partial x} f(x(\tau_k), k(x(\tau_k)), 0) \quad \text{if } V(x(\tau_k)) \ge \rho_e \ \text{ or } \ t \ge t_s \quad (30f)$$

where t_s is a switching time of the controller which is discussed below and the other notation is similar to that of Eq. (9).

In the optimization problem of Eq. (30a), the objective function (Eq. (30a)) is the integral of the economic cost function (Eq. (13)) over the prediction horizon. The model of Eq. (30b) is used to predict the future evolution of the process system over the prediction horizon and is initialized through a state measurement at the current sampling period (Eq. (30c)). The input constraint of Eq. (30d) bounds the computed piecewise constant input trajectory to be in the set of available control actions. The remaining Lyapunov-based constraints distinguish the two modes of operation of the LEMPC. Namely, the Lyapunov-based constraints of Eq. (30e) and Eq. (30f) define mode 1 and mode 2 operation of the LEMPC, respectively. Under mode 1 operation, the trajectory $\tilde{x}(t)$ may dynamically evolve in a bounded set $\Omega_{\rho_e} \subset \Omega_\rho$. The size of Ω_{ρ_e} depends on the stability properties of the system, the sampling period, and the bound on the disturbance and has the property such that if a disturbance forces the state outside of Ω_{ρ_e} over the sampling period, the state will be maintained in Ω_ρ. This may be mathematically summarized through the following: if $x(\tau_k) \in \Omega_{\rho_e}$, then $x(\tau_{k+1}) \in \Omega_\rho$. Under mode 2 operation, the constraint of Eq. (30f) enforces that the time-derivative of the Lyapunov function under the LEMPC be less than the time-derivative of the Lyapunov function under the Lyapunov-based controller $k(x)$. Through this constraint, the Lyapunov function under LEMPC is guaranteed to decrease over the sampling period τ_k to τ_{k+1} for any state $x(t_k) \in \Omega_\rho$ (thus, $x(t_{k+1}) \in \Omega_\rho$). Under mode 2 operation of the LEMPC, the Lyapunov function is guaranteed to decrease until the state trajectory converges to a small neighborhood of the origin as a result of the closed-loop stability properties of the controller $k(x)$. The mode 2 constraint is enforced to either steer the state to the set Ω_{ρ_e} or to enforce convergence to the origin (i.e., x_s^*). An example illustration of the possible evolution under the two modes of operation of the LEMPC is shown in Fig. 3.

Mode 1 is active when $x(\tau_k) \in \Omega_{\rho_e}$ and $\tau_k < t_s$, while mode 2 is active when $x(\tau_k) \notin \Omega_{\rho_e}$ or $\tau_k \ge t_s$. The switching time t_s warrants more explanation. The LEMPC scheme may dictate a dynamic operating policy. Therefore, continuous forcing of the system through the control actuators may be required to dictate this type of operation. Therefore, the switching time may be chosen to manage the trade-off between dynamically optimal operation and excess control actuator usage. The two extremes, $t_s = 0$ and $t_s = \infty$, correspond to the case when it is desirable to enforce convergence to the origin and to the case when time-varying operation is desirable for the entire length of operation. Notice that the economic cost does not need to be modified to enforce convergence to the origin. On the other hand, the cost function is typically modified to achieve guaranteed convergence to the origin under EMPC with a terminal constraint so that the closed-loop system satisfies Assumption 2 (accomplished by adding convex or quadratic terms, e.g., [6]).

The LEMPC has unique feasibility and stability properties compared to EMPC formulated with a terminal constraint. The set Ω_ρ is a characterizable set in state-space and is an estimate of the

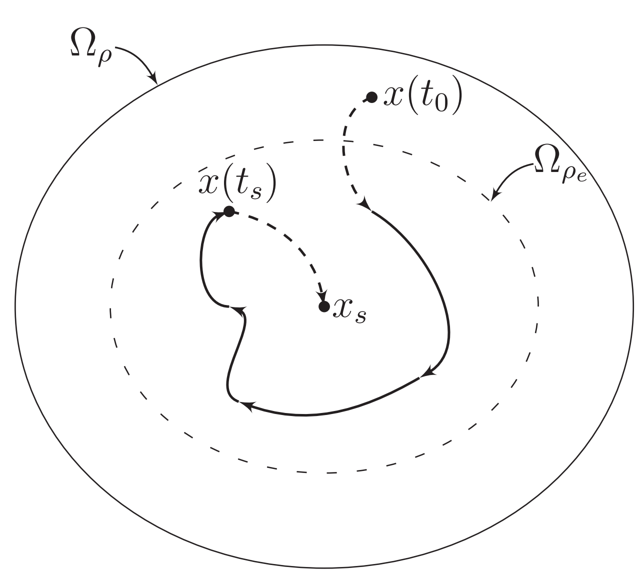

Fig. 3. An illustration of the state trajectory under the two-mode LEMPC of Eq. (30a). The state trajectory under mode 1 operation of the LEMPC is the solid trajectory, while the dashed trajectory is under mode 2 operation of the LEMPC.

stability region and feasible set. For any state $x \in \Omega_\rho$, the state is guaranteed to be maintained in Ω_ρ for all times for sufficiently small disturbances and a sufficiently small sampling period (Ω_ρ is forward invariant). Furthermore, the set Ω_ρ does not depend on the choice of prediction horizon length. Regarding feasibility of Eq. (30a), the optimization problem is recursively feasible because the input trajectory obtained from the Lyapunov-based controller is a feasible solution to the optimization problem regardless of whether $x \in \Omega_\rho$ or $x \notin \Omega_\rho$. However, stability in the sense of convergence to Ω_ρ (and then, boundedness in Ω_ρ) cannot be guaranteed for any $x \notin \Omega_\rho$ because the time-derivative of the Lyapunov function under the Lyapunov-based controller may be positive. The set where convergence to Ω_ρ under the LEMPC is guaranteed (i.e., the region in state-space where the time-derivative of the Lyapunov function is negative under the controller $k(x)$) is denoted as Φ_u and $\Omega_\rho \subseteq \Phi_u$. Thus, Ω_ρ is also an estimate of the feasible set. The detailed analysis of the stability properties of this control scheme can be found in [68].

Regarding imposing state constraints within LEMPC, one can extend the concepts from LMPC (e.g., [115]) for imposing state constraints in LEMPC. Specifically, define the set Φ_u as the set in state-space that includes all the states where $\dot{V} < 0$ under the controller $k(x)$. Consider the case where $\Phi_u \subseteq \mathbb{X}$. This means that any initial state starting in the region $\mathbb{X} \setminus \Phi_u$ will satisfy the state constraint. However, the time-derivative of the Lyapunov function may be positive and thus, it may not be possible to stabilize the closed-loop system starting from this initial condition. The stability region used in the formulation of the LEMPC for this case is $\Omega_\rho = \Omega_{x,u} = \{x \in \mathbb{R}^{n_x} \mid V(x) \le \rho_{x,u}\}$ where $\rho_{x,u}$ is the largest number for which $\Omega_{x,u} \subseteq \Phi_u$. On the other hand, consider the case where $\mathbb{X} \subset \Phi_u$. This case is depicted in Fig. 4. For any initial state starting outside $\mathbb{X}$, the state constraint will be violated from the outset. Also, for any initial state in the set $\mathbb{X}$, it is not possible, in general, to guarantee that the set $\mathbb{X}$ is forward invariant because there may exist a stabilizing state trajectory (i.e., a trajectory where $\dot{V}(x) < 0$) that goes outside of the set $\mathbb{X}$ before it enters back into the set to converge to the origin. For this case, the determination must be made whether the state constraints are hard constraints (i.e., cannot be violated) or soft constraints (i.e., may be violated for some periods of time). For the case with hard constraints, define the set Ω_ρ as $\Omega_\rho = \Omega_{x,u} = \{x \in \mathbb{R}^{n_x} | V(x) \le \rho_{x,u}\}$ where $\rho_{x,u}$ is the largest number for which $\Omega_{x,u} \subseteq \mathbb{X}$. For the case where the state constraints may be treated as soft constraints (i.e., may be violated over certain periods of time), one can extend the switching constraints of [115] in the formulation of the LEMPC, since Φ_u cannot be computed in practice. Instead, the set $\Omega_u = \{x \in \mathbb{R}^{n_x} | V(x) \le \rho_u\}$) where ρ_u is the largest number for which $\dot{V} < 0$ (under the controller $k(x)$ which accounts for the input constraint only) may be used. An illustration of the set definitions is provided in Fig. 4. Furthermore, the sets $\Omega_{x,u}$ and Ω_u in Fig. 4 are computed for the example used in Section 5.

Other theoretical developments on LEMPC include designing a state-estimation-based LEMPC using high-gain observers and moving horizon estimation [69,55], formulating an LEMPC scheme for switched systems [71], utilizing LEMPC or Lyapunov-based design concepts to design two-layer control structures featuring EMPC or LEMPC [50,51], designing a composite controller with LEMPC for nonlinear singularly perturbed systems [54], accounting for time-varying pricing in the economic cost function [49], and integrating preventive control actuator maintenance, process economics, and process control into a unified framework with LEMPC [92]. In all the cases, a stability analysis was provided for the system of Eq. (1) with bounded disturbances. Additionally, the LEMPC techniques were applied to parabolic PDE systems along with model reduction techniques [91,90].

Remark 3. General methods for constructing Lyapunov functions for nonlinear systems with constraints (e.g., state and input constraints) remain an open research topic. The construction of Lyapunov functions for unconstrained nonlinear systems may be accomplished by exploiting the system structure like the use of quadratic Lyapunov functions for feedback linearizable systems and the use of back-stepping techniques. Some methods exist for the design of Lyapunov functions for nonlinear systems with constraints which include techniques based on Zubov's method [46] and based on the sum of squares decomposition [134]. In practice, quadratic Lyapunov functions have been widely used and have yielded good estimates of the closed-loop stability regions (e.g., [32]). While the resulting estimates do not necessarily capture the entire domain of attraction, it is possible to obtain improved estimates of the domain of the attraction by using, for example, a family of quadratic Lyapunov functions (e.g., [32,49]).

4. Closed-loop economic performance under EMPC

The economic performance of the closed-loop system under EMPC is typically measured with the total economic cost index defined by:

$$J_e := \int_{t_0}^{t_f} l_e(x(t), u^*(t))\,dt \tag{31}$$

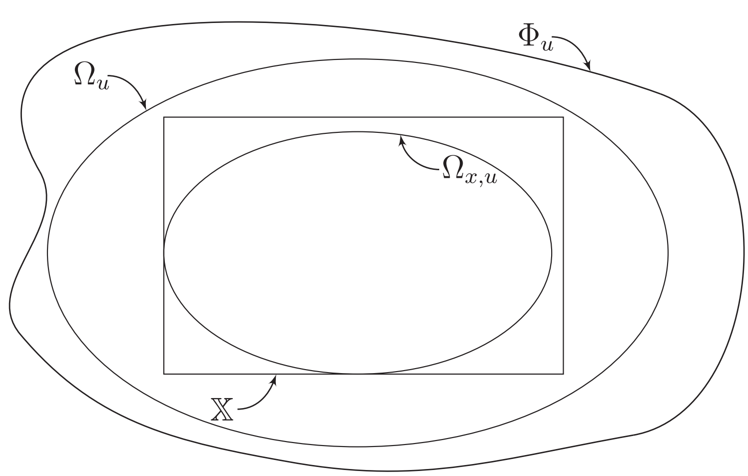

Fig. 4. An illustration of the various state-space sets described for enforcing state constraints with LEMPC. The case when $\mathbb{X} \subset \Phi_u$ is depicted in this illustration.

or the average economic cost index defined by:

$$\bar{J}_e := \frac{1}{t_f - t_0} \int_{t_0}^{t_f} l_e(x(t), u^*(t))\, dt \quad (32)$$

where $x(t)$ is the actual closed-loop state trajectory under the input profile $u^*(t)$ computed by EMPC (i.e., $x(t)$ is the solution of Eq. (1) with the input profile $u^*(t)$ and a given realization of the process disturbances $w(t)$ over the time t_0 to t_f). While the EMPC works to optimize the process economics, closed-loop performance has been the subject of much recent research on EMPC. To the casual observer, it may seem like applying EMPC to a system will result in improved closed-loop economic performance over traditional control methodologies (e.g., tracking MPC). Unfortunately, this is not the case in general. To better illustrate this, consider the simple example below.

Example 1. Consider the scalar system described by

$$\dot{x}(t) = x(t) + u(t) \quad (33)$$

where the input is bounded by $-10 \le u(t) \le 10$ and the economic cost for the system is

$$l_e(x, u) = 2(u + 10) + (x - 2)^2. \quad (34)$$

The optimal steady-state and steady-state input are $x_s^* = 3.0$ and $u_s^* = -3.0$ which correspond to a steady-state economic cost of $l_e(x_s^*, u_s^*) = 15.0$. If one formulates an EMPC using the general form of Eq. (9) for the system of Eq. (33), the resulting EMPC would have the following formulation:

$$\begin{aligned} &\underset{u \in S(\Delta)}{\text{minimize}} \int_0^{\tau_N} \left[2(u(t) + 10) + (\tilde{x}(t) - 2)^2\right] dt \\ &\text{subject to } \dot{\tilde{x}}(t) = \tilde{x}(t) + u(t), \quad \tilde{x}(0) = x(\tau_k), \\ &\qquad -10 \le u(t) \le 10, \quad \forall\, t \in [0, \tau_N) \end{aligned} \quad (35)$$

where the notation is consistent with the notation used in Eq. (9). The EMPC is applied to the scalar system with a sampling period of $\Delta = 0.05$ and two different prediction horizons ($N = 5$ and $N = 100$) are considered for a length of operation of $t = 5.0$. The system is initialized at $x(0) = 1.0$ and the closed-loop trajectories are given in Fig. 5 over the time period $t = 0$ to $t = 0.5$ to better illustrate the difference in the transient operation between the two cases.

From the closed-loop trajectories (Fig. 5), the closed-loop system under EMPC with the prediction horizons $N = 5$ and $N = 100$ responds differently which is reflected in the total economic cost (defined in Eq. (31)). Under the two cases, the total economic costs are 29.0 ($N = 5$) and 15.8 ($N = 100$), respectively; the performance with the horizon $N = 5$ is 84% worse than with $N = 100$. For the given system and economic cost, steady-state operation is likely the optimal operating strategy. The steady-state to which the closed-loop state under the EMPC with $N = 5$ converges has an economic cost of 30.0, which is worse than that of the economically optimal steady-state which has an economic cost of 15.0. For this initial condition, any controller that stabilizes around the economically optimal steady-state would at least asymptotically outperform the EMPC with a prediction horizon of $N = 5$ from an economic perspective. Furthermore, if we apply the EMPC to the system with various prediction horizons, performance improvement with increasing prediction horizon is observed (Table 1).

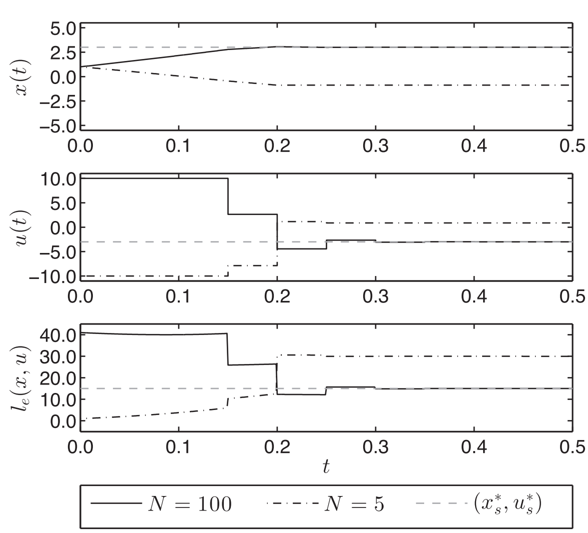

Fig. 5. Closed-loop trajectories of the system of Eq. (33) under the EMPC of Eq. (35).

The closed-loop trajectory of the system under EMPC may seem unexpected or even undesirable. However, the EMPC with $N = 5$ is performing exactly as it should according to the dynamic optimization problem of Eq. (35). The reason for this behavior is best explained by observing the open-loop predicted trajectories of the system under EMPC; that is, the state trajectory under the input trajectory computed by the EMPC at one sampling period. The open-loop predicted trajectories for $N = 5$ and $N = 100$ are given in Fig. 6. Comparing the total economic cost over the open-loop predicted trajectory from $t = 0$ to $t = 0.25$, the total economic costs are 5.47 with $N = 5$ and 32.27 with $N = 100$. While the actions taken by the EMPC with $N = 5$ are better near-term (over the time period from $t = 0$ to $t = 0.25$) compared to the actions taken by the EMPC with $N = 100$, these actions are not optimal over a larger horizon. This type of behavior has been observed in many applications (e.g., [64,130]) and was described as myopic behavior in [130] which was originally a term used in the scheduling literature to describe the solution of a scheduling problem derived from an optimal control problem that exhibited similar behavior [98].

Another point to be observed from the open-loop predicted trajectory of Fig. 6b is the three distinct segments of the open-loop predicted trajectory. The first segment from $t = 0$ to approximately $t = 0.5$ is the process transients (i.e., the effect of the initial condition). From approximately $t = 0.5$ to $t = 4.5$, the state trajectory converges to a neighborhood of the optimal steady-state. In the last segment, the state trajectory is driven away from the optimal steady-state to achieve an improvement in the economic cost. This property is referred to as a turnpike property [27,40,110,111] since the state passes through the optimal steady-state until it finally moves away to achieve further economic benefit (like a vehicle getting on and then, off a turnpike or highway). The turnpike property is a common property amongst many optimal control problems and dynamic optimization problems. Unsurprisingly, this property has been found to be a useful property in the context of EMPC [64,141].

In Example 1, steady-state operation is likely the optimal operation strategy for the system and the economic cost (in fact, it will be shown below that it is). Aligned with current practice, one may consider adding and tuning quadratic terms to the economic cost function or additional stabilizing constraints to the EMPC in an attempt to achieve stabilization at the economically optimal steady-state. In this case, one potentially helpful tuning methodology is to observe and understand the open-loop predicted

Table 1
Total economic cost J_e with the prediction horizon N.

N	J_e	N	J_e
1	61928.67	10	15.94
2	361.49	20	15.85
3	70.45	50	15.85
4	41.27	100	15.85
5	28.98		

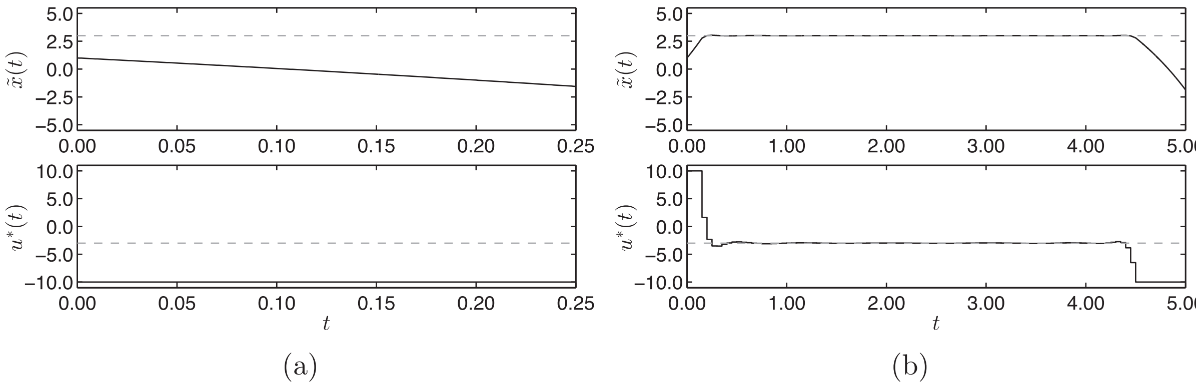

Fig. 6. The open-loop predicted trajectories $\tilde{x}(t)$ (solid lines) under the optimal input trajectory computed by the EMPC of Eq. (35) with a prediction horizon of (a) $N = 5$ and (b) $N = 100$ (dashed lines are the optimal steady-state and corresponding input).

trajectory of the system under its optimal input solution when tuning the stage cost (i.e., the economic cost plus the additional terms) or assessing what constraints should be added to the EMPC to improve performance.

From a theoretical perspective, currently two methodologies exist for closed-loop performance guarantees under EMPC: (1) to employ a sufficiently large horizon [64] and (2) the application of a terminal constraint. The former methodology allows for (approximate) closed-loop performance guarantees for both transient and infinite-time operation intervals with respect to the economically optimal steady-state (see [64] for a complete discussion of these points). For the latter methodology, the type of closed-loop performance guarantee that may be made depends on the type of terminal constraint used (see Sections 4.1–4.2). The use of a sufficiently large horizon is clearly evident in the above example (Table 1). Two methods for constructing a terminal constraint are described below, but it is important to emphasize these results on closed-loop performance only work under nominal operation. Closed-loop performance under EMPC in the presence of disturbances is an open issue.

4.1. Terminal constraint or cost

One approach for closed-loop performance improvement under EMPC is to consider how to approximate the economic cost that is not covered in the prediction horizon (i.e., over the time interval $[\tau_k, \infty)$). To do this, a point-wise terminal constraint based on the economically optimal steady-state $\mathbb{X}_f = \{x_s^*\}$ may be used. At any sampling time k (using discrete time), the EMPC is solved under the constraint that the predicted state at the end of the horizon converges to the economically optimal steady-state $\tilde{x}(N) = x_s^*$. If the input trajectory computed at k was applied in closed-loop over the next N sampling periods, the actual closed-loop state would converge to x_s^* at time step $k + N$ where it could be maintained thereafter under nominal conditions. Therefore, the average closed-loop economic performance under EMPC formulated with the economically optimal steady-state is guaranteed to be no worse than the economically optimal steady-state over a sufficiently long operating time because over any N time steps, it is possible to force the system to x_s^* where it can be maintained thereafter (thus, essentially canceling out the effect of the transients). Of course, this does not imply that the EMPC formulated with a point-wise terminal constraint will compute control actions that force the closed-loop state to the steady-state over time (recall the discussion on stability for EMPC formulated with a terminal constraint above). This result on the average closed-loop economic performance under EMPC can be mathematically stated as follows: at the next sampling instance $k + 1$, the EMPC is re-solved. The input solution $\mathbf{u}_{\text{feas}} = \{u^*(1|k), u^*(2|k), \ldots u^*(N-1|k), u_s^*\}$, where $u^*(j|k)$ denotes the jth input along the prediction horizon computed at time step k, is a feasible solution to the optimization problem at $k + 1$. If there exists any better input solution with respect to the economic cost, the EMPC would return that input solution. However, using the feasible solution $\mathbf{u}_{\text{feas}}$, the difference in objective function values of the EMPC over two consecutive time steps can be bounded by:

$$\sum_{j=0}^{N-1} l_e(x(j|k+1), u^*(j|k+1)) - \sum_{j=0}^{N-1} l_e(x(j|k), u^*(j|k)) \leq l_e(x_s^*, u_s^*) - l_e(x(k), u^*(j|k)). \quad (36)$$

Using Eq. (36), it was shown in [6] that for nominal operation the asymptotic average performance under EMPC formulated with a point-wise terminal constraint is bounded above by the economically optimal steady-state where the asymptotic average is given by (defined in discrete time):

$$\limsup_{T\to\infty} \frac{\sum_{k=0}^{T} l_e(x(k), u(k))}{T+1} \leq l_e(x_s^*, u_s^*). \quad (37)$$

Asymptotic average performance (Eq. (37)) under EMPC compared to the economically optimal steady-state has been extended to EMPC with a terminal region constraint and terminal cost in [4] and to EMPC with a generalized terminal constraint [57,120].

Another important result in the context of performance of EMPC with a terminal constraint is that when the closed-loop system under the EMPC with a point-wise terminal constraint is dissipative with supply function $s(x, u) = l_e(x, u) - l_e(x_s^*, u_s^*)$, steady-state operation is the economically optimal operating strategy [6] (i.e., no other type of operation will give better average economic cost). Dissipativity also comes close to being a necessary condition for economically optimal steady-state operation [122]. Unfortunately, for large-scale systems dissipativity is hard to verify in general.

Since the strong duality condition (Assumption 3) implies dissipativity, it can be shown that steady-state operation of the system of Eq. (33) with the economic cost of Eq. (34) is the economically optimal operating strategy.

Example 2. For the example system of Eq. (33) under the EMPC of Eq. (35), the assumption of strong duality (Assumption 3) can be analytically verified. The state at the next sampling period $x(\tau_{k+1})$ is

$$x(\tau_{k+1}) = e^{\Delta}x(\tau_k) + (e^{\Delta} - 1)u(\tau_k) \quad (38)$$

Applying the definition of the rotated economic cost (Eq. (22)) yields:

$$L(x,u) := l_e(x(\tau_k), u(\tau_k)) + \lambda_s^T(x(\tau_k) - x(\tau_{k+1})) = 2u(\tau_k) + 20 + (x(\tau_k) - 2)^2 + \lambda_s(x(\tau_k) + u(\tau_k))(1 - e^{\Delta}) \quad (39)$$

The function $L(\cdot)$ is convex and the multiplier λ_s and unique steady-state minimizer of $L(x, u)$ are

$$\lambda_s = \frac{-2}{1 - e^{\Delta}}, \quad x^*(\tau_k) = x_s^* = 3 \quad (40)$$

The property of Eq. (21) is satisfied since $L(x, u) \geq 15 = l(x_s^*, u_s^*)$ for any $x \in \mathbb{R}$ and $u \in \mathbb{U}$. Therefore, this satisfies the strong duality of the steady-state optimization problem which implies that steady-operation at (x_s^*, u_s^*) is the optimal operating strategy.

Remark 4. Methods exist for approximating the cost-to-go (the economic cost over the time interval $[\tau_{k+N}, \infty)$) such as estimating this through an infinite-horizon economic linear optimal control problem as in [38,130]. The cost-to-go was added to an EMPC formulated without a terminal constraint in [38,130]. The approach demonstrated improved closed-loop performance over an EMPC without adding the approximated cost-to-go.

4.2. Performance constraints based on auxiliary controllers

As another way to formulate a terminal constraint for closed-loop performance guarantees under EMPC, one may consider how to construct a constraint which accounts for the closed-loop performance over a finite operating window [70,53]. Namely, one may use an auxiliary stabilizing controller to compute both its input profile and open-loop predicted state trajectory over some operating window. Then, send the terminal state of the computed state trajectory (i.e., the state at the end of the operating window with the auxiliary controller) to the EMPC as a terminal constraint. Utilizing a shrinking prediction horizon, the EMPC computes the economically optimal path to the terminal state. With this EMPC algorithm, the closed-loop economic performance under EMPC is at least as good as the closed-loop economic performance under the auxiliary stabilizing controller on both the finite-time and infinite-time intervals [53]. Another idea is to compute the total control energy used by the auxiliary stabilizing controller and enforce that the EMPC computes an input trajectory that uses no more control energy than the auxiliary controller input profile over the operating window [70]. This may be particularly important when the economic cost function does not penalize the use of control energy.

5. Evaluation of EMPC using a chemical process example

In this section, various EMPC formulations are demonstrated. Specifically, applications of the various EMPC schemes to a chemical process example are considered in this section. The specific chemical process example has been chosen because the understanding of its dynamic evolution is tractable to most engineers familiar with chemical processes. This section is not meant to apply all available EMPC formulations/algorithms presented in the literature to a chemical process example, but rather, to discuss how one can design an EMPC of the form of Eq. (9) for a specific application by taking advantage of several of the theoretical developments of EMPC presented in the literature.

5.1. CSTR description

Consider a non-isothermal continuously stirred tank reactor (CSTR) where an elementary, exothermic second-order reaction takes place that converts the reactant A to the desired product B. The reactant is fed to the reactor through a feedstock stream with concentration C_{A0}, flow rate F, and temperature T_0. The CSTR contents are assumed to have a uniform temperature and composition, and the CSTR is assumed to have a constant liquid hold-up. A jacket provides/removes heat to/from the reactor at rate Q. Applying first principles and standard modeling assumptions (e.g., constant fluid density and heat capacity, Arrhenius rate dependence of the reaction rate on temperature, etc.), the following system of ordinary differential equations (ODEs) is derived that describes the evolution of the CSTR reactant concentration and temperature:

$$\frac{dC_A}{dt} = \frac{F}{V_R}(C_{A0} - C_A) - k_0 e^{-E/RT} C_A^2 \quad (41a)$$

$$\frac{dT}{dt} = \frac{F}{V_R}(T_0 - T) - \frac{\Delta H k_0}{\rho_R C_p} e^{-E/RT} C_A^2 + \frac{Q}{\rho_R C_p V_R} \quad (41b)$$

where C_A denotes the concentration of A in the reactor, T denotes the temperature of the reactor contents, and the remaining notation definitions and process parameter values are given in Table 2. The CSTR has two manipulated inputs: the inlet concentration of A with available control energy $u_1 = C_{A0} \in [0.5, 7.5]$ kmol m^{-3} and the heat rate supplied to the reactor $u_2 = Q$ with available control energy $Q \in [-50.0, 50.0]$ MJ h^{-1}.

The process economics are assumed to be adequately described by the production rate of the desired product. Therefore, the control objective of the CSTR is to maximize the production rate of the desired product while maintaining safe operation of the process (i.e., boundedness of the state). The instantaneous economic cost function to accomplish this objective is:

$$l_e(x, u) = k_0 e^{-E/RT} C_A^2 \quad (42)$$

which describes the operating profit and thus, is maximized in the EMPC formulations below. Two traditional strategies to increase the production rate are (1) to increase the temperature of the reactor contents and (2) to increase the concentration of A by feeding more A. In the example, the greatest steady-state reaction rate occurs at $C_{As} = 0.143$ kmol m^{-3} and $T_s = 711.1$ K corresponding to the steady-state inputs $C_{A0s} = 7.5$ kmol m^{-3} and $Q_s = 50.0$ MJ h^{-1} with steady-state reaction rate $l_e(x_s, u_s) = 36.8$ kmol m^{-3} h^{-1} (the units on the reaction rate are dropped in the remainder). However, from a practical perspective, it may not be desirable to operate at such a high temperature and/or operate at a steady-state that uses the maximum available control energy. To address this, consider a constraint on the time-average reactant material of the form:

$$\frac{1}{t_{avg}} \int_0^{t_{avg}} C_{A0}(t)\, dt = C_{A0,avg} \quad (43)$$

where t_{avg} is the time over which to enforce the material constraint and $C_{A0,avg.}$ is an average amount of reactant which is taken to be the median value $C_{A0,avg} = 4.0$ kmol m^{-3}. This fixes the optimal inlet concentration steady-state value to $C_{A0}^* = C_{A0,avg}$. Both cases where the operating period t_{avg} is chosen to be finite and infinite are considered below.

Table 2
Process parameters of the CSTR.

Symbol	Description	Value
F	Feedstock flow rate	5.0 m^3 h^{-1}
T_0	Feedstock temperature	300 K
V_R	Reactor fluid volume	1.0 m^3
E	Activation energy	5.0×10^4 kJ kmol^{-1}
k_0	Pre-exponential rate factor	8.46×10^6 m^3 kmol^{-1} h^{-1}
ΔH	Reaction enthalpy change	-1.16×10^4 kJ kmol^{-1}
C_p	Heat capacity	0.231 kJ kg^{-1} K^{-1}
ρ_R	Density	1000 kg m^{-3}
R	Gas constant	8.314 kJ kmol^{-1} K^{-1}

In the simulations below, the open-source interior point nonlinear optimization solver Ipopt [163,19] was used to solve the EMPC problems at every sampling instance. To numerically integrate the ODEs of Eq. (41) forward in time, the explicit Euler method was used with an integration time step of 0.001 h. The sampling period of the EMPC schemes presented below is $\Delta = 0.01$ h.

5.2. Closed-loop performance under EMPC

In the first set of simulations, closed-loop economic performance under EMPC is considered where two different EMPC schemes are formulated and applied to the nominally operated CSTR ($w(t) \equiv 0$). The EMPC is allowed to operate the CSTR in a large operating envelope, and no state constraints are imposed. Explicit considerations of the practicality of the operating policy dictated by the EMPC (e.g., consideration of state constraints) is left to the subsequent subsection which discusses closed-loop stability under EMPC (Section 5.3). However, to provide some limit to the operating range of the CSTR the available control energy for the heat rate input to the CSTR is restricted to $Q \in [0.0, 20.0]$ MJ h^{-1}. This restriction limits the temperature range over which the EMPC operates the CSTR because the optimal operating strategy is to provide the upper limit heat rate to the CSTR for all time to make the reaction rate as large as possible (as discussed above). Therefore, the economically optimal steady-state is defined as $C_{As}^* = 0.719$ kmol m^{-3} and $T_s^* = 481.4\,K$ with corresponding optimal steady-state inputs of $C_{A0s}^* = 4.0$ kmol m^{-3} and $Q_s^* = 20.0\,MJ\,h^{-1}$ and with a steady-state production rate of $l_e(x_s^*, u_s^*) = 16.4$.

Recall that the time-averaged amount of material fed to the CSTR is fixed. One method to ensure that the average constraint of Eq. (43) is satisfied over the entire length of operation is to construct a constraint that ensures that the constraint is satisfied over each consecutive operating period t_{avg}. This may be accomplished by using a simple inventory balance accounting for the total amount of input energy available over each operating period compared to the total amount of input energy already used in the operating period. The main advantages of enforcing the average constraint in this fashion are that (1) only a limited number of constraints are required to be added to the EMPC, and (2) it ensures that the average constraint is satisfied on both the finite-time and infinite-time intervals. The enforcement of the constraint is carried out as follows: if the prediction horizon covers the entire operating period, then the average constraint can be enforced directly; that is, impose the following constraint in the optimization problem of the EMPC:

$$\frac{\Delta}{\tau_M}\sum_{i=0}^{M-1} u(\tau_i) = u_{\text{avg}} \tag{44}$$

where τ_M is the operating period length that the average input constraint is imposed (i.e., $M = \tau_M/\Delta$ is the number of sampling periods in the operating period) and u_{avg} is the average input constraint value. The integral of Eq. (43) has been converted to a sum in Eq. (44) because the input trajectory is piecewise constant. If the prediction horizon does not cover the entire operating period, then the remaining part of the operating period not covered in the prediction horizon must be accounted for in the constraints. Namely, at a sampling period $\tau_k \in [\tau_0, \tau_M)$, the following must be satisfied:

$$Mu_{\text{avg}} - \sum_{j=k}^{\min\{k+N,\,M-1\}} u(\tau_j) - \sum_{i=0}^{k-1} u^*(\tau_i) \le \max\{M-N-k, 0\}u_{\max}, \tag{45a}$$

$$Mu_{\text{avg}} - \sum_{j=k}^{\min\{k+N,\,M-1\}} u(\tau_j) - \sum_{i=0}^{k-1} u^*(\tau_i) \ge \max\{M-N-k, 0\}u_{\min}. \tag{45b}$$

Together these constraints ensure that the average constraint of Eq. (44) is satisfied. Specifically, Eq. (45) means that the difference between the total available input energy (Mu_{avg}) and the total input energy used from the beginning of the operating period through the end of the prediction horizon must be equal to or less/greater than the total input energy if the maximum/minimum allowable input was applied over the remaining part of the operating period from τ_{k+N} to τ_M. If the prediction horizon extends over multiple consecutive operating periods, a combination of the constraints of Eq. (44) and (45) can be employed. For example, if the prediction horizon extends over two operating periods, the constraint to be enforced at a sampling instance, τ_k, becomes:

$$\sum_{i=k}^{M-1} u(\tau_i) + \sum_{i=0}^{k-1} u^*(\tau_i) = Mu_{\text{avg}}, \tag{46a}$$

$$Mu_{\text{avg}} - \sum_{j=M}^{\min\{k+N,\,2M-1\}} u(\tau_j) \le \max\{2M-N-k, 0\}u_{\max}, \tag{46b}$$

$$Mu_{\text{avg}} - \sum_{j=M}^{\min\{k+N,\,2M-1\}} u(\tau_j) \ge \max\{2M-N-k, 0\}u_{\min}. \tag{46c}$$

For simplicity of notation, k is reset (i.e., $k = 0$ at the sampling period τ_M) at the beginning of each operating period.

The following EMPC is applied to the CSTR system:

$$\underset{u \in S(\Delta)}{\text{maximize}} \quad \int_0^{\tau_N} k_0 e^{-E/R\tilde{T}(t)} \tilde{C}_A^2(t)dt \tag{47a}$$

$$\text{subject to} \quad \dot{\tilde{C}}_A(t) = \frac{F}{V}(u_1(t) - \tilde{C}_A(t)) - k_0 e^{-E/R\tilde{T}(t)} \tilde{C}_A^2(t) \tag{47b}$$

$$\dot{\tilde{T}}(t) = \frac{F}{V}(T_0 - \tilde{T}(t)) - \frac{\Delta H k_0}{\rho C_p} e^{-E/R\tilde{T}(t)} \tilde{C}_A^2(t) + \frac{u_2(t)}{\rho C_p V} \tag{47c}$$

$$\tilde{C}_A(0) = C_A(\tau_k), \quad \tilde{T}(0) = T(\tau_k) \tag{47d}$$

$$u(t) \in U, \ \forall\, t \in [0,\ \tau_N) \tag{47e}$$

$$\sum_{j=0}^{M-k-1} u_1(\tau_j) + \sum_{i=0}^{k-1} u_1^*(\tau_i) = Mu_{1,\text{avg}} \tag{47f}$$

$$Mu_{1,\text{avg}} - \sum_{j=M-k}^{N} u_1(\tau_j) \le \max\{2M-N-k, 0\}u_{1,\max} \tag{47g}$$

$$Mu_{1,\text{avg}} - \sum_{j=M-k}^{N} u_1(\tau_j) \ge \max\{2M-N-k, 0\}u_{1,\min} \tag{47h}$$

where the notation used is similar to the previous EMPC formulations. We note that the EMPC of Eq. (47) resets its initial time to zero at each sampling period (i.e., the real-time horizon τ_k to τ_{k+N} corresponds to the prediction horizon from 0 to τ_N in the controller). Therefore, the time indices of the constraints of Eq. (47f)–(47h) are shifted to account for this point. The closed-loop simulation results are shown in Figs. 7 and 8 for the initial condition $C_A(0) = 2.0$ kmol m^{-3} and $T(0) = 425.0$ K, an operating period of 100 sampling periods (i.e., $M = 100$ and $\tau_M = 1.0$ h) and prediction horizon $N = 10$. The closed-loop state trajectories converge to a limit cycle over several periods of operation (Fig. 8). The closed-loop performance of the system under the EMPC of Eq. (47) is evaluated

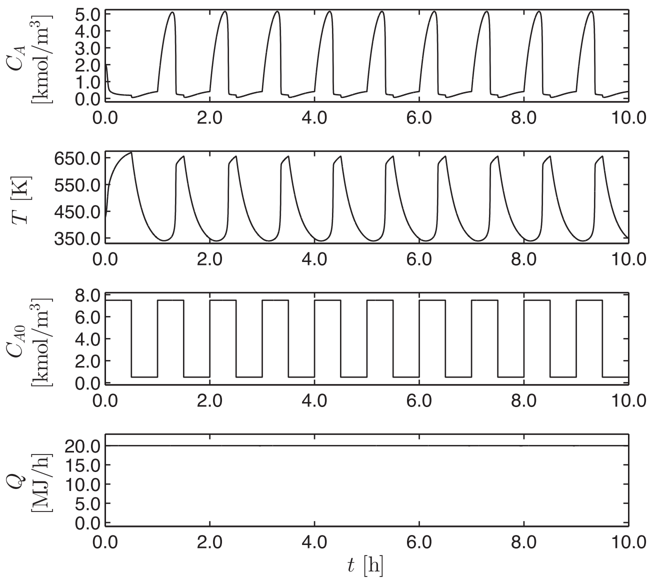

Fig. 7. Closed-loop trajectories of the CSTR over 10 h of operation under the EMPC with the operating period average input constraints.

using the average economic cost index of Eq. (32). Over the first operating period under the EMPC of Eq. (47), the average economic cost is 19.95, while the average economic cost over one hour starting from the same initial condition and under a constant input of u_s^* is 17.46 (operation under EMPC has 14.25% better performance). Over the 50.0 h length of operation, the average economic cost under the EMPC is 12.68, while under the constant steady-state input value it is 16.43. The average closed-loop economic performance under EMPC is 22.78% worse than the performance under the constant input u_s^*. It is important to emphasize that several chemical process examples under EMPC formulated without the use of a point-wise terminal constraint and without a terminal cost have demonstrated improved economic closed-loop performance over traditional control methods (e.g., [71,51]).

One solution to guarantee performance improvement over steady-state operation for long term (infinite-time) operation is to add a terminal constraint based on the economically optimal steady-state or the open-loop predicted trajectory under an auxiliary controller which was described in Section 4 above. Therefore,

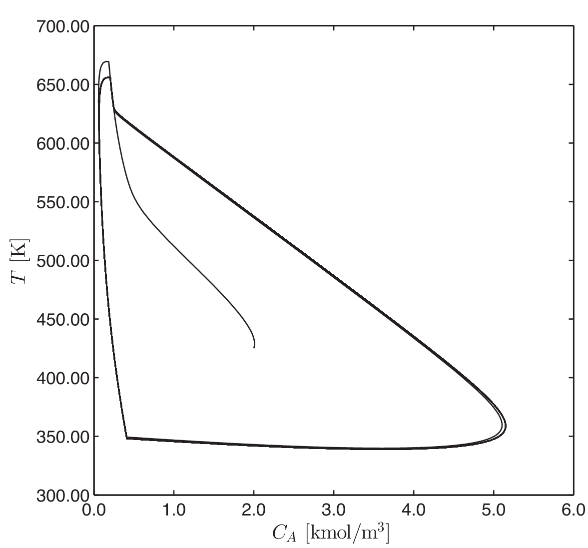

Fig. 8. The closed-loop state trajectory in state-space of the CSTR over 10 h of operation under the EMPC with the operating period average input constraints.

Table 3
Closed-loop performance over 50 h of operation under the EMPC with a terminal constraint and the constraint of Eq. (48) with various prediction horizon lengths.

N	$\bar{J}_E$
10	16.76
20	17.03
30	17.05
40	17.09
50	17.14

a terminal (point-wise) constraint based on the economically optimal steady-state is added to the EMPC. However, the average input constraint must be carefully constructed. Applying constraints of the form of Eq. (46) will likely cause the EMPC, formulated with a terminal constraint, to become infeasible owing to the fact that the input constraint may become tight near the end of the operating window. The terminal constraint may no longer be a reachable steady-state with the remaining control energy and thus, increasing the prediction horizon may not resolve this issue for this type of average constraint. For instance, initializing the CSTR with the initial condition $C_A(0) = 2.0$ kmol m^{-3} and $T(0) = 425.0$ K, the EMPC of Eq. (47) with the terminal constraint $\tilde{x}(\tau_N) = x_s^*$ becomes infeasible at 0.78 h. One method to resolve this issue is to handle the average constraint asymptotically [6].

One type of constraint to enforce such that the input average constraint is asymptotically satisfied is the constraint of Eq. (27) (presented in [6]). Formulating constraints of this form which ensure that the average input constraint is asymptotically satisfied for the CSTR example yields the following constraints:

$$\sum_{j=k}^{k+N} u(\tau_j) \geq N u_{\min} + (k+N) u_{\text{avg}} - \sum_{j=0}^{k-1} u^*(\tau_j) \tag{48a}$$

$$\sum_{j=k}^{k+N} u(\tau_j) \leq N u_{\max} + (k+N) u_{\text{avg}} - \sum_{j=0}^{k-1} u^*(\tau_j) \tag{48b}$$

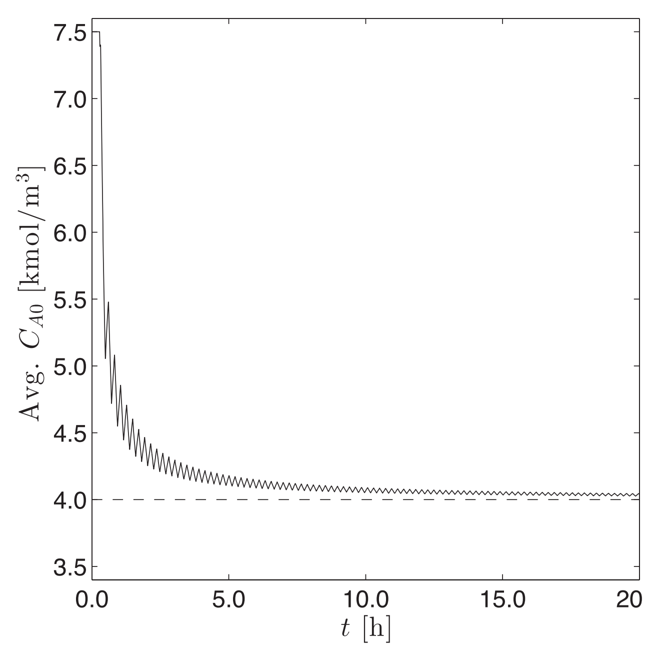

Fig. 9. The average of C_{A0} computed by the EMPC with a terminal constraint, a prediction horizon length of $N = 20$ and the asymptotic average input constraints (solid trajectory). The desired average, $u_{1,\text{avg}}$ is the dashed line.

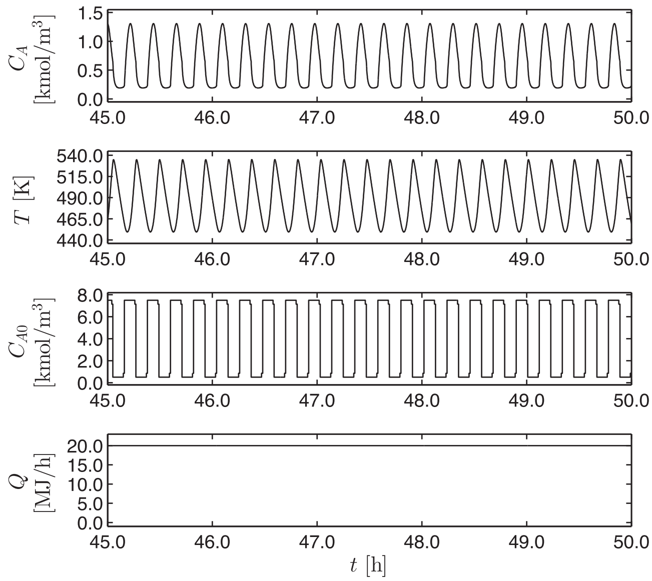

Fig. 10. The closed-loop trajectories of the CSTR over 50 h of operation under the EMPC with a terminal constraint, a prediction horizon length of $N = 20$ and the asymptotic average input constraints.

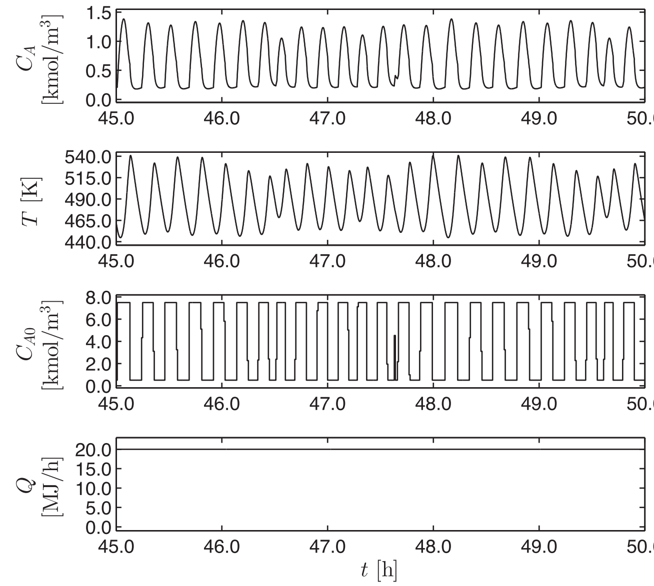

Fig. 12. The closed-loop trajectories of the CSTR over 50 h of operation under the EMPC with a terminal constraint, a prediction horizon length of $N = 50$ and the asymptotic average input constraints.

which replace the average constraints of Eq. (47f)–(47h) in the EMPC of Eq. (47). Also, the terminal constraint $\tilde{x}(\tau_N) = x_s^*$ is added to the EMPC. The resulting EMPC is applied to the CSTR. The closed-loop performance is given in Table 3 for the initial condition $C_A(0) = 2.0\,\text{kmol m}^{-3}$ and $T(0) = 425.0\,\text{K}$ and for several prediction horizon lengths. Overall, a similar trend in performance with prediction horizon length is observed in Table 3 as the one observed in Example 1.

The average C_{A0} value with time for the input trajectory computed by the EMPC with a prediction horizon of $N = 20$ is given in Fig. 9 which demonstrates that the average constraint on C_{A0} is asymptotically satisfied. The closed-loop trajectories under the EMPC with a terminal constraint are shown in Figs. 10 and 11 for $N = 20$ and Figs. 12 and 13 for $N = 50$. From the closed-loop trajectories of the CSTR under the EMPC with $N = 20$ (Fig. 10), a periodic operating policy is dictated by the EMPC; a more complex periodic-like operating policy is observed with $N = 50$ (Fig. 12). Owing to the enforcement of the terminal constraint, the EMPC with a terminal constraint operates the CSTR in a much smaller operating range compared to the CSTR under the EMPC of Eq. (47) (Fig. 7). Interestingly, however, operation over a much larger operating region as is the case under the EMPC of Eq. (47) (with $N = 10$) does not yield better closed-loop economic performance compared to operating in a smaller region as is the case under the EMPC with a terminal constraint and with $N = 10$. Specifically, the average reaction rate over the 50.0 h simulation is 16.76 with the EMPC with a terminal constraint and with $N = 10$. This is 1.93% better than steady-state operation (constant input u_s^*) and 31.07% better than the EMPC without the terminal constraint.

From the simulations of the CSTR under the EMPC with the terminal constraint, the computed C_{A0} profile is approximately a periodic profile. From these simulations, the period of the periodic switching policy dictated by EMPC may be approximated. On average, the EMPC with a terminal constraint switches between $C_{A0,\max}$ and $C_{A0,\min}$ approximately every 10 sampling periods and thus, the optimal period of switching is approximated as 20 sampling

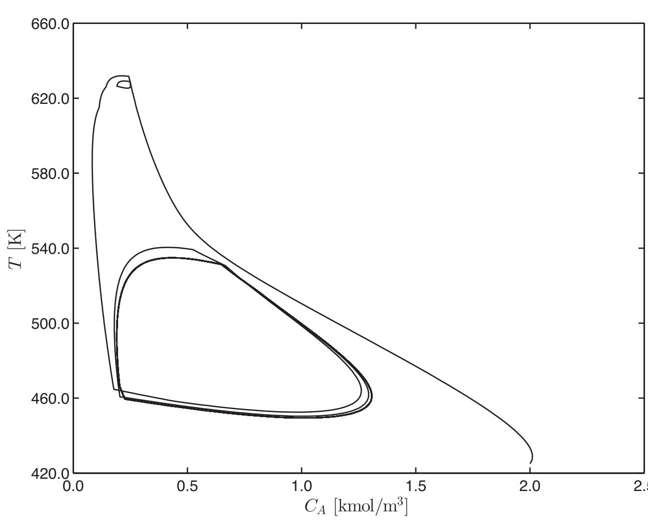

Fig. 11. The closed-loop state trajectory in state-space of the CSTR over 50 h of operation under the EMPC with a terminal constraint, a prediction horizon length of $N = 20$ and the asymptotic average input constraints.

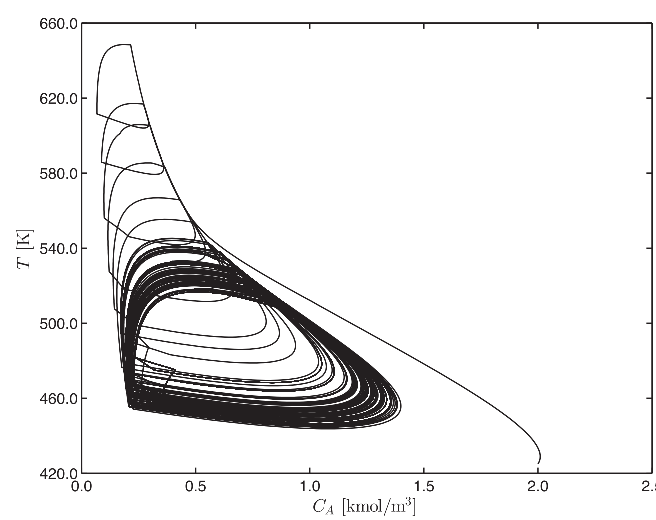

Fig. 13. The closed-loop state trajectory in state-space of the CSTR over 50 h of operation under the EMPC with a terminal constraint, a prediction horizon length of $N = 50$ and the asymptotic average input constraints.

periods. Utilizing the computed optimal period consisting of 20 sampling periods, we revisit the EMPC of Eq. (47) which uses an average input constraint enforced over operating windows of size τ_M.

Consider the EMPC of Eq. (47) where the operating period is now 20 sampling periods (M = 20) and the prediction horizon is N = 20. The EMPC is applied to the CSTR with two initial conditions: $x_{0,1}^T = [2.0 \;\; 425.0]$ and $x_{0,2}^T = [1.0 \;\; 490.0]$. The closed-loop performances under the EMPC of Eq. (47) (with M = 20 and N = 20) for these two initial conditions are 16.95 and 16.94, respectively, while the closed-loop performances under the EMPC with a terminal constraint, the asymptotic average constraint of Eq. (48), and N = 20 are 17.03 and 16.99 for these two initial conditions, respectively. The differences in performance between the two EMPC schemes are 0.48% for the first initial condition and 0.29% for the second, with the EMPC with a terminal constraint having the better performance in each case. However, recall that this comparison does not amount to comparing two equivalent scenarios. Since the EMPC under the asymptotic average constraint only needs to asymptotically satisfy the average constraint, it may use more (or less) input energy over the finite-time (transient) interval. In fact, for this example, the EMPC with the terminal constraint and asymptotic average constraint uses an average of 4.01 kmol m^{-3} for both initial conditions which is slightly more than the EMPC with operating period average constraints. This analysis suggests that it may be useful, when using an operating period average input constraint, to construct and solve a dynamic optimization problem to determine the optimal period (τ_M) to enforce the operating period average input constraint since the closed-loop performance under EMPC with an average constraint enforced over operating periods may be dependent on the choice of operating period length.

5.3. Stability under EMPC

In the previous section, closed-loop performance was studied comparing the closed-loop performance of the CSTR under EMPC formulated with a terminal constraint and under an EMPC without a terminal constraint. For this particular example, closed-loop economic performance of the EMPC without a terminal constraint and an average input constraint formulated for successive operating windows and an EMPC formulated with a terminal constraint and asymptotic average input constraint yield similar closed-loop performance (for an appropriately chosen operating period τ_M of the first EMPC). Furthermore, both cases demonstrated a closed-loop economic performance improvement under dynamic operation compared to steady-state operation. In the previous study, strictly nominal operation was considered (i.e., operation under no plant-model mismatch, no disturbances, and no other uncertainty). Practically speaking, it is never possible to achieve nominal operation. Furthermore, work in the direction of closed-loop performance under actual operation (e.g., in the presence of process noise, external forcing and unmeasured disturbances, communication disruptions between components of the control architecture, etc.) remains an open research topic.

In this section, operation under process noise and plant-model mismatch is considered. As argued in [4,141], the use of a (terminal) region is superior to the point-wise terminal constraint. Arguing for or against this point for EMPC is not within the scope of this work, but instead, the discussion proceeds using this point to motivate the use of an EMPC with a region constraint instead of a point-wise terminal constraint for operation in the presence of process noise. Specifically, an LEMPC is chosen to be formulated and applied to the CSTR model due to several of its unique properties compared to EMPC with a terminal region and terminal cost (e.g., the EMPC presented in [4]). With LEMPC, a region constraint can be constructed without the need to add a terminal penalty to the economic cost while still having provable stability guarantees in the presence of bounded disturbances (i.e., boundedness of the closed-loop state in the region). The region constraint, Ω_ρ, is characterized with an explicit stabilizing controller, $k(x)$, and therefore, is an estimate of the region of attraction for the system under the input constraints. Furthermore, different Lyapunov-based constraints can be formulated to achieve multiple objectives which will be discussed and demonstrated below.

Before the auxiliary explicit stabilizing controller can be designed for the LEMPC, the control objective for the CSTR is modified. Since the economic cost does not penalize the use of control energy, the optimal operating strategy is to operate at the maximum allowable heat rate supplied to the reactor. However, this may lead to a large temperature operating range (Fig. 8) which may be impractical or undesirable. Therefore, consider a modified control objective for more practical closed-loop operation of the CSTR under EMPC. The modified control objective is to maximize the reaction rate while feeding a time-averaged fixed amount of the reactant A to the process and while forcing and maintaining operation to/at a pre-specified set-point temperature. Additionally, the temperature of the reactor contents must be maintained below the maximum allowable temperature $T(t) \leq T_{\max}$ = 470.0 K, which is treated as a hard constraint and thus, $\mathbb{X} = \{x \in \mathbb{R}^2 | x_2 \leq 470.0\}$. The optimal steady-state, in this case, is obtained from the need to satisfy the new control objective and therefore, is not directly derived from an optimization problem. Specifically, the heat rate input is allowed to take values in its full set of available control energy ($Q \in [-50.0, 50.0]$ MJ h^{-1}) and the optimal steady-state inputs are set as follows: $C_{A0}^* = C_{A0,\text{avg}} = 4.0$ kmol m^{-3} and Q_s^* is the average available heat rate which is $Q_s^* = 0$ MJ h^{-1}. The reasoning for the latter choice is to have an equal amount of positive and negative control energy. The steady-state in the operating range of interest corresponding to steady-state input values of $C_{A0}^* = 4.0$ kmol m^{-3} and $Q_s^* = 0$ MJ h^{-1} is $C_{As}^* = 1.18$ kmol m^{-3} and $T_s^* = 440.9$ K and is open-loop asymptotically stable.

A stabilizing state feedback controller $k(x)$ is designed for the CSTR with respect to the optimal steady-state. The first input C_{A0} in the stabilizing controller is fixed to the average inlet concentration to satisfy the average input constraint. The second input Q is designed via feedback linearization with a controller gain of γ = 1.4 (see [68] for more details regarding the controller design). A quadratic Lyapunov function is considered of the form $V(x) = \bar{x}^T P \bar{x}$

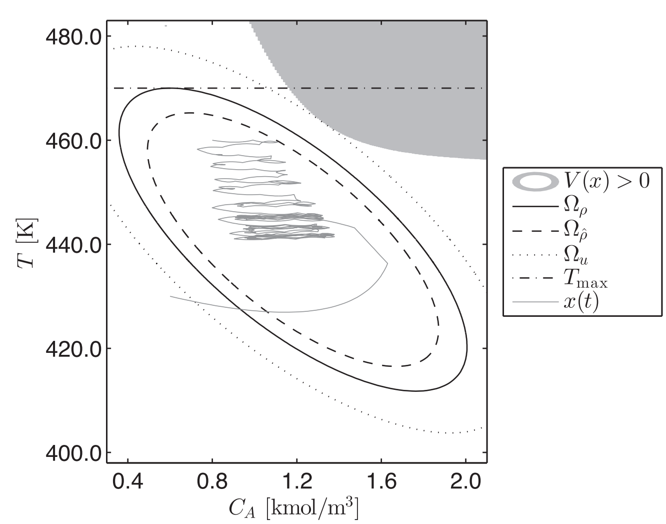

Fig. 14. Two closed-loop state trajectories under the LEMPC in state-space.

where $\bar{x}$ is the deviation of the states from their corresponding steady-state values and P is the following positive definite matrix:

$$P = \begin{bmatrix} 250 & 5 \\ 5 & 0.2 \end{bmatrix}. \quad (49)$$

The stability region of the CSTR under the controller $k(x)$ is characterized with a level set of the Lyapunov function where the time-derivative of the Lyapunov function along the closed-loop state trajectories is negative and is denoted as $\Omega_u = \{x \in \mathbb{R}^2 | V(x) \le \rho_u\}$ where $\rho_u = 138$. However, $\mathbb{X} \subset \Omega_u$ which is shown in Fig. 14 and thus, we define the set Ω_ρ where $\rho = 84.76$ to account for the state constraint. Bounded Gaussian process noise is added to the CSTR with a standard deviation of $\sigma = [0.3\ 5.0]^T$ and bound $\theta = [1.0\ 20.0]^T$. Specifically, a new noise vector is generated and applied additively to the right-hand side of the ODEs of Eq. (41) over the sampling period ($\Delta = 0.01$ h) and the bounds are given for each element of the noise vector ($|w_i| \le \theta_i$ for $i = 1, 2$). Through extensive closed-loop simulations of the CSTR under the controller $k(x)$ and under the LEMPC (described below) and with many realizations of the process noise, the set Ω_{ρ_e}, a set where time-varying operation is allowed while boundedness in Ω_ρ is maintained, was determined to be $\rho_e = 59.325$.

The first differential equation of Eq. (41) (C_A) is input-to-state-stable (ISS) with respect to T. Therefore, a contractive Lyapunov-based constraint can be applied to the LEMPC to ensure that the temperature converges to a neighborhood of the optimal steady-state temperature value. Namely, we define: $V_T(\tau_k) := (T(\tau_k) - T_s^*)^2$. The LEMPC formulation is the same as Eq. (47) (modified as noted above to account for noise) with the following added constraints:

$$T(t) \le T_{\max} \quad (50a)$$

$$V(\tilde{x}(t)) \le \rho_e \quad \forall \quad t \in [0, \tau_N) \quad (50b)$$

$$\frac{\partial V_T(\tau_k)}{\partial T} f_2(\tilde{x}(0), u(0), 0) \le \frac{\partial V_T(\tau_k)}{\partial T} f_2(\tilde{x}(0), k(\tilde{x}(0)), 0) \quad (50c)$$

where $f_2(\cdot)$ is the right-hand side of the second ODE of Eq. (47). The CSTR was initialized at many states distributed throughout state-space including some cases where the initial state is outside Ω_u. The LEMPC described above was applied to the CSTR with an operating period over which to enforce the average input constraint of $M = 20$ and a prediction horizon of $N = 20$. Several simulations of 50.0 h length of operation were completed. In all cases, the LEMPC was able to force the system to Ω_ρ and maintain operation inside Ω_ρ without violating the state constraint. The closed-loop state trajectories over the first 1.0 h are shown in Fig. 14 for one initial condition starting inside Ω_ρ and one starting outside Ω_u. Moreover, the CSTR was simulated with the same realization of the process noise and same initial condition under the controller $k(x)$ applied in a sample-and-hold fashion and under a constant input equal to the steady-state input. The average economic cost over each of these simulations is reported in Table 4. From these results, an average of 0.6% closed-loop performance benefit was observed with the LEMPC over the controller $k(x)$ and the constant input u_s^*. It is important to note that for one of the simulations that was initialized outside Ω_u the CSTR under the constant input u_s^* settled on an offsetting steady-state which is denoted with an asterisk in Table 4.

Table 4
Average economic cost over several simulations under the LEMPC, the Lyapunov-based controller applied in a sample-and-hold fashion, and the economically optimal input u_s^*. For the case denoted with a "*", the system under the constant input u_s^* settled at a steady-state different from the economically optimal steady-state.

$\bar{J}_E$ under LEMPC	$\bar{J}_E$ under $k(x)$	$\bar{J}_E$ under u_s^*
14.17	14.10	14.09
14.18	14.11	14.09
14.17	14.10	14.08
14.17	14.09	14.06
14.18	14.10	14.10
14.17	14.09	14.08
14.18	14.09	14.10
14.18	14.08	14.08
14.19	14.08	14.10
14.18	14.07	14.07
14.18	14.11	14.11
14.18	14.08	14.07
14.17	14.06	0.36*
14.19	14.06	14.10

6. Discussion of current status of EMPC

In this section, we reflect on the current status of EMPC developments as well as unify some of the results on EMPC to compare and contrast the various approaches.

6.1. RTO and EMPC

Throughout the EMPC literature, EMPC has been widely reported as a method that merges process economic optimization and process control. Indeed, it does have this property. However, among all the current theoretical work on EMPC, only the work of Grüne [64] does not employ the use of a precomputed steady-state or periodic operating trajectory as a terminal constraint or stability region. Notice that the EMPC schemes of Eq. (14) and (30), each with provable stability properties, use precomputed information as constraints in the EMPC formulation. This information must come from some higher level information technology system. Moreover, a wide variety of assumed *a priori* knowledge exists amongst the various EMPC schemes. Some EMPC schemes only require the economically optimal steady-state (e.g., EMPC with a terminal constraint), while others require much more knowledge like the optimal cyclical (perhaps periodic) operating strategy [79] and sufficient understanding of the behavior of the future economic pricing to model it appropriately [130]. Given the availability of inexpensive computation, EMPC should take advantage of any available information (like information provided by RTO) to improve the closed-loop performance. Furthermore, EMPC does not replace all the tasks completed by the RTO layer (e.g., data validation and reconciliation and model updating). Thus, it is important to keep in mind that EMPC does not entirely replace RTO.

Remark 5. While EMPC may not completely replace RTO, it is important to understand that this does not imply that one should not apply EMPC. Using EMPC will yield different closed-loop operating trajectories compared to using (frequent) RTO (i.e., RTO that is executed more frequently than that of traditional RTO systems) with a traditional control structure (e.g., tracking MPC). This is because the RTO layer typically uses a steady-state process model, while the EMPC uses a dynamic process model. If Dynamic RTO (D-RTO) is used (i.e., RTO with a dynamic process model), it would be expected that the closed-loop economic performance under EMPC is better than that with D-RTO since D-RTO is typically executed at a slower frequency than EMPC. If D-RTO is executed at the same rate as EMPC, then D-RTO, in this case, is essentially (one-layer) EMPC (see Section 7.2 for more discussion on this point).

6.2. Closed-loop operation under EMPC

From our experiences applying EMPC to various applications, we have observed that the most closed-loop performance benefit under EMPC occurs when EMPC dictates a time-varying operating policy which may range from a periodic or cyclical operating policy to more complex non-periodic time-varying operation. When steady-state operation is the optimal operating strategy, one should carefully consider the applicability of EMPC. Specifically, when operating conditions or economic factors are updated infrequently and a system can be maintained near the optimal steady-state (i.e., a sufficient time-scale separation exists between the frequency of economic factors update and the time constants of the process dynamics), it is unexpected that much benefit will be observed under EMPC since the system spends little time in the transient phase relative to the total length of operation. For large-scale systems optimally operated at steady-state, one must also consider the computational burden required to solve EMPC on-line which may be significant. For instance, the application of EMPC to a large-scale chemical process network used in the production of vinyl acetate led to comparable closed-loop performance when compared to a well-tuned MPC formulated with a quadratic cost, as economically optimal steady-state operation was likely optimal [162]. However, the computation requirement of EMPC compared to MPC was considerably higher. If significant disturbances are present such that it is difficult to achieve operation near the steady-state (and steady-state operation is the optimal operating strategy), one may consider using a local approximation of the nonlinear model (e.g., linear dynamic model) and a local approximation of the economic cost when these approximations provide sufficient accuracy of the process dynamics and economic cost, respectively. This may convert the optimization to an easier problem (i.e., milder nonlinearities, possibly convert the optimization problem of EMPC to a convex optimization problem which is readily solvable, etc.) helping to ease the computational burden of EMPC, while achieving some possible closed-loop performance benefit.

6.3. Economic assessment of EMPC

For the chemical process example presented in this work, a 2% greater production rate was achieved under EMPC with nominal operation than with steady-state operation, while only a 0.6% greater production rate was achieved when process noise affected the evolution of the process system. Whether it is an economically viable option to apply EMPC to this particular process largely depends on how valuable the product is and what the production rate currently being realized in practice under the current control methodology is. Therefore, a careful economic assessment would need to be completed when determining if the benefit of applying EMPC is worth the engineering, capital, and related investment costs (see the survey paper [14] for approaches for carrying out such an evaluation).

7. Future research directions

In this section, we discuss some topics for future research work in the area of EMPC based on our experiences, observations, and motivations. The topics reflect our own bias and the list is certainly not complete.

7.1. State-estimation-based EMPC

Almost all of the proposed EMPC schemes rely on state feedback. However, in practice, only measured output feedback may be available. Since there exists no separation principle for general nonlinear systems, it is hard to prove stability of the closed-loop system under a state-estimator or state-observer with a state feedback controller. An approach within nonlinear systems for the design of an output feedback controller is to use a high-gain observer with a stabilizing state feedback controller. In this case, one can apply singular perturbation arguments to prove stability of the closed-loop system [86,87]. In a previous work [69], the LEMPC design was extended to the case of output feedback based on a high-gain observer. While using high-gain observers may allow for proving closed-loop stability, it may also be sensitive to measurement noise.

As another approach to output feedback EMPC, moving horizon estimation (MHE) based on least squares techniques has become a popular state estimation technique because of its ability to handle nonlinear systems, account for the presence of disturbances, and account for constraints on decision variables leading to improved estimation performance [124,138,139,78,144]. One particular formulation of MHE, robust MHE (RMHE), has been proposed in [101,175] which is based on an auxiliary nonlinear observer that asymptotically tracks the nominal system state. The auxiliary deterministic nonlinear observer is taken advantage of to calculate a confidence region that contains the actual system state taking into account bounded model uncertainties at every sampling time. The region is then used to design a constraint on the state estimate in the RMHE. The RMHE brings together deterministic and optimization-based observer design techniques. It was proved to give bounded estimation error in the case of bounded model uncertainties. In [55], an RMHE-based output feedback LEMPC was presented and stability in the presence of measurement and process noise was proved. Future work in this direction should embark on considering the rigorous design of other types of output feedback EMPC schemes.

7.2. Distributed EMPC, hierarchical EMPC, and distributed economic optimization

In general, it has been pointed out that the computational burden of EMPC over conventional MPC may be significantly higher since EMPC may use a general nonlinear, non-convex cost function with a sufficiently long prediction horizon for good closed-loop performance (e.g., [162]). In fact, the computational time required to solve the EMPC may be greater than the time available (i.e., greater than the sampling period). Furthermore, it has been argued in [166] that the use of a one-layer EMPC which fully combines the RTO and supervisory (MPC) layers in Fig. 1 is undesirable within the context of industrial application without the use of some additional safety control layer. Three potentially attractive choices to handle computational concerns include using distributed EMPC, hierarchical EMPC, distributed optimization techniques, and/or any combination of these three approaches.

It is clear in the context of MPC of large-scale process networks that distributed MPC (DMPC) schemes may significantly reduce the on-line computational load of MPC and thus, make MPC a feasible control methodology for large-scale, nonlinear process networks (e.g., see, for instance, [145,103,102,34] and the references therein). While early work in this direction in the context of distributed EMPC (DEMPC) has shown promising results [29,42,95,96], more work is necessary which includes work on the development of novel DEMPC algorithms, rigorous theoretical stability analysis, and introducing control loop decomposition methodologies for DEMPC. One potentially interesting research direction is to define new control loop decomposition methods on the basis of process economics. This idea has some similarities to the so-called self-optimizing control methodology for control structure design for steady-state operated processes [154,155,10,136]. However, it remains to be seen how these methods can be extended to dynamically operated systems.

Another alternative to using a single-layer EMPC system to compute the control actions directly for the manipulated inputs, which may be computationally taxing, is to use EMPC in a hierarchical control structure which is also commonly referred to as Dynamic-RTO [168,166]. Here, the idea is to maintain the current hierarchical control structure (Fig. 1), but replace the RTO layer with essentially an EMPC that computes an optimal operating trajectory. The optimal trajectory is sent to the lower control layers to steer the process system to operate along these trajectories. Similar to current RTO, the upper-layer EMPC would recompute its operating trajectory for the system infrequently to lower computational requirements (i.e., not every sampling period), and thus, a loss of economic performance may be expected over single-layer EMPC. Furthermore, stability analysis of the entire closed-loop hierarchical EMPC structure is in order although only limited work has been completed in this direction (e.g., [50,51]). Future research in this direction should include detailed stability analysis of the closed-loop structure and novel algorithms that work to minimize the performance loss of hierarchical EMPC compared to single-layer EMPC.

Lastly, a continued desire within many fields that solve large-scale, non-convex, nonlinear optimization problems is to continue to push the boundaries of nonlinear optimization solver capabilities and computational efficiency. Thus, it will be important to continue research efforts in parallel and distributed computation (e.g., [18]) with a specific focus on distributed and parallel dynamic optimization methods for EMPC.

7.3. Real-time calculation and network considerations

In practice, an optimization-based controller takes a finite amount of time to solve which may be significant or insignificant depending on the time constants of the process dynamics. Therefore, there is a (theoretical) maximum amount of time that the nonlinear solver may spend in computation and must return a control action by this maximum amount of time to ensure closed-loop stability. Even if novel EMPC algorithms are presented that reduce the on-line computational load, it is still possible that the nonlinear optimization solver will not converge to a solution in the time allotted. This may happen, for instance, if a poor initial guess is supplied to the solver. In this case, the solver will return a suboptimal input solution and thus, this type of MPC implementation is often referred to as suboptimal MPC (e.g., [146]). Future research should understand the stability properties of the input solution computed by suboptimal EMPC. Furthermore, it may also be desirable to look at methods that efficiently store previous input solutions in a database as data storage is becoming increasingly inexpensive and take advantage of the database to provide the solver with a potentially better initial guess.

Also of interest within the context of solving EMPC in real-time is the fact that the components of the control architecture are connected through wired and/or wireless communication connections. Communication delays between components may occur. Furthermore, asynchronous measurements (i.e., asynchronous sampling) may occur in certain practical applications. For example, species concentration may be asynchronously measured. Therefore, EMPC schemes that explicitly account for network considerations and asynchronous sampling are important practical challenges of EMPC. Within the context of conventional MPC, several results have been obtained in this direction (e.g., [33]). It will be important to leverage these results and extend these results to EMPC.

7.4. EMPC of distributed parameter and hybrid systems

Almost all of the work on control of distributed parameter systems modeled by PDEs has focused on steady-state stabilization and operation [30] especially in the context of predictive control formulated for PDE systems (e.g. [47,45,43,44]). To this end, only recently has some work been done on applying EMPC to PDE systems [91,90] which has primarily focused on the construction of reduced-order models for EMPC by applying Galerkin's method using analytical or empirical eigenfunctions as basis functions. It is important to note that using a high-order spatial discretization of the PDE model to obtain a system of ODEs describing the temporal evolution of the PDE system in an EMPC framework may result in a computationally-intractable optimization problem to solve on-line. However, despite the demonstration of the computational benefits of using reduced-order models in the formulation of EMPC [91,90], rigorous theoretical stability analysis of PDEs under EMPC remains an open topic.

Another class of systems that remains an open research topic within the context of EMPC is hybrid systems. Hybrid systems are systems that are modeled with states that evolve on the continuous time-scale as well as states that evolve on a discrete time-scale like discrete events. Historically, hybrid systems have attracted much attention within the control community (e.g., [17,32]). However, EMPC schemes for hybrid systems have received very limited attention. Within chemical process control, hybrid systems arise due to, for instance, grade changes in the desired product (i.e., changes in product specifications), raw material changes, and variable energy source pricing. Therefore, it is important to introduce EMPC methods with guaranteed stability properties that may be applied to hybrid systems. Future research in the direction of distributed parameter and hybrid systems may include proposing novel EMPC schemes for these classes of systems and deriving conditions under which stability and improved closed-loop performance of the system under EMPC may be guaranteed.

7.5. EMPC with input/output models

As its name implies, EMPC requires the availability of a dynamic model to compute its control actions. For the cases where the development of a sufficiently accurate first-principles dynamic model is not possible, system identification techniques may need to be used to obtain an accurate empirical input/output model of the process dynamics. In particular, nonlinear autoregressive moving average with exogenous inputs (NARMAX) models may be one of many types of nonlinear system identification techniques employed and used to construct an empirical model [20]. Future research should incorporate input/output models into the formulation of EMPC and investigate the capabilities and limitations of applying these models in the context of EMPC.

7.6. EMPC and safety/robustness considerations

Operating in a continuously dynamic fashion, as EMPC may dictate, may have considerable safety implications both positive and negative. On the positive side, a dynamically operated system may offer some insight into the health of the components. For instance, one common method for fault detection within steady-state operation is to excite the process (induce a transient phase) and observe its response in an effort to detect and isolate faulty components. Since a system under EMPC may be under a constant "excited" state, future research effort may focus on harnessing this for fault detection, isolation, and control reconfiguration to handle various types of process faults. A potential drawback within the context of safety is that the operating policy dictated by the EMPC must be robust to component failures. For example, the EMPC should compute operating trajectories that are safe with respect to potential component failures. Furthermore, EMPC and process monitoring tools should be developed and deployed to help assess the overall process safety.

While a few EMPC formulations with provable stability properties in the presence of disturbances have been proposed (e.g., [68,77,39,67]), more work in this direction is in order. Future work on EMPC should strive to provide provable stability and performance of EMPC in the presence of disturbances as well as present novel EMPC algorithms and formulations that account for disturbances of known form and process noise with known statistics.

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