I&EC research
Industrial & Engineering Chemistry Research

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pubs.acs.org/IECR

Performance Monitoring of Economic Model Predictive Control Systems

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ABSTRACT: A framework for performance monitoring of economic model predictive control (EMPC) systems is presented which includes the computation of an acceptable operating region, which is a well-defined region in state-space, for EMPC systems to operate a process in a time-varying fashion to optimize process economics while meeting input constraints and stabilizability requirements. To capture the interplay between sources of common cause variance caused by various sources like sensor noise, imperfect actuator operation, and model inaccuracy, a residual variable taken to be the difference of actual real-time economic cost and the predicted (expected) economic cost is defined. Utilizing exponentially weighted moving average (EWMA) and historical closed-loop process data, an upper control limit and a lower control limit are established which defines normal operation (i.e., operation with common cause variation). The limits are utilized to monitor the performance of EMPC by comparing real-time process operation data under EMPC and the corresponding regions of acceptable EMPC operation computed in the normal operation dynamic data generation step. The proposed monitoring framework is demonstrated and evaluated using a chemical process example.

INTRODUCTION

Economic model predictive control (EMPC), which utilizes a process economics-based, typically nonquadratic, cost function, has recently been proposed to dynamically optimize economic process performance in the context of feedback control.[1−13] The early work on EMPC primarily focused on addressing unreachable set points generated by traditional steady-state process economic optimization referred to as real-time optimization (RTO). Specifically, the use of an economic cost function in the formulation of MPC was proposed to replace steady-state or target optimization which converts an unreachable set point computed by RTO into a reachable steady-state.[1] Numerous technical and stability details of EMPC have subsequently been studied including: addressing changing economic criterion by formulating EMPC with terminal constraint to enforce the predicted state to converge to an admissible steady-state set by the end of the prediction horizon,[2] proposing an EMPC formulated with a terminal region constraint based on a terminal state (point constraint) and adding a terminal cost in the cost function,[3] establishing a suitable Lyapunov function for EMPC formulated with terminal constraint by imposing a strong duality assumption,[4] proving Lyapunov stability of EMPC for cyclic processes,[5] proposing an EMPC, referred to as Lyapunov-based EMPC (LEMPC), designed utilizing Lyapunov-based techniques,[6] using a generalized terminal constraint for use with EMPC where the terminal constraint is allowed to be an optimization decision variable in the EMPC optimization problem,[9] studying EMPC formulated without a terminal constraint by utilizing a turnpike property and controllability properties to prove convergence to a neighborhood of the optimal steady-state,[10] and presenting an adaptive EMPC for uncertain nonlinear systems.[11] Dissipativity of the closed-loop (nominal) system under EMPC was proved to be a sufficient condition for optimal operation at the economically optimal steady-state[12] and was subsequently extended to show, under mild additional assumptions, to be a necessary condition for steady-state operation to the optimal operating policy.[13] However, in the most general sense, EMPC optimizes process economic performance by dynamic regulation. Specifically, when the process economics are time-varying, when certain economics-based constraints are imposed (e.g., the amount of raw materials that can be fed to the process over an operating window is constrained), or if time-varying disturbances are significant such that maintaining the process in a small neighborhood of the optimal steady-state is difficult, a time-varying (transient) operating policy is likely to be economically optimal. LEMPC, one such EMPC that operate process systems in a time-varying fashion, has been demonstrated to yield improved closed-loop economic performance over steady-state operation for several chemical process examples.[6−8] An open fundamental (yet motivated by practical application considerations) challenge to time-varying operation under EMPC is introducing online methods that can assess and monitor the performance of EMPC schemes.

Existing results on monitoring of model predictive control (MPC) deal with operation at steady-state and are based on the use of historical fault-free operation data to construct state-space regions of acceptable operation around the desired operating steady-state.[14−20] The traditional method of monitoring of MPC takes in historical data of high dimension and projects the data to a lower dimension through the application of principle component analysis (PCA) or partial least-squares (PLS). The squared prediction error (SPE) and/or Hotelling's T^2 statistic are used to establish a region of acceptable operation

Special Issue: Alírio Rodrigues Festschrift

Received: October 14, 2013
Revised: November 13, 2013
Accepted: November 14, 2013
Published: November 14, 2013

and to perform online and off-line process monitoring.[15] These regions account for common cause variance present in the process and the control system due to model inaccuracy, sensor noise, and actuator imperfect operation. However, the regions of acceptable process operation are computed in the context of steady-state operation and may not be suitable to be used to infer acceptable control system performance when the control system operates the process in an inherently time-varying fashion to optimize the economic performance as EMPC does. Furthermore, while some of these methods have been extended to monitoring of nonstationary and time-varying processes,[21] a more convenient and more economics-oriented approach to monitoring EMPC systems may be to take advantage of the economic cost function since it is considered to be a direct measure of the real-time process economics (operational cost and/or profit).

Motivated by this issue, we present a framework for the computation of acceptable operating regions for LEMPC systems that operate the process in a time-varying fashion within a well-defined region of the state-space. To this end, it is critical to capture the interplay between sources of common cause variance in the process and dynamic process operation. Therefore, given a well-defined region in the state-space of the process where time-varying operation is allowed to take place to optimize economics, to satisfy input constraints and to meet stabilizability requirements, several closed-loop simulations of normal operation with common cause variation are carried out to collect process dynamic operation data. Utilizing the nominal process model, the expected (predicted) economic cost value at each sampling period is computed and compared with the actual economic cost value. The difference between the two economic cost values defines the residual which is subsequently used in establishing acceptable operation limits by computing the exponentially weighted moving average (EWMA) of the residual. These regions are utilized to monitor the performance of LEMPC by comparing real-time process operation data under LEMPC and the corresponding regions of acceptable LEMPC operation computed in the fault-free, dynamic data generation step. The proposed framework is developed and evaluated using a chemical process example.

PRELIMINARIES

Notation. The notation $|\cdot|$ denotes the Euclidean norm of a vector. The symbol Ω_ρ denotes a level set of a function V: $\mathbf{R}^{n_x} \to \mathbf{R}$ (i.e., $\Omega_\rho = \{x \in \mathbf{R}^{n_x} \mid V(x) \le \rho\}$). A function α: $[0,a) \to [0,\infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$.

Class of Systems. The class of dynamical systems considered is the class of nonlinear systems that have the following state-space form:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \quad (1)$$

where $x \in \mathbf{R}^{n_x}$ is the state vector, $u \in U \subset \mathbf{R}^{n_u}$ is the input vector, and $w \in W \subset \mathbf{R}^{n_w}$ is the disturbance vector. The sets that bound the inputs and disturbance vectors are assumed to have the following form:

$$U = \{u \in \mathbf{R}^{n_u} \mid |u_i| \le u_{\max,i},\ i = 1, ..., n_u\}$$
$$W = \{w \in \mathbf{R}^{n_w} \mid |w| \le w_b\} \quad (2)$$

The vector field f is assumed to be a locally Lipschitz vector field of its arguments, and the origin is assumed to be the equilibrium (i.e., $f(0, 0, 0) = 0$). State measurements are assumed to be available synchronously at time instances $t_k = t_0 + k\Delta$, $k = 0, 1, ...$ To describe the real-time economics (e.g., operating profit or cost) of the system of eq 1, a scalar valued function of form: $l_e(x(t), u(t))$ is assumed.

Lyapunov-Based Controller. Central in the development of a Lyapunov-based EMPC (LEMPC) scheme for the system of eq 1 is the development of an invariant set whereby the closed-loop state remains bounded. A stabilizability assumption is required for the system of eq 1 for the construction of such a set. Namely, the existence of a Lyapunov-based controller $h(x)$ is assumed that renders the origin of the closed-loop system of eq 1 asymptotically stable with the inputs continuously computed by the Lyapunov-based controller (i.e., $u = h(x)$). Using converse theorems,[22,23] this assumption implies the existence of a Lyapunov function for the system of eq 1 which satisfies the inequalities:

$$\alpha_1(|x|) \le V(x) \le \alpha_2(|x|)$$
$$\frac{\partial V}{\partial x} f(x, h(x), 0) \le -\alpha_3(|x|)$$

for $x \in D$ where D is an open neighborhood of the origin and $\alpha_i(\cdot)$, and $i = 1, 2, 3$ are class $\mathcal{K}$ functions. Utilizing the controller $h(x)$, a level set of the Lyapunov function $\Omega_\rho \subset D$ can be computed where the Lyapunov function is decreasing along the state trajectory. The region Ω_ρ, referred to as the stability region, is an invariant set for the system of eq 1 under the controller $h(x)$ (see, for example, ref 24 for the details of this point).

Lyapunov-Based Economic Model Predictive Control. In a previous work,[6] an EMPC, referred to as Lyapunov-based EMPC (LEMPC), was designed by taking advantage of the explicit Lyapunov-based controller, $h(x)$. LEMPC is used to compute optimal control actions for the nonlinear system of eq 1 with respect to the economic cost $l_e(x, u)$. Since it is not assumed that the economic cost takes its optimal value at steady-state, LEMPC may operate the system in a time-varying fashion. As pointed out above, the stability region Ω_ρ is used in the formulation of the LEMPC to maintain the closed-loop state trajectory. LEMPC is given by the following optimization problem:

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} l_e(\tilde{x}(\tau), u(\tau))\, d\tau \quad (3a)$$

$$\text{st } \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \quad (3b)$$

$$\tilde{x}(t_k) = x(t_k) \quad (3c)$$

$$u(t) \in U, \quad \forall\, t \in [t_k, t_{k+N}] \quad (3d)$$

$$V(\tilde{x}(t)) \le \rho_e, \quad \forall\, t \in [t_k, t_{k+N}), \quad \text{if } V(x(t_k)) \le \rho_e \quad (3e)$$

$$\frac{\partial V}{\partial x} f(x(t_k), u(t_k), 0) \le \frac{\partial V}{\partial x} f(x(t_k), h(x(t_k)), 0), \quad \text{if } V(x(t_k)) > \rho_e \quad (3f)$$

where Δ is the sampling period, $S(\Delta)$ is the family of piecewise constant functions with period Δ, $\tilde{x}$ denotes the predicted state evolution with the nominal model ($w(t) \equiv 0$) of eq 3b initialized by eq 3c which is obtained through a state feedback measurement. The input constraint (eq 3d) is used to compute an input trajectory that is within the available input bounds. The two Lyapunov-based constraints of eqs 3e and 3f are used to maintain the closed-loop state inside Ω_ρ. If the current state is within a predefined subset of the stability region (i.e., $\Omega_{\rho_e} \subset \Omega_\rho$), LEMPC may dictate a time-varying operation while maintaining the predicted state within $\Omega_{\rho e}$ which is enforced through constraint of eq 3e. This defines mode 1 operation of LEMPC. If the current state is outside the subset of the stability region, LEMPC operates in mode 2 and the constraint of eq 3f is active so that LEMPC computes a control action that decreases the Lyapunov function by at least the rate of that the Lyapunov-based control would if it was implemented in a sample-and-hold fashion over t_k to t_{k+1}. The detailed analysis of the stability properties of this control scheme can be found in ref 6.

Remark 1. The subset of the stability region $\Omega_{\rho_e} \subset \Omega_\rho$ accounts for common cause variation which is incorporated in the bounded vector $w(t)$ (e.g., small modeling uncertainty, sensor noise, and nonideal actuator operation). In practice, $w(t)$ is unknown. The important property of Ω_{ρ_e} is that it is chosen to be sufficiently small such that if the state starts within Ω_{ρ_e} and the predicted state $\tilde{x}(t_{k+1}) \in \Omega_\rho$ under LEMPC, then $x(t_{k+1}) \in \Omega_\rho$. If the state starts within $\Omega_\rho \backslash \Omega_{\rho_e}$, then $x(t_{k+1}) \in \Omega_\rho$. Thus, Ω_ρ is also the stability region of the closed-loop system under the LEMPC of eq 3 (i.e., an invariant set where the state is maintained).

■ MONITORING OF ECONOMIC MODEL PREDICTIVE CONTROL SYSTEMS

The development and formulation of a monitoring scheme for EMPC systems is demonstrated through application on a nonisothermal continuous stirred tank reactor (CSTR) under LEMPC. While the monitoring methods is presented with the CSTR example under LEMPC, it can be generalized to any system of form described by eq 1, and the monitoring may be applied to other EMPC designs that lead to time-varying process operation.

Description of a Chemical Process Example. Consider a continuously stirred tank reactor (CSTR) where the contents of the reactor are assumed to be well-mixed meaning the reactor temperature T and reactant concentration C_A are spatially uniform. A second-order, endothermic reaction takes place in the chemical reactor that converts the reactant A to the product B. Since the reactor is nonisothermal and the reaction is endothermic, a jacket is used to supply heat to the reactor with heat rate Q. Applying first-principles and standard modeling assumptions, a dynamic model describing the evolution of the reactor temperature, T, and concentration of A, C_A in the reactor is obtained and is given by the following ordinary differential equations (ODEs):

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - k_0 e^{-E/RT} C_A{}^2 \quad (4)$$

$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) - \frac{\Delta H k_0}{\rho_L C_p} e^{-E/RT} C_A{}^2 + \frac{Q}{\rho_L C_p V} \quad (5)$$

where the definition and values of the process parameters are given in Table 1. The two states of the CSTR are the temperature and the concentration of A (i.e., $x = [C_A\ T]$), and the two inputs of the CSTR are the inlet concentration of A and the heat rate supplied to the reactor (i.e., $u = [C_{A0}\ Q]$) with available control energy: $u_1 \in [0.5, 7.5]$ kmol m^{-3} and $u_2 \in [0.0, 2.0 \times 10^5]$ kJ h^{-1}.

The control objective is to maximize the operation profit of the CSTR process through dynamic operation which is considered to be directly proportional to the amount of B produced minus the energy consumption:

$$p_1 k_0 e^{-E/RT} C_A{}^2 - p_2(Q - Q_s) \quad (6)$$

Table 1. Parameter Notation and Values

notation/value	description
$F = 5.0$ m^3 h^{-1}	inlet flow rate
$T_0 = 330$ K	inlet temperature
$V = 1.0$ m^3	reactor volume
$\Delta H = 6.55 \times 10^3$ kJ kmol^{-1}	heat of reaction
$k_0 = 13.93$ m^3 kmol^{-1} h^{-1}	pre-exponential factor
$E = 5.0 \times 10^3$ kJ kmol^{-1}	activation energy
$C_p = 0.231$ kJ kg^{-1} K^{-1}	heat capacity
$R = 8.314$ kJ kmol^{-1} K^{-1}	gas constant
$\rho_L = 1000$ kg m^{-3}	liquid density

where $p_1 > 0$ and $p_2 > 0$ are weighting factors corresponding to the profit generated from the production of B and the energy price. As demonstrated throughout the literature, periodic switching of the inlet reactant concentration can lead to improved time-averaged production rates over feeding in a constant concentration of A (e.g., ref 25). The reactor is assumed to be nominally operated at Q_s which is taken to be the median heat rate supplied to the reactor in the set of available control energy. If the LEMPC computes control inputs that supply less heat to the reactor than Q_s, it credits operation profit for using less heat. Depending on the value of the weights p_1 and p_2, LEMPC will compute one of two input trajectories for Q if eq 6 was to be used as the cost function in LEMPC: (1) periodic switching between the Q_{max} and Q_{min} when $C_{A0} = C_{A0,max}$ and $C_{A0} = C_{A0,min}$, respectively, or (2) the heat rate would be constant at its maximum or minimum value owing to the linear dependence of the heat rate on the economic cost. The first input behavior results when both terms are significant. When $C_{A0} = C_{A0,max}$, the reaction rate increases and the first term dominates the cost function for some p_1 and p_2. To further increase the reaction rate, it is also desirable to heat the reactor as much as possible to increase the temperature. When $C_{A0} = C_{A0,min}$, the reaction rate decreases so the second term becomes the dominant term so the heat rate decreases to its minimum value. The second input behavior results when one of the term always dominates over the other term.

In this example, the case where p_1 and p_2 are both significant is considered. However, as a result of the periodic switching of Q, the reactor may be operated over a large temperature range. Instead, a quadratic term that penalizes the deviation of the temperature from the median value is added to the pure process economics to define the economic cost function that LEMPC maximizes and is given by the following function:

$$l_e(x(t), u(t)) = p_1 k_0 e^{-E/RT} C_A{}^2 - p_2(Q - Q_s) - p_3(T - T_s)^2 \quad (7)$$

where p_1, p_2, and p_3 ($p_3 > 0$) are weighting factors chosen such that each term is significant (i.e., cover comparable order of magnitudes), and the quadratic term is least significant. From an economics perspective, a constant time-averaged amount of the reactant material A is available to be fed to the CSTR and is described by the constraint:

$$\frac{1}{t_f}\int_0^{t_f} F C_{A0}(\tau)\, d\tau = F C_{A0s} \quad (8)$$

which the LEMPC enforces over a period, t_f and is chosen to be 1 h (i.e., the period $t_f = 1.0$ h).

The economic weights p_i are chosen to be $p_1 = 10$, $p_2 = 1.62 \times 10^{-4}$, and $p_3 = 0.03$. Maximizing the operating profit (eq 6) over the steady-state CSTR model (i.e., the right-hand sides of eqs 4 and 5 set equal to zero) subject to the constraint of eq 8, the economically optimal steady-state is computed and is open-loop asymptotically stable: $C_{As}^* = 2.35$ kmol m^{-3}, $T_s^* = 341.5$ K, $C_{A0s}^* = 5.0$ kmol m^{-3}, $Q_s^* = 1.0 \times 10^5$ kJ h^{-1}. Deviation variables are used to express the inputs: $u_1 = C_{A0} - C_{A0}^*$ and $u_2 = Q - Q_s^*$. To characterize the stability region Ω_ρ, a Lyapunov-based controller is designed with $u_1 = h_1(x) = 1.0$ kmol m^{-3} and $u_2 = h_2(x) = -500(T - 341.5)$ (i.e., a proportional control based on the temperature with a gain of 500). A quadratic Lyapunov function (i.e., $V(x) = x^T P x$) with

$$P = \begin{bmatrix} 230 & -5.5 \\ -5.5 & 0.55 \end{bmatrix} \tag{9}$$

is considered and the level sets that are used for the stability regions are $\rho_e = 900$ and $\rho = 1200$.

In the simulation results below, explicit Euler method is used to numerically integrate the ODEs of eqs 4 and 5 with a fixed integration step size of $h = 0.001$ h. Bounded Gaussian white noise is added to the right-hand side of the ODEs of eqs 4 and 5 to model the sources of common cause variation. The bounded noise variables have a zero mean, standard deviation of $\sigma_{C_A} = 0.70$ kmol m^{-3} and $\sigma_T = 6.0$ K and bounds of $w_{b,C_A} = 2.25$ kmol m^{-3} and $w_{b,T} = 20.0$ K, respectively. The LEMPC uses a sampling period of $\Delta = 0.02$ h and a shrinking horizon that covers each hour of operation. For example, at the beginning of each one hour operating window, the LEMPC is initialized with a prediction horizon $N = 50$. At each subsequent sampling period, the prediction horizon is decreased by one. For the remainder, nominal operation will refer to operation without the presence of process noise (i.e., $w(t) \equiv 0$), and normal operation will refer to operation with the added noise terms described above which is used to model the common cause variation. To solve the nonlinear program of eq 3, the software package Ipopt[26] is used.

Remark 2. Regarding the selection of the inlet concentration of the reactant material to the reactor as an input to the CSTR, local or lower tier control (e.g., proportional—integral control) could be used to achieve a desired inlet feed concentration of the reactant material requested by the LEMPC. Specifically, one particular process design where manipulating the inlet concentration of the reactant material could be to have a pure solvent stream combine with a pure reactant stream with flow valves on each stream in closed-loop with lower tier control to achieve the desired (constant) inlet volumentric flow rate and inlet feed concentration of the reactant material.

Design of Monitoring Filter and Residuals. Since the scalar-valued economic cost function of eq 7 is assumed to describe the economics of the CSTR, the economic cost function is a convenient choice as performance metric to use in the design of a monitoring scheme that assesses the performance of the LEMPC scheme of eq 3. However, one assumption that is not placed on $l_e(x, u)$ is that it attains its optimum at the optimal steady-state $l_e(x_s^*, u_s^*)$ (i.e., other state and input pairs that do not satisfy the steady-state equation may satisfy $l_e(x, u) > l_e(x_s^*, u_s^*)$). As a result, the LEMPC operates the CSTR in a completely dynamic manner to achieve better instantaneous cost values over the steady-state cost. Since these points are transient states, the system may also instantaneously pass through points where $l_e(x,u) < l_e(x_s^*, u_s^*)$. Therefore, when considering the closed-loop economic performance of systems under LEMPC, the time-averaged economic performance is considered. From a monitoring stand-point, the instantaneous economic cost under normal operation (i.e., with common cause variance) exhibits variation with time for two reasons: (1) the system operates in a time-varying fashion and (2) the common cause variation. Figure 1 displays the time-varying evolution of the closed-loop trajectories and instantaneous economic cost ($l_e(x, u)$) with time for the closed-loop CSTR under LEMPC; while Figure 2 is the distribution of instantaneous economic cost values. From Figure 2, the distribution of the instantaneous cost is a bimodal distribution for the CSTR under LEMPC owing to the time-varying operation. Another problem with using the economic cost directly as the monitoring variable is that it is autocorrelated with time as shown in Figure 3.

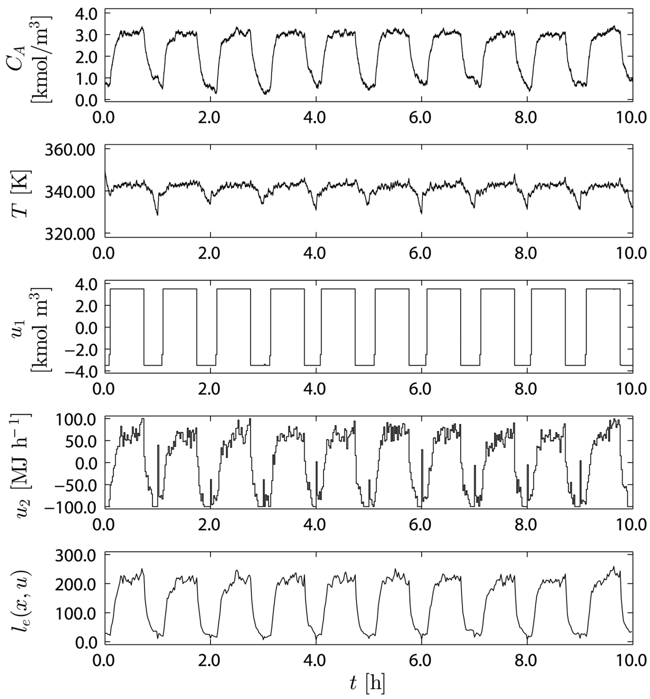

Figure 1. Closed-loop trajectories and instantaneous economic cost of the CSTR under LEMPC.

To define a monitoring variable that is not autocorrelated and has a normal distribution (which is demonstrated below), a residual variable is defined to assess the performance of LEMPC systems which is similar to our previous work[14] and is defined as

$$r_{l_e}(t_k) = l_e(x(t_k), u(t_k)) - l_e(\tilde{x}(t_k), u(t_k)) \tag{10}$$

where $x(t_k)$ is the actual state at t_k and $\tilde{x}(t_k)$ is the predicted state at t_k. The predicted state at time t_k, which is used in the evaluation of the predicted economic cost, is computed by solving the nominal model ($w(t) \equiv 0$) of eq 1 initialized with the previously obtained state measurement $x(t_{k-1})$ with the control action $u^*(t_{k-1})$ computed from the LEMPC at the previous sampling period applied in a sample-and-hold fashion. To account for past data which is important because the LEMPC operates systems in a time-varying fashion, exponentially weighted moving average (EWMA) is used as the monitoring statistic which captures smaller drifts in system and provides some protection against occasional spikes.[27] The exponentially weighted moving average (EWMA) is defined as

$$r_E(t_k) = \lambda r_{l_e}(t_k) + (1 - \lambda) r_E(t_{k-1}) \tag{11}$$

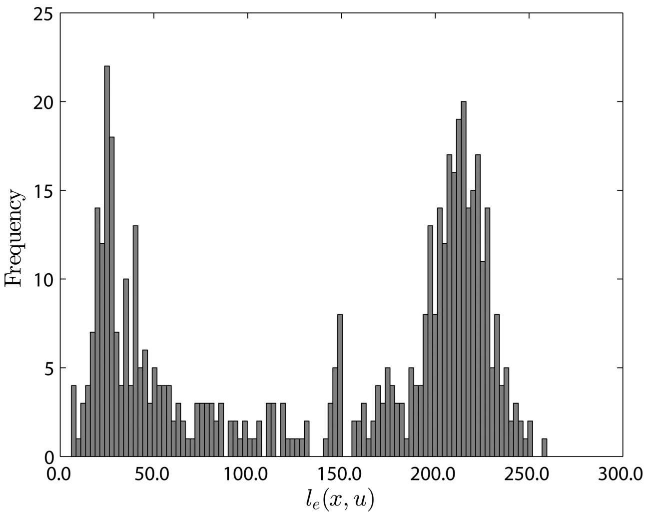

Figure 2. Histogram of the instantaneous closed-loop economic cost of the CSTR under LEMPC.

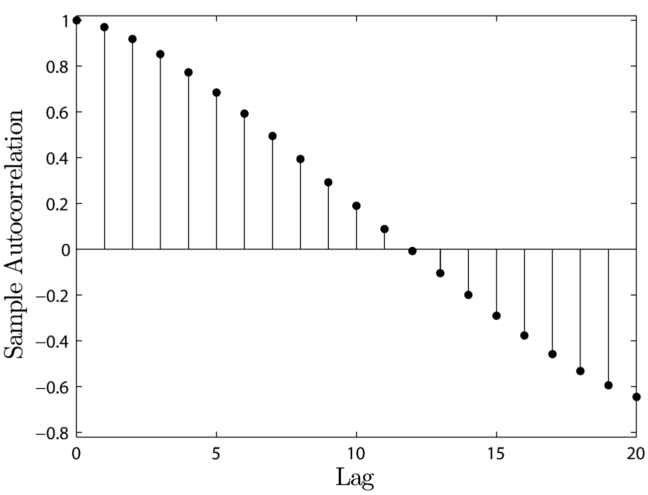

Figure 3. Sample autocorrelation of the instantaneous economic cost reveals the economic cost is clearly autocorrelated.

where λ is a parameter describing the how much past data enters into the calculation and $r_E(t_0) = r_{l_e}(t_0)$. The upper control limit (UCL) and lower control limit (LCL) or the thresholds of EWMA under normal operation is

$$\text{UCL/LCL} = \bar{r} \pm Ls\sqrt{\frac{\lambda}{2 - \lambda}} \tag{12}$$

where $\bar{r}$ and s are mean and standard deviation based on historical operation data of the closed-loop system (i.e., normal operation with common cause variation). The parameter L is chosen to be 3 to represent the typical six sigma processing limit. As another way to protect against Type I error (i.e., false alarms), the EWMA must exceed the LCL or UCL for a sufficiently long window determined through historical data. This window is defined as Δt_p.

Monitoring Algorithm. The implementation strategy of the performance monitoring of LEMPC systems is summarized below:

1. At t_k, a measurement of the state $x(t_k)$ is received.
2. If $V(x(t_k)) \leq \rho_e$, go to step 2.1. Else, go to step 2.2.
   2.1. The LEMPC operates in mode 1 whereby it dynamically operates the system to maximize the economic cost, go to step 3.
   2.2. The LEMPC operates in mode 2 to steer the state into the region Ω_{ρ_e}, go to step 3.
3. The EWMA $r_E(t_k)$ of the residual $r_{l_e}(t_k)$ defined in eq 10 is computed. If $r_E(t_k)$ > LCL or $r_E(t_k)$ > UCL, go to step 4. Else, go to step 5.
4. If the EWMA has exceeded the LCL or the UCL for a period of time greater than Δt_p, the closed-loop performance is deemed to be poor.
5. Wait until the next sampling period. Go to step 1; $k \rightarrow k + 1$.

Remark 3. This monitoring algorithm can be expanded further if the performance is deemed to be poor. Specifically, in the example below, a process parameter change is considered as the source for the closed-loop performance degradation under LEMPC. Therefore, once the monitoring algorithm deems the LEMPC is not functioning correctly, the monitoring scheme could trigger an online or off-line model update.

Phase I: Normal Operation. In the first set of simulations, several closed-loop simulations of the CSTR under LEMPC over 10 h is completed to compute the average $\bar{r}$ and standard deviation s of the residual under normal operation and to determine parameter values λ and Δt_p. Since the short-term economic cost under LEMPC can be influenced by the effect of the initial condition, each of the simulations are initialized at various points equally distributed in the region Ω_{ρ_e} to account for this source of variability. The operation length of 10 h was chosen such that the closed-loop economic performance does not significantly depend on the effect of the initial condition. The metric that assesses the overall real-time economic performance is the total (sum) economic cost and is given by

$$J_E = \frac{1}{10}\sum_{k=0}^{499} l_e(x(t_k), u(t_k))\Delta \tag{13}$$

The total economic cost function of the ten simulations is provided in Table 2 along with average and standard deviation of $r_{l_e}(t)$ over each simulation. From this set of training data, the average and standard deviation of the residual were determined to be $\bar{r} = -0.034$ and $s = 8.989$, respectively. Additionally, as a baseline comparison on the economic performance with LEMPC, starting from the steady-state and maintaining operation at the steady-state thereafter under nominal operation has a total economic cost of 136.2; while, initializing

Table 2. Results of Several 10 h Simulations of the CSTR under LEMPC and under Normal Operation (i.e., with Common Cause Variation)

sim	J_E	avg $r_{l_e}(t)$	std $r_{l_e}(t)$
1	140.2	0.455	9.032
2	139.6	−0.345	9.173
3	139.0	−0.277	8.903
4	139.9	−0.537	8.664
5	140.9	0.501	8.616
6	141.5	0.058	8.660
7	141.2	−0.170	8.505
8	140.3	0.309	9.375
9	140.0	0.069	9.593
10	138.0	−0.404	9.366
avg	$\bar{J}_E$ = 140.0	$\bar{r}$ = −0.034	s = 8.989

the CSTR at the steady-state and the closed-loop system under LEMPC and nominal operation the CSTR achieves a total economic cost of 140.3, a 3% improvement.

Figures 4 and 5 display the distribution and sample autocorrelation, respectively, of the residual variable $r_{l_e}(t)$ for one of

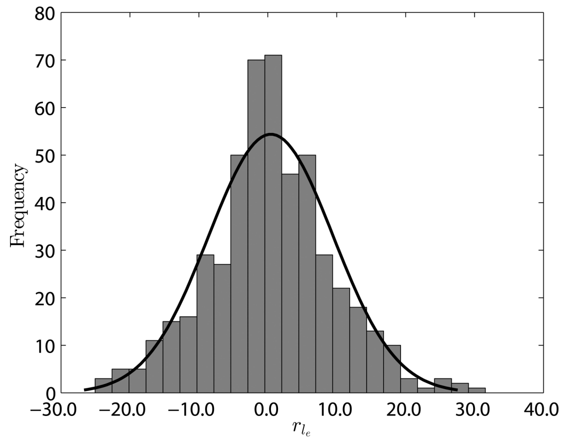

Figure 4. Distribution of the residual variable $r_{l_e}(t)$ of the CSTR under LEMPC over 10 h of operation.

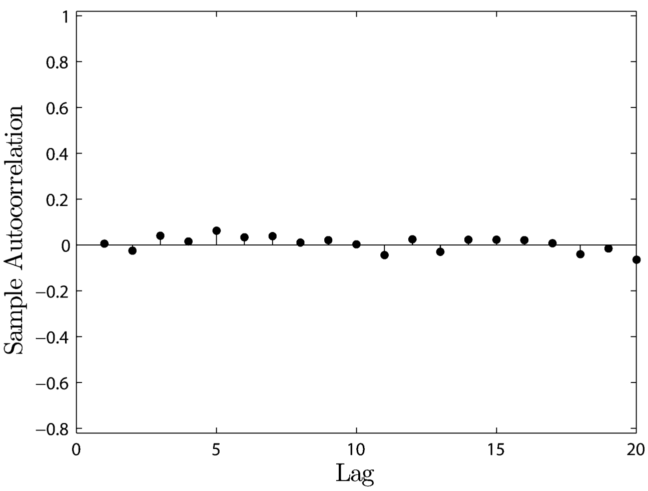

Figure 5. Sample autocorrelation of the residual variable $r_{l_e}(t)$ shows that the residual variable is not significantly autocorrelated.

the 10 h simulations of the CSTR under LEMPC. From these figures, the residual variable $r_{l_e}(t)$ is approximately normally distributed and is not significantly autocorrelated with time. While the behavior of the common cause variation is modeled as bounded Gaussian noise, it is important to point out that the noise affects the monitoring variable nonlinearly. However, the residual variable still approximates a normal distribution. This is not unexpected. In fact, it would be expected that expressing the noise as a linear (or nonlinear in this case) combination of several random variables with varying probability distributions, the overall behavior of the cumulative variable will be Gaussian for the closed-loop system (see, for example, ref 28). If the residual variable is not normally distributed, the monitoring procedure may still be applied owing to the robustness of EWMA to non-normality of the data.[17]

To test the proposed monitoring method, another simulation of normal operation is completed where the monitoring scheme is verified and assessed for false alarms. The EWMA parameter and window are chosen to be $\lambda = 0.05$ and $\Delta t_p = 0.04$, $h = 2.4$ min (i.e., two sampling periods) based on the training data set as to not give any false alarms. Also, λ has been chosen as to be the smallest value in the suggested range of $0.05 \leq \lambda \leq 0.2$[17] to place a large weight on previous data in the computation of the EWMA and, thus, detect small shifts in the residual. Figure 6 displays the EWMA of the residual for ten

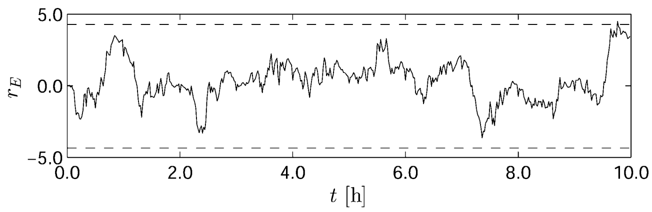

Figure 6. EWMA of the residual with time over a 10 h simulation of the CSTR under LEMPC and under normal operation.

hours of normal operation of the CSTR under LEMPC. The EWMA is maintained with in the LCL and the UCL except for one sampling instance. However, it does not exceed the threshold for two consecutive sampling periods (Δt_p), so the performance of the LEMPC scheme over the course of the 10 h is deemed to be within the acceptable limits.

Remark 4. Systematic bias in the training data (e.g., plant—model parameter mismatch) may lead to a nonzero mean of the residual variable. For this case, ideally the model could be updated to remove this systematic bias in the training data. If this is not possible, it may still be possible to successfully apply the proposed monitoring scheme using the nonzero mean by carefully tuning monitoring parameters λ and Δt_p. However, if the bias is significant such that the autocorrelation of the residual becomes significant, the number of false alarms may increase. For this case, one may need to fit an appropriate time-series model and/or use an EWMA procedure for correlated data (see, for example, ref 17).

Phase II: Monitoring of Real-Time Performance under EMPC. To test the performance monitoring scheme, a step change in the rate constant k_0 is considered which can happen in practice owing to catalyst deactivation. At $t = 3.4$ h, the rate constant decreases from 13.93 to 10.00 $m^3\ kmol^{-1}\ h^{-1}$. If the EWMA of the residual is within the bounds established through normal operation data the monitor returns "Within Thresholds". If the EWMA of the residual is outside the bounds for at least Δt_p, the monitoring system returns "Exceeds Thresholds" to denote the residual is outside the threshold for normal operation. Visually inspecting the closed-loop trajectories (Figure 7) of the CSTR under LEMPC after a step change in k_0, some noticeable differences occur in the closed-loop trajectories after 3.4 h; however, it is difficult to determine if these differences are associated with a problem. If, instead, the EWMA control chart for the residuals is inspected (Figure 8), it shows that the EWMA of the residuals is clearly outside the threshold and the monitoring system detects poor performance of the LEMPC at 3.58 h.

Owing to the inherent dynamic operation under LEMPC, not all model parameter changes may result in performance degradation since the LEMPC may be able to compensate for the parameter change and/or the economic cost may not be sensitive to this model parameter. This is perhaps one of the most significant difference between traditional control system

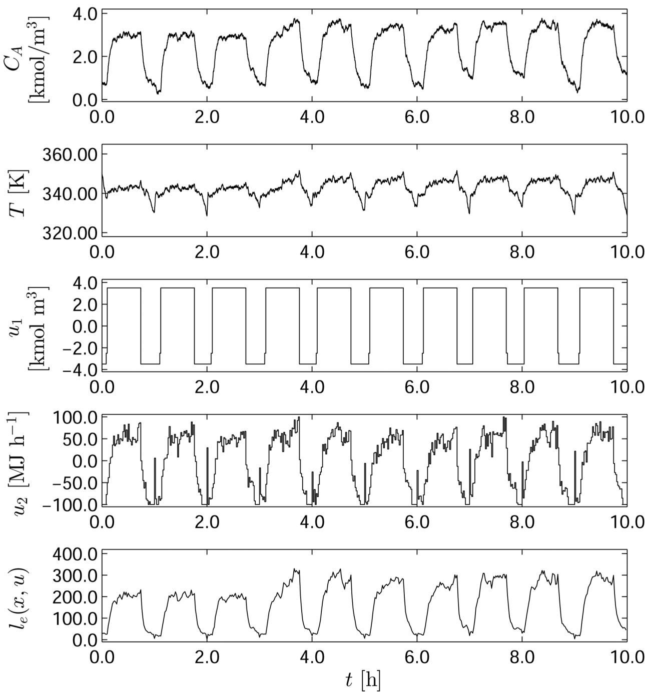

Figure 7. Closed-loop trajectories of the CSTR under LEMPC for a step change in the model parameter k_0 (i.e., the rate constant).

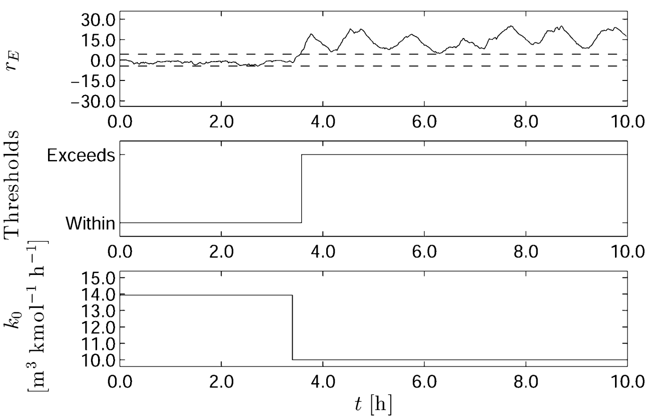

Figure 8. EWMA control chart for the residual variable along with the performance monitoring output and the step change in k_0.

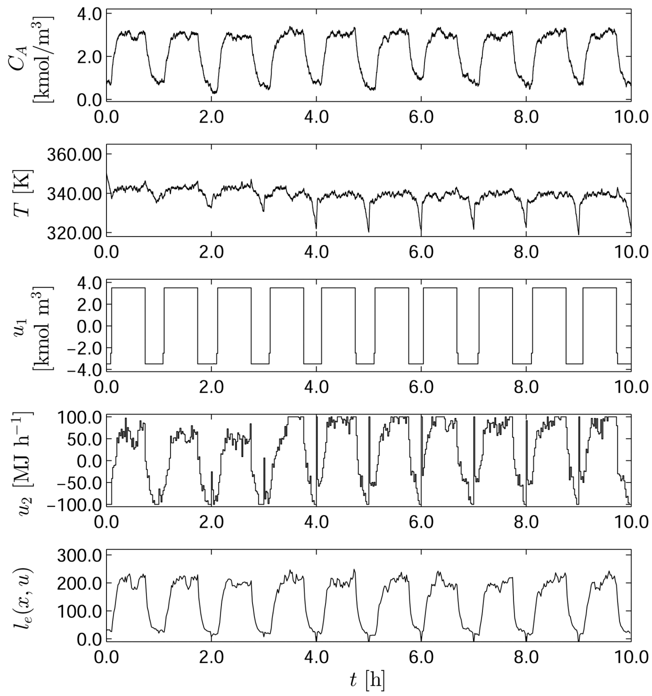

Figure 9. Closed-loop trajectories of the CSTR under LEMPC for a step change in the model parameter T_0 (i.e., the inlet temperature).

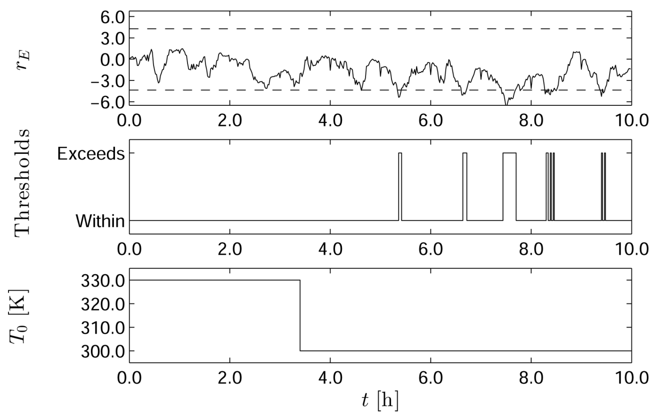

Figure 10. EWMA control chart for the residual variable along with the performance monitoring output and the step change in T_0.

monitoring and monitoring of EMPC systems. Generally, traditional control systems are used to force processes to operate at a steady-state (set-point) and to maintain operation at the steady-state (set-point). The system may settle at an offsetting steady-state when the model is subject to a constant bias (e.g., constant model parameter error) particularly when using model-based control. Under steady-state type operation, offset is generally considered undesirable and may lead to suboptimal operation. However, when operating in a dynamic fashion, the effect of a constant bias may not be as detrimental to closed-loop performance as in steady-state operation. Figures 9 and 10 demonstrate this point which show the closed-loop trajectories and the EWMA of the residuals, respectively of a 10 h closed-loop simulation of the CSTR under LEMPC with a step decrease in the inlet temperature T_0 from 330.0 to 300.0 K. The parameter changes at t = 3.4 h. From the u_2 trajectory, the LEMPC increases the amount of heat rate supplied to the reactor after the step change. Clearly, the LEMPC is able to compensate for this change in this case. This behavior is confirmed in the EWMA control chart. Namely, the EWMA does not exceed the LCL until 5.36 h and even after it exceeds the LCL, the EWMA of the residual does not continue to exceed the LCL.

■ CONCLUSIONS

In this work, performance monitoring of LEMPC systems was considered. Since LEMPC systems operate systems in a time-varying or dynamic fashion, a residual variable based on the economic cost function was introduced to capture the sources of common cause variance; while essentially eliminating the variation caused by dynamic process operation. Several closed-loop simulations were carried out to determine monitoring parameters. Additional simulations were carried out under both normal operation and under abnormal operation (i.e., model parameter step changes) which showed the applicability of the performance monitoring scheme for LEMPC systems. The proposed monitoring framework and principles can be applied to other EMPC schemes that lead to time-varying operation.

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Notes

The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

Financial support from the National Science Foundation and the Department of Energy is gratefully acknowledged.

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