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Brief paper

On finite-time and infinite-time cost improvement of economic model predictive control for nonlinear systems ☆

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ABSTRACT

A novel two-layer economic model predictive control (EMPC) structure that addresses provable finite-time and infinite-time closed-loop economic performance of nonlinear systems in closed-loop with EMPC is presented. In the upper layer, a Lyapunov-based EMPC (LEMPC) scheme is formulated with performance constraints by taking advantage of an auxiliary Lyapunov-based model predictive control (LMPC) problem solution formulated with a quadratic cost function. The lower layer LEMPC uses a shorter prediction horizon and smaller sampling period than the upper layer LEMPC and involves explicit performance-based constraints computed by the upper layer LEMPC. Thus, the two-layer architecture allows for dividing dynamic optimization and control tasks into two layers for a computationally manageable control scheme at the feedback control (lower) layer. A chemical process example is used to demonstrate the performance and stability properties of the two-layer LEMPC structure.

1. Introduction

Within process control, economic model predictive control (EMPC) has ignited wide-spread interest because of its unique ability to dynamically regulate processes to achieve closed-loop economic performance not attainable through traditional tracking control techniques (Adetola & Guay, 2010; Amrit, Rawlings, & Angeli, 2011; Angeli, Amrit, & Rawlings, 2012; Baldea & Touretzky, 2013; Diehl, Amrit, & Rawlings, 2011; Fagiano & Teel, 2013; Ferramosca, Rawlings, Limon, & Camacho, 2010; Grüne, 2013; Guay & Adetola, 2013; Heidarinejad, Liu, & Christofides, 2012, 2013; Huang, Biegler, & Harinath, 2012; Idris & Engell, 2012; Ma, Qin, Salsbury, & Xu, 2012; Müller, Angeli, & Allgöwer, 2013; Omell & Chmielewski, 2013). The fundamental difference between EMPC and conventional model predictive control (MPC) is the cost function used in the formulations of these two control schemes. Typically, in conventional MPC schemes, a quadratic cost function that penalizes a weighted error of states and inputs from their economically optimal steady-state values is typically used, while, EMPC schemes use a general cost function that is derived from the process economics (e.g., operating cost or profit). As a result of the type of cost function used, EMPC can handle both dynamic process economic optimization and process control. To utilize EMPC for the computation of optimal inputs in real-time, EMPC is formulated with a finite prediction horizon.

An important, albeit not well understood property, is the closed-loop performance of systems under EMPC since EMPC is formulated with a finite prediction horizon. The main results on closed-loop performance with EMPC include: (1) EMPC formulated with a terminal constraint has asymptotic (infinite-time) average performance at least as good as the economically optimal steady-state (Angeli et al., 2012) (others have extended asymptotic average performance to various EMPC formulations Amrit et al., 2011, Fagiano & Teel, 2013, Müller et al., 2013), (2) EMPC formulated without any (terminal) constraints was shown to be approximately optimal for both finite-time (i.e., transient) and infinite-time when a sufficiently long prediction horizon is used and certain controllability assumptions are satisfied (Grüne, 2013), and (3) a Lyapunov-based EMPC (LEMPC) which uses performance constraints derived from an auxiliary conventional (tracking) MPC and a shrinking horizon to guarantee that over a finite operating window the closed-loop performance under LEMPC is at least as good as the auxiliary conventional MPC (Heidarinejad et al., 2013). In Angeli

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et al. (2012), the effect of the initial condition is essentially neglected since it is insignificant when considering operation for an infinite-time period. Given the power of EMPC to yield dynamically optimal regulation of systems operating away from steady-state, the importance of considering the effect of the initial condition on closed-loop performance should be considered as this is an important property of EMPC. In Heidarinejad et al. (2013), on the other hand, guarantees on closed-loop performance can only be made over finite operating windows. Therefore, introducing an EMPC structure that provides provable finite-time (i.e., accounts for the effect of the initial condition) and infinite-time performance guarantees on closed-loop economic performance is an important issue.

Another challenge of EMPC is that the achievable closed-loop economic performance benefit of EMPC over conventional tracking MPC may strongly depend on the prediction horizon length (e.g., Grüne, 2013). A long prediction horizon (i.e., many decision variables), however, may make it difficult to solve the EMPC optimization problem for real-time applications. To address guaranteed closed-loop economic performance while formulating a computationally efficient control structure, a novel two-layer LEMPC structure is proposed in this work. The core idea of the two-layer EMPC is to solve the upper layer LEMPC infrequently (i.e., not every sampling period) over a long horizon. Then, take advantage of the solution generated by the upper layer LEMPC in the formulation of a lower layer LEMPC used for feedback control. Specifically, in the upper layer, an LEMPC, formulated with a sufficiently large prediction horizon, is used to compute economically optimal trajectories which are sent down to the lower layer LEMPC. The lower layer LEMPC uses a shorter prediction horizon and smaller sampling time than the upper layer LEMPC to compute control actions for the process in real-time while maintaining operation around the economically optimal trajectories computed in the upper layer. For guaranteed performance improvement with the proposed LEMPC scheme, both layers are formulated with explicit performance-based constraints computed from an auxiliary Lyapunov-based model predictive control (LMPC) problem solution formulated with a quadratic cost which allows for provable finite-time and infinite-time closed-loop economic performance and effectively merges the provable performance guarantees on finite-time and infinite-time performance compared to a conventional (tracking) MPC. The two-layer LEMPC structure is applied to a chemical process example to demonstrate the closed-loop performance, stability, and robustness properties of the two-layer LEMPC structure.

2. Preliminaries

2.1. Class of systems

The class of continuous-time nonlinear systems considered is described by the following state-space form:

$$\dot{x}(t) = f(x(t), u(t)) \tag{1}$$

where the state is $x(t) \in \mathbf{R}^n$ and the input is $u(t) \in \mathbf{R}^m$. The vector function $f : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^n$ is a locally Lipschitz vector function on $\mathbf{R}^n \times \mathbf{R}^m$. The available control effort is defined by the convex set $U = \{u_{\min} \le u \le u_{\max}\} \subset \mathbf{R}^m$. The state x of the system is synchronously sampled at time instances $t_0 + k\Delta$ with $k = 0, 1, 2, \ldots$ where t_0 is the initial time and Δ is the sampling period. Without loss of generality, the initial time is taken to be zero ($t_0 = 0$). To distinguish between the continuous time and the discrete sampling instances, the notation t will be used for the continuous time and the time sequence $\{\tau_k\}_{k=0}^{\infty}$ is the partitioning of t with $\tau_k = k\Delta$.

A time-invariant economic cost measure $l_e(x, u)$ is assumed to describe the real-time economics of the system of equation (1) and is assumed to be continuous on $X \times U$ where $X \subset \mathbf{R}^n$ is the set of admissible operating states. The optimal steady-state x_s^* and steady-state input u_s^* with respect to the economic cost function is $(x_s^*, u_s^*) = \arg\max_{u_s \in U,\, x_s \in X} \{l_e(x_s, u_s) : f(x_s, u_s) = 0\}$. For the sake of simplicity, the optimal steady-state is assumed to be unique and to be $(x_s^*, u_s^*) = (0, 0)$. Furthermore, the notation $|\cdot|$ denotes the Euclidean norm of a vector, the notation $|\cdot|_Q$ denotes the square of a weighted Euclidean norm of a vector (i.e., $|x|_Q = x^T Q x$ where Q is a positive definite matrix), and the symbol Ω_ρ denotes a level set of a Lyapunov function (i.e., $\Omega_\rho = \{y \in \mathbf{R}^n : V(y) \le \rho\}$).

2.2. Existence of a stabilizing controller

Assumption 1. There exists a locally Lipschitz feedback controller $u = h(x)$ with $h(0) = 0$ for the system of equation (1) that renders the origin of the closed-loop system under continuous implementation of the controller $h(x)$ locally exponentially stable. More specifically, there exist constants $\rho > 0$, $c_i > 0$, $i = 1, 2, 3, 4$ and a continuously differentiable Lyapunov function $V : \mathbf{R}^n \to \mathbf{R}_+$ such that the following inequalities hold for all $x \in \Omega_\rho$:

$$c_1 |x|^2 \le V(x) \le c_2 |x|^2, \tag{2a}$$

$$\frac{\partial V(x)}{\partial x} f(x, h(x)) \le -c_3 |x|^2, \tag{2b}$$

$$\left| \frac{\partial V(x)}{\partial x} \right| \le c_4 |x|, \tag{2c}$$

for all $x \in \Omega_\rho$.

Explicit feedback controllers that may be designed to satisfy Assumption 1 are, for example, feedback linearizing controller and some Lyapunov-based controllers (e.g., Khalil, 2002, Kokotović & Arcak, 2001). With the controller $h(x)$, the following results hold for the closed-loop system of equation (1) under the controller $h(x)$ implemented in a zero-order sample-and-hold fashion with sampling period Δ (i.e., $h(x)$ is applied as an emulation controller).

Proposition 2. *Suppose Assumption 1 holds. Then, there exists $\Delta^* > 0$ and $M,\ \sigma > 0$ such that for the partition $\{\tau_i\}_{i=0}^{\infty}$ of $\mathbf{R}_+$ with $\tau_{i+1} - \tau_i = \Delta \le \Delta^*$ the closed-loop system of equation (1) with the input trajectory*

$$u(t) = h(x(\tau_i)) \quad \text{for } t \in [\tau_i, \tau_{i+1}) \text{ and integers } i \ge 0 \tag{3}$$

and arbitrary initial condition $x(0) = x_0 \in \Omega_\rho$ satisfies the estimate:

$$|x(t)| \le M \exp(-\sigma t) |x_0| \tag{4}$$

for all $t \ge 0$.

The proof of Proposition 2 may be found in Ellis et al. (2014, Corollary 1) and shows that V is a Lyapunov function for the closed-loop sampled-data system in the sense that there exists a constant $\hat{c}_3 > 0$ such that

$$\frac{\partial V(x(t))}{\partial x} f(x(t), h(x(\tau_i))) \le -\hat{c}_3 |x(t)|^2 \tag{5}$$

for all $t \in [\tau_i, \tau_{i+1})$ and integers $i \ge 0$ where $x(t)$ is the solution of Eq. (1) starting from $x(\tau_i) \in \Omega_\rho$ and with the input $u(t) = h(x(\tau_i))$ for $t \in [\tau_i, \tau_{i+1})$. The stability region of the closed-loop system under the controller $h(x)$ is defined as $\Omega_\rho \subseteq X$.

Remark 3. Assumption 1 is stronger than the one imposed in our previous works (e.g., Christofides, Liu, & Muñoz de la Peña, 2011, Heidarinejad et al., 2012). In the present works, the existence of a controller $h(x)$ that renders the origin of the closed-loop system locally exponentially stable under continuous implementation is assumed whereas, in Christofides et al. (2011) and Heidarinejad et al.

(2012), the existence of a controller $h(x)$ is assumed that renders the closed-loop system only asymptotically stable under continuous implementation. The stronger assumption is needed to ensure that the controller $h(x)$ renders the origin of the closed-loop system exponentially stable under sample-and-hold implementation and thus, be able to use it to design a Lyapunov-based MPC that renders the origin of the closed-loop asymptotically stable. The latter requirement is needed to consider infinite-time operation under MPC. Specifically, asymptotic convergence to the origin and not just convergence to a neighborhood of the steady-state (i.e., practical stability) is required to study closed-loop economic performance under conventional MPC on the infinite-time interval. In the context of sampled-data system, the interested reader is referred to Karafyllis and Kravaris (2009), Nesić and Teel (2001) and the references therein for more issues regarding designing an emulation controller for the system of equation (1).

2.3. Lyapunov-based MPC

Lyapunov-based model predictive control (LMPC) is a conventional MPC scheme (i.e., formulated with a quadratic cost function) that is used to steer the system to the origin (Christofides et al., 2011). The LMPC scheme is given by the following optimization problem:

$$\min_{u \in S(\Delta)} \int_0^{\tau_N} \left(\left|\tilde{x}(t)\right|_{Q_c} + |u(t)|_{R_c} \right) dt \tag{6a}$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t)) \tag{6b}$$

$$\tilde{x}(0) = x(\tau_k) \tag{6c}$$

$$u(t) \in U, \quad \forall\, t \in [0, \tau_N) \tag{6d}$$

$$\frac{\partial V(x(\tau_k))}{\partial x} f(x(\tau_k), u(0)) \leq \frac{\partial V(x(\tau_k))}{\partial x} f(x(\tau_k), h(x(\tau_k))) \tag{6e}$$

where $\tilde{x}$ is the predicted state trajectory, $S(\Delta)$ is the set of piecewise constant functions with period Δ, and N is the number of sampling periods of the prediction horizon. Given that the cost function is positive definite with respect to the optimal steady-state, the global minimum of the cost function occurs at the optimal steady-state. The optimal input trajectory to the optimization problem of the LMPC is $u^*(t)$ and is defined for $t \in [\tau_k, \ \tau_{k+N})$. The control action computed for the first sampling period of the prediction horizon $u^*(\tau_k)$ is sent to the control actuators to be applied over the sampling period and the LMPC is resolved at the next sampling period.

In the optimization problem of Eq. (6), the model of the system is used to predict the future state of the system with control input computed by the LMPC (Eq. (6b)). The dynamic model is initialized through a state measurement obtained at the current sampling instance τ_k and is given by the constraint of Eq. (6c). The input constraint (Eq. (6d)) ensures the computed control input is within the bounds of the available control action. Lastly, the Lyapunov-based constraint (Eq. (6e)) is used to guarantee closed-loop stability. The constraint of Eq. (6e) is enforced at the sampling instance τ_k to ensure that the Lyapunov function of the closed-loop system under the LMPC decreases by at least the rate given by the Lyapunov-based controller over $[\tau_k, \tau_{k+1})$. When the controller $h(x)$ is applied in a sample-and-hold fashion with a sufficiently small sampling period, the rate of decrease of the Lyapunov function due to the control action $h(x)$ applied in a sample-and-hold fashion over the sampling period is enough to overcome the rate of possible increase of the Lyapunov function due to the fact that the controller $h(x)$ is applied in a sample-and-hold fashion. Owing to the constraint (Eq. (6e)), the control action computed by the LMPC for the sampling period τ_k to τ_{k+1} will have the same property and thus, the LMPC inherits the closed-loop stability region Ω_ρ of the controller $h(x)$ applied in a sample-and-hold fashion.

2.4. Lyapunov-based EMPC

Lyapunov-based economic model predictive control (LEMPC) directly optimizes the economic cost (profit) function $l_e(\cdot, \cdot)$. The LEMPC scheme (Heidarinejad et al., 2012) is given by the following optimization problem:

$$\max_{u \in S(\Delta)} \int_0^{\tau_N} l_e(\tilde{x}(t), u(t))\, dt \tag{7a}$$

$$\text{s.t. } \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t)) \tag{7b}$$

$$\tilde{x}(0) = x(\tau_k) \tag{7c}$$

$$u(t) \in U, \quad \forall\, t \in [0, \tau_N) \tag{7d}$$

$$V(\tilde{x}(t)) \leq \rho, \quad \forall\, t \in [0, \tau_N) \tag{7e}$$

where the notation and constraints Eqs. (7b)–(7d) are equivalent to the notation and constraints of Eqs. (6b)–(6d) of the LMPC. To allow for the LEMPC to dictate a time-varying operating policy to optimize the system economics, a Lyapunov-based constraint is used to maintain the predicted state in the stability region Ω_ρ (i.e., Ω_ρ is an invariant set for the system of equation (1) which has been shown in Heidarinejad et al. 2012). With the Lyapunov-based constraint of Eq. (7e), the provable stability property of the closed-loop system is boundedness in Ω_ρ and not asymptotic convergence to the steady-state.

3. Two-layer EMPC structure

In this section, a detailed description of the proposed two-layer EMPC structure is provided which includes descriptions of the implementation strategy, the formulations of the upper and lower layer LEMPC schemes, and the provable stability and performance properties.

3.1. Implementation strategy

To address guaranteed performance improvement with EMPC compared to that with conventional MPC over both finite-time and infinite-time, performance-based constraints are used in the formulation of an LEMPC which are computed from an auxiliary LMPC. In general, the achievable economic performance improvement with EMPC is closely associated with the length of the prediction horizon of the EMPC. However, an EMPC with a long prediction horizon may be unsuitable for use in real-time implementation because of the computational time required to solve the optimization problem of the EMPC. To address this challenge, a two-layer LEMPC structure is proposed. In the upper layer, an LEMPC is used to optimize dynamic operation over a long horizon while accounting for the performance-based constraints from the auxiliary LMPC. Both the auxiliary LMPC and the upper layer LEMPC compute their optimal input trajectory once over some operating window. In the lower layer, an LEMPC, using a shorter prediction horizon and a smaller sampling period than the upper layer LEMPC, computes control inputs that are applied to the process. Constraints that have been generated from the upper layer LEMPC are used to ensure that the lower layer LEMPC guides the system along the optimal solution computed in the upper layer since it uses a shorter prediction horizon and a smaller sampling period. In this manner, the lower layer LEMPC is used to improve robustness of the closed-loop system (recomputes its optimal trajectory every sampling period to incorporate feedback) as well as for providing additional economic cost improvement over the upper layer LEMPC solution owing to the use of a smaller sampling time.

Since the upper layer LEMPC and auxiliary LMPC use a different sampling period and prediction horizon than the lower layer LEMPC, the notation $\hat{}$ will be used for the upper layer MPCs (i.e., the

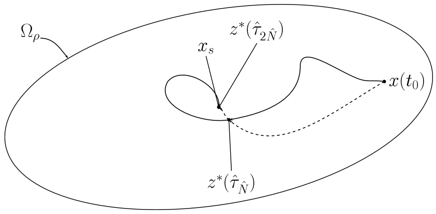

Fig. 1. A state-space illustration of the evolution of the closed-loop system in the stability region Ω_ρ over two operating periods with inputs computed by the upper layer LEMPC (solid line) and the auxiliary LMPC (dashed line).

auxiliary LMPC and the upper layer LEMPC): $\hat{N}$ is the number of sampling periods of size $\hat{\Delta}$ of each operating window and it is also the prediction horizon used in the upper layer MPCs. The time sequence $\{\hat{\tau}_j\}_{j=0}^{\infty}$ corresponds to the sampling periods of the upper layer ($\hat{\tau}_j = j\hat{\Delta}$) and M is a positive integer corresponding to the number of elapsed operation windows. At the beginning of each operating window, the upper layer MPCs receive a state measurement of the system $x(\hat{\tau}_{(M-1)\hat{N}})$ and each MPC computes a piecewise constant input trajectory with period $\hat{\Delta}$ over the horizon $\hat{\tau}_{(M-1)\hat{N}}$ to $\hat{\tau}_{M\hat{N}}$.

The upper layer period $\hat{\Delta}$ is divided into N subintervals of length Δ ($\Delta = \hat{\Delta}/N$ where N is a positive integer). The subintervals define the sampling period of the lower layer and correspond to the sequence $\{\tau_k\}_{k=0}^{\infty}$. The lower layer LEMPC recomputes its optimal input trajectory employing a shrinking horizon. Namely, at the beginning of each sampling period of the upper layer $\hat{\Delta}$, the lower layer is initialized with a prediction horizon N and computes an input trajectory from τ_k to τ_{k+N}. At each subsequent sampling period of the lower layer LEMPC, the prediction horizon decreases (i.e., shrinking horizon approach) until the next sampling period of the upper layer LEMPC when the prediction horizon is reset to N.

The implementation strategy is summarized below and an illustration of the closed-loop system is given in Fig. 1.

1. At the beginning of the Mth operating period, the auxiliary LMPC and the upper layer LEMPC are initialized with the state measurement $x(\hat{\tau}_{(M-1)\hat{N}})$.
2. The auxiliary LMPC computes its optimal input trajectory denoted as $v^*(t)$ and defined for $t \in [\hat{\tau}_{(M-1)\hat{N}}, \hat{\tau}_{M\hat{N}})$, the state trajectory denoted as $z^*(t)$ under the input trajectory $v^*(t)$ according to Eq. (1), and the total economic cost:
$$L^*_{\text{LMPC},M} = \int_{\hat{\tau}_{(M-1)\hat{N}}}^{\hat{\tau}_{M\hat{N}}} l_e(z^*(t), v^*(t))\,dt. \tag{8}$$
3. The upper layer LEMPC receives $L^*_{\text{LMPC},M}$, and $z^*(\hat{\tau}_{M\hat{N}})$ from the auxiliary LMPC.
4. The upper layer LEMPC computes its optimal input $\hat{u}^*(t)$ and state $\hat{x}^*(t)$ trajectories defined for $t \in [\hat{\tau}_{(M-1)\hat{N}}, \hat{\tau}_{M\hat{N}})$ and sends them down to the lower layer LEMPC.
5. The lower layer LEMPC receives the optimal trajectories computed in the upper layer LEMPC and initializes the prediction horizon $N = \hat{\Delta}/\Delta$.
   5.1 At the lower layer sampling period τ_k, the lower layer LEMPC receives a state measurement $x(\tau_k)$.
   5.2 The total economic cost $\hat{L}^*_k$ with the upper layer LEMPC input trajectory is computed over the horizon τ_k to τ_{k+N} where
$$\hat{L}^*_k = \int_{\tau_k}^{\tau_{k+N}} l_e(\hat{x}^*(t), \hat{u}^*(t))\,dt, \tag{9}$$
   and the state at $\hat{x}^*(\tau_{k+N})$ with the upper layer LEMPC input trajectory is also computed.
   5.3 Using $\hat{L}^*_k$ and $\hat{x}^*(\tau_{k+N})$, the lower layer LEMPC computes its optimal input trajectory.
   5.4 The lower layer LEMPC sends the input $u^*(\tau_k)$ to the control actuators to be applied in a sample-and-hold fashion from τ_k to τ_{k+1}.
   5.5 If $N > 1$, $k \leftarrow k+1$, $N \leftarrow N-1$ and go to Step 5.1. Else if $N = 1$ and $\tau_{k+1} < \hat{\tau}_{M\hat{N}}$, $k \leftarrow k+1$, $N \leftarrow \hat{\Delta}/\Delta$ and go to Step 5.1. Else, go to Step 1, $M \leftarrow M+1$ and $k \leftarrow k+1$.

Remark 4. The lower layer LEMPC recomputes its solution at the beginning of each sampling period because the lower layer uses a smaller sampling period than the upper layer. Recomputing the lower layer EMPC input at every subsequent sampling time is necessary regardless if the solution to the lower level EMPC is the same or not. The incorporation of feedback allows for stabilization of open-loop unstable systems that cannot be accomplished with an open-loop implementation and ensures the robustness of the control solution with respect to infinitesimally small disturbances/uncertainty. For further explanation on this point, see, for example, Sontag (1998).

3.2. Formulation

In this subsection, the formulations of the auxiliary LMPC and the two LEMPC schemes are given. First, the closed-loop state trajectory over the Mth operating window ($\hat{\tau}_{(M-1)\hat{N}}$ to $\hat{\tau}_{M\hat{N}}$) under the explicit controller $h(x)$ is defined as the state trajectory $x_h(t)$ satisfying
$$\dot{x}_h(t) = f(x_h(t), u_h(t)) \tag{10}$$
initialized with a state measurement of the system $x_h(0) = x(\hat{\tau}_{(M-1)\hat{N}})$ and the input trajectory defined as
$$u_h(t) = h(x_h(\hat{\tau}_j)) \tag{11}$$
for all $t \in [\hat{\tau}_j, \hat{\tau}_{j+1})$ where $j = 0, 1, \ldots, \hat{N}-1$. Utilizing $x_h(t)$ and $u_h(t)$ according to Eqs. (10)–(11), the auxiliary LMPC solution is computed at the beginning of each operating window. The auxiliary LMPC is given by the following optimization problem:
$$\min_{v \in S(\hat{\Delta})} \int_0^{\hat{\tau}_{\hat{N}}} (|z(t)|_{Q_c} + |v(t)|_{R_c})\,dt \tag{12a}$$
$$\text{s.t. } \dot{z}(t) = f(z(t), v(t)) \tag{12b}$$
$$z(0) = x(\hat{\tau}_{(M-1)\hat{N}}) \tag{12c}$$
$$v(t) \in U, \quad \forall\, t \in [0, \hat{\tau}_{\hat{N}}) \tag{12d}$$
$$\frac{\partial V(z(\hat{\tau}_j))}{\partial z} f(z(\hat{\tau}_j), v(\hat{\tau}_j)) \le \frac{\partial V(x_h(\hat{\tau}_j))}{\partial x_h} f(x_h(\hat{\tau}_j), u_h(\hat{\tau}_j)), \quad j = 0, 1, \ldots, \hat{N}-1 \tag{12e}$$
where $z(t)$ is the state trajectory of the system with input trajectory $v(t)$ calculated by the auxiliary LMPC. The Lyapunov-based constraint of Eq. (12e) differs from the Lyapunov-based constraint of Eq. (6e) as it is imposed at each sampling period along the prediction horizon of the LMPC to ensure that the state trajectory with input computed by the LMPC converges to the steady-state. The optimal solution of Eq. (12) is denoted as $v^*(t)$ and is defined for $t \in [\hat{\tau}_{(M-1)\hat{N}}, \hat{\tau}_{M\hat{N}})$. From the optimal input trajectory, the optimal state trajectory $z^*(t)$ and the total economic cost $L^*_{\text{LMPC},M}$ (Eq. (8)) can be computed for the Mth operating window.

The formulation of the upper layer LEMPC for the Mth operating period is similar to the optimization problem of Eq. (7) formulated with the following additional constraints:
$$\int_{\hat{\tau}_{(M-1)\hat{N}}}^{\hat{\tau}_{M\hat{N}}} l_e(\hat{x}(t), \hat{u}(t))\,dt \ge L^*_{\text{LMPC},M}, \tag{13}$$
$$\hat{x}(\hat{\tau}_{M\hat{N}}) = z^*(\hat{\tau}_{M\hat{N}}), \tag{14}$$

where $\hat{x}$ is the predicted state trajectory with the input trajectory $\hat{u}$ computed by the upper layer LEMPC. The constraint of Eq. (13) guarantees that the total economic cost over the entire operating period with the input trajectory from the upper layer LEMPC is no less than that given by the total economic cost of the input trajectory from the auxiliary LMPC. To ensure the existence of an input trajectory that has at least as good economic performance as the auxiliary LMPC input trajectory over the entire length of operation, a terminal constraint based on the auxiliary LMPC is used (Eq. (14)). The terminal constraint differs from traditional terminal constraints because $z^*(\hat{\tau}_{M\hat{N}})$ is not necessarily the steady-state. The optimal solution to the optimization problem of the upper layer LEMPC is denoted as $\hat{u}^*(t)$ and is defined for $t \in [\hat{\tau}_{(M-1)\hat{N}}, \hat{\tau}_{M\hat{N}})$.

At each sampling period of the lower layer LEMPC, the total economic cost of the solution obtained from the upper layer is recomputed over the prediction horizon of the lower layer LEMPC and is defined by Eq. (9). The lower layer LEMPC uses $\hat{L}_k^*$ in its formulation and a terminal constraint which are given by:

$$\int_{\tau_k}^{\tau_{k+N}} l_e(\tilde{x}(t), u(t))\, dt \geq \hat{L}_k^*, \tag{15}$$

$$\tilde{x}(\tau_{k+N}) = \hat{x}^*(\tau_{k+N}) \tag{16}$$

where the performance-based constraints of Eqs. (15)–(16) are computed from the upper layer LEMPC, but have the same purpose as the constraints of Eqs. (13)–(14). The optimal solution to the lower layer LEMPC is denoted as $u^*(t)$ which is defined for $t \in [\tau_k, \tau_{k+N})$. The control input $u^*(\tau_k)$ is sent to the control actuators to be applied to the system of equation (1) in a sample-and-hold fashion until the next sampling period.

Remark 5. When each optimization problem can be solved to optimality, the terminal constraints of Eqs. (14) and (16) imply Eqs. (13) and (15) by the principle of optimality. Given the nonlinearity and likely non-convexity of each optimization problem, this may be difficult to guarantee. Even if the solver converges, it will likely return a local optimum. In the context of the theoretical developments of the present work the optimization solver is treated as a black box with a minimum requirement that it at least returns the higher level input solution (e.g., the auxiliary LMPC at worst returns the Lyapunov-based controller solution, the LEMPC at worst returns the auxiliary LMPC solution, etc.). Although the solution returned by the solver of the various optimization problems is denoted as the optimal solution, it does not need to be optimal for the results of the subsequent subsection to hold (stability and performance). This point is especially important considering that any optimization-based control algorithm may take a non-negligible time to solve for real-time applications. In other words, there is a (theoretical) maximum amount of computation time alloted to the optimization-based controller to ensure closed-loop stability. Our results hold for premature termination (i.e., when the solver returns a solution before it converges to the local optimum) as long as the solver returns a feasible solution which is always possible by design.

3.3. Closed-loop stability and performance

The following proposition proves that the closed-loop system state under the two-layer EMPC structure is always bounded in the invariant set Ω_ρ and the economic performance is at least as good as the closed-loop state with the auxiliary LMPC over each operating period.

Proposition 6. *Consider the system of equation (1) in closed-loop under the lower layer LEMPC with the performance constraints based on the upper layer LEMPC and the auxiliary LMPC of Eq. (12) and Lyapunov-based constraints based on the controller $h(x)$ that satisfies Assumption 1. Let $\hat{\Delta} \in (0, \Delta^*]$, $\hat{N} \geq 1$, $N \geq 1$, and $\Delta = \hat{\Delta}/N$. If $x(\hat{\tau}_{(M-1)\hat{N}}) \in \Omega_\rho$, then the state remains bounded in Ω_ρ over the entire operating period with $x(\hat{\tau}_{M\hat{N}}) = z^*(\hat{\tau}_{M\hat{N}}) \in \Omega_\rho$, the upper and lower LEMPCs and the auxiliary LMPC remain feasible for all $t \in [\hat{\tau}_{(M-1)\hat{N}}, \hat{\tau}_{M\hat{N}})$, and the following inequality holds:*

$$\int_{\hat{\tau}_{(M-1)\hat{N}}}^{\hat{\tau}_{M\hat{N}}} l_e(x(t), u^*(t))\, dt \geq \int_{\hat{\tau}_{(M-1)\hat{N}}}^{\hat{\tau}_{M\hat{N}}} l_e(z^*(t), v^*(t))\, dt. \tag{17}$$

Proof. If the optimization problems of the controllers remain feasible, stability (i.e., state remains bounded in Ω_ρ) follows (by construction of the LEMPCs). As a result of imposing the conditions of Eqs. (13) and (15) as constraints in the computed control actions, the inequality of Eq. (17) directly follows when all upper and lower layer LEMPCs and the auxiliary LMPC are feasible optimization problems. Since $z^*(t)$, $x(t) \in \Omega_\rho$ and $v^*(t)$, $u^*(t) \in U$ for all $t \in [\hat{\tau}_{(M-1)\hat{N}}, \hat{\tau}_{M\hat{N}})$ and $l_e(\cdot, \cdot)$ is continuous on $\Omega_\rho \times U$, both integrals of Eq. (17) are bounded.

The remaining part of the proposition focuses on the feasibility of the optimization problems. At the beginning of an operation period $\hat{\tau}_{(M-1)\hat{N}}$, the auxiliary LMPC of Eq. (12) is feasible if $x(\hat{\tau}_{(M-1)\hat{N}}) \in \Omega_\rho$ because the input trajectory obtained from the controller $h(x)$ when applied in a sample-and-hold fashion is a feasible solution to the auxiliary LMPC optimization problem. Feasibility of the upper layer LEMPC is maintained if $x(\hat{\tau}_{(M-1)\hat{N}}) \in \Omega_\rho$. At $\hat{\tau}_{(M-1)\hat{N}}$, the upper layer LEMPC receives the total economic cost and the terminal constraint from the auxiliary LMPC. Therefore, one feasible solution to the upper layer LEMPC is the auxiliary LMPC solution since it satisfies the performance constraint of Eq. (13) and the terminal constraint of Eq. (14).

Recursive arguments, utilizing the previously obtained solution, are used to construct a feasible solution for the lower layer LEMPC at each sampling period Δ. Given that the lower layer LEMPC is implemented with a shrinking horizon until the next sampling period $\hat{\Delta} \in (0, \Delta^*]$ of the upper layer LEMPC, the lower layer is always feasible because the upper layer LEMPC solution is a feasible solution to the lower layer LEMPC. At the beginning of the upper layer LEMPC sampling period $\hat{\Delta}$, there is a constant input $\hat{u}^*(\hat{\tau}_j)$ that forces the state from $\hat{x}(\hat{\tau}_j)$ to $\hat{x}(\hat{\tau}_{j+1})$. Namely, the upper layer LEMPC sampling period is divided into $N = \hat{\Delta}/\Delta$ subintervals corresponding to the sampling period Δ of the lower layer LEMPC. At $\tau_k = \hat{\tau}_j$, the constant input trajectory $u(t) = \hat{u}^*(\hat{\tau}_j)$ for $t \in [\tau_k, \tau_{k+N})$ is a feasible solution for the optimization problem since it satisfies the performance constraint of Eq. (15) and the terminal constraint of Eq. (16). If the lower layer LEMPC computes a different input trajectory, it must still force the state to $\hat{x}(\tau_{k+N})$ while satisfying the performance constraint. At the next sampling period τ_{k+1}, the prediction horizon decreases, so the previous solution defined for τ_{k+1} to τ_{k+N} is a feasible solution to the lower layer LEMPC. □

The following theorem provides sufficient conditions such that the two-layer EMPC structure maintains the closed-loop state inside the region Ω_ρ and the closed-loop economic performance is at least as good as if the auxiliary LMPC was applied to the system of equation (1) over the entire length of operation which may be finite or infinite.

Theorem 7. *Consider the system of equation (1) in closed-loop under the lower layer LEMPC with the performance constraints based on the upper layer LEMPC and the auxiliary LMPC of Eq. (6) and Lyapunov-based constraints based on the controller $h(x)$ that satisfies Assumption 1 and let the assumptions of Proposition 6 hold. If $x(0) \in \Omega_\rho$,* *then $x(t) \in \Omega_\rho$ for all $t \geq 0$ and the following inequality holds for finite-time operation:*

$$\int_0^T l_e(x(t), u^*(t))\, dt \geq \int_0^T l_e(z^*(t), v^*(t))\, dt \tag{18}$$

for any $T = i\hat{N}\hat{\Delta}$ where i is any positive integer, and the following inequality holds for infinite-time operation:

$$\liminf_{T\to\infty} \frac{1}{T}\int_0^T l_e(x(t), u^*(t))\, dt \geq l_e(x_s^*, u_s^*). \tag{19}$$

Proof. Applying the results of Proposition 6 recursively over M operating periods, recursive feasibility of the optimization problems follows, and the closed-loop state is always bounded in Ω_ρ if $x(0) \in \Omega_\rho$, and $x(\hat{\tau}_{j\hat{N}}) = z^*(\hat{\tau}_{j\hat{N}})$ for $j = 1, 2, \ldots, M$. To show the result of Eq. (18), the length of operation is divided into M operating periods and $T := \hat{\tau}_{M\hat{N}}$:

$$\int_0^T l_e(x(t), u^*(t))\, dt = \int_0^{\hat{\tau}_{\hat{N}}} l_e(x(t), u^*(t))\, dt + \cdots + \int_{\hat{\tau}_{(M-1)\hat{N}}}^{\hat{\tau}_{M\hat{N}}} l_e(x(t), u^*(t))\, dt. \tag{20}$$

By Proposition 6, the inequality of Eq. (17) holds over each operating window when $x(\hat{\tau}_{j\hat{N}}) = z^*(\hat{\tau}_{j\hat{N}})$, $j = 1, 2, \ldots, M$ and thus, the inequality of Eq. (18) follows.

Owing to the result of Eq. (18), the average finite-time economic cost is given by:

$$\frac{1}{T}\int_0^T l_e(x(t), u^*(t))\, dt \geq \frac{1}{T}\int_0^T l_e(z^*(t), v^*(t))\, dt \tag{21}$$

for any $T = i\hat{N}\hat{\Delta}$ where i is any positive integer. Recall, the economic cost function $l_e(\cdot,\cdot)$ is continuous on the compact set $\Omega_\rho \times U$ and $x(t), z^*(t) \in \Omega_\rho$ and $u^*(t),\ v^*(t) \in U$ for all $t \geq 0$. Thus, both integrals of Eq. (21) are bounded for any $T > 0$. As a result of the Lyapunov-based constraint of Eq. (12e) imposed on the input trajectory of the auxiliary LMPC and the fact that the system of equation (1) in closed-loop with the Lyapunov-based controller $h(x)$ implemented in a sample-and-hold fashion is exponentially stable for $\hat{\Delta} \in (0, \Delta^*]$, the state and input computed by the LMPC (i.e., $z^*(t)$ and $v^*(t)$) asymptotically converge to the steady-state (x_s^*, u_s^*).

If we consider the limit of the right-hand side of Eq. (21) as T tends to infinity, the limit exists and is equal to $l_e(x_s^*, u_s^*)$ owing to the fact that $z^*(t)$ and $v^*(t)$ asymptotically converge to optimal steady-state (x_s^*, u_s^*) while remaining bounded for all $t \geq 0$. To prove this limit, it is sufficient to prove that given any $\epsilon > 0$, there exists a $T^* > 0$ such that for $T > T^*$, the following holds (Amann & Escher, 2005):

$$\left| \frac{1}{T}\int_0^T l_e(z^*(t), v^*(t))\, dt - l_e(x_s^*, u_s^*) \right| < \epsilon. \tag{22}$$

Define $I(0, T)$ as the following integral:

$$I(0, T) := \int_0^T l_e(z^*(t), v^*(t))\, dt \tag{23}$$

where the arguments of I represent the lower and upper limits of integration, respectively. Since $z^*(t)$ and $v^*(t)$ asymptotically converge to x_s^* and u_s^*, respectively, as t tends to infinity, $l_e(z^*(t), v^*(t)) \to l_e(x_s^*, u_s^*)$ as t tends to infinity. Furthermore, $z^*(t) \in \Omega_\rho$ and $v^*(t) \in U$ for all $t \geq 0$, so for every $\epsilon > 0$, there exists a $\tilde{T} > 0$ such that

$$|l_e(z^*(t), v^*(t)) - l_e(x_s^*, u_s^*)| < \epsilon/2 \tag{24}$$

for $t \geq \tilde{T}$. For any $T > \tilde{T}$, we have:

$$\begin{aligned}
&\left|I(0, T) - Tl_e(x_s^*, u_s^*)\right| \\
&\quad = \left|I(0, \tilde{T}) + I(\tilde{T}, T) - Tl_e(x_s^*, u_s^*)\right| \\
&\quad \leq \int_0^{\tilde{T}} \left|l_e(z^*(t), v^*(t)) - l_e(x_s^*, u_s^*)\right| dt \\
&\qquad + \int_{\tilde{T}}^T \left|l_e(z^*(t), v^*(t)) - l_e(x_s^*, u_s^*)\right| dt \\
&\quad \leq \tilde{T}\tilde{M} + (T - \tilde{T})\epsilon/2
\end{aligned} \tag{25}$$

where $\tilde{M} := \sup_{t\in[0,\tilde{T}]} \left\{\left|l_e(z^*(t), v^*(t)) - l_e(x_s^*, u_s^*)\right|\right\}$. For any $T > T^* = 2\tilde{T}(\tilde{M} - \epsilon/2)/\epsilon$, the following inequality is satisfied:

$$\left|I(0, T)/T - l_e(x_s^*, u_s^*)\right| \leq (1 - \tilde{T}/T)\epsilon/2 + \tilde{T}\tilde{M}/T < \epsilon \tag{26}$$

which proves that the asymptotic average economic cost under the auxiliary LMPC is $l_e(x_s^*, u_s^*)$.

Considering the left hand side of Eq. (21), the limit as $T \to \infty$ may not exist owing to the possible time-varying system operation under the proposed two-layer LEMPC scheme. Therefore, a lower bound on the asymptotic average performance under the proposed LEMPC scheme is considered. Since the limit inferior is equal to the limit when the limit exists (Amann & Escher, 2005), we obtain:

$$\begin{aligned}
&\liminf_{T\to\infty} \frac{1}{T}\int_0^T l_e(x(t), u^*(t))\, dt \\
&\quad \geq \liminf_{T\to\infty} \frac{1}{T}\int_0^T l_e(z^*(t), v^*(t))\, dt = l_e(x_s^*, u_s^*)
\end{aligned} \tag{27}$$

which is the desired result of Eq. (19). □

Remark 8. Besides introducing a two-layer structure, the current work goes beyond what was presented in Heidarinejad et al. (2013). In fact, one cannot apply the results of Heidarinejad et al. (2013) recursively to get the results of the present work as guarantees on closed-loop performance improvement with LEMPC over a LMPC can only be made over each (individual) finite time operating window in Heidarinejad et al. (2013). Using the approach in Heidarinejad et al. (2013), one could guarantee improved closed-loop performance over some operating window $\hat{\tau}_0$ to $\hat{\tau}_{\hat{N}}$ and over the next operating window from $\hat{\tau}_{\hat{N}}$ to $\hat{\tau}_{2\hat{N}}$. No performance conclusion can be made if one were to consider the performance of the closed-loop system under the LMPC applied to the system from $\hat{\tau}_0$ to $\hat{\tau}_{2\hat{N}}$ compared to the closed-loop performance under the LEMPC from $\hat{\tau}_0$ to $\hat{\tau}_{2\hat{N}}$ because at the beginning of each operating window, both the auxiliary LMPC and the LEMPC are re-initialized with a state measurement obtained from the system under LEMPC. However, the closed-loop system under the auxiliary LMPC may have evolved to a different state at end of each operating window if the auxiliary LMPC was applied in a feedback control fashion to the system.

4. Application to the chemical process example

Consider a three vessel chemical process network consisting of two non-isothermal continuously stirred tank reactors (CSTRs) in series followed by a flash separator. In each of the reactors, an irreversible second-order reaction of the form $A \to B$ takes place in an inert solvent D (A is the reactant and B is the desired product). The bottom stream of the flash tank is the product stream of the process network. Part of the overhead vapor stream from the flash separator is purged from the process, while, the remainder is fully condensed and recycled back to the first reactor. Each of the vessels have a heating/cooling jacket to supply/remove heat from the liquid contents of the vessel. To simplify the notation, the following indices are used to refer to each vessel: $j = 1$ denotes CSTR-1, $j = 2$ denotes CSTR-2, and $j = 3$ denotes SEP-1. The heat rate supplied/removed from the jth vessel is Q_j, $j = 1, 2, 3$. Furthermore, each reactor is fed with fresh feedstock containing A in the solvent D with concentration C_{Aj0} where $j = 1, 2$. Applying first principles and standard modeling assumption, a ninth-order dynamic model of the reactor–separator process network can be obtained (neglecting the dynamics of the condenser). The ODEs and process parameters are given in Ellis et al. (2014).

There are nine state variables that are used to describe the evolution of the process network which includes vessel temperatures and vessel concentrations of A and B. The manipulated inputs to the system are the heat inputs to the three vessels, Q_1, Q_2, and Q_3, and the concentration of A in the inlet streams, C_{A10} and C_{A20}: $u^T = [Q_1\ Q_2\ Q_3\ C_{A10}\ C_{A20}]$. The control objective is to regulate the process in an economically optimal time-varying fashion to maximize the average amount of product B in the product stream F_3. We consider that the average amount of reactant material with time is fixed motivating the need to operate this process network under EMPC. In addition, supplying/removing heat to/from the vessels is considered undesirable. To accomplish these economic considerations, the proposed two-layer LEMPC structure is applied and the upper layer and lower layer LEMPCs are formulated with the following cost function and constraint, respectively:

$$l_e(x, u) = F_3 C_{B3} - A_1 Q_1^2 - A_2 Q_2^2 - A_3 Q_3^2 \tag{28}$$

$$\frac{1}{t_f} \int_{\hat{\tau}_{(M-1)\hat{N}}}^{\hat{\tau}_{M\hat{N}}} (C_{A10} + C_{A20})\, dt = 8.0 \text{ kmol m}^{-3} \tag{29}$$

where C_{B3} is the concentration of the desired product in the product stream and $t_f = 1.0$ h is the operating period length and $A_i = 10^{-6}$, $i = 1, 2, 3$ are the penalty weights for using energy. The value for the heat rate penalty has been chosen to account for the different numerical range of the heat rate and the first term in the economic cost. The economically optimal steady-state with respect to the economic cost function of Eq. (28) is open-loop asymptotically stable and is the only steady-state in the operating region of interest. Therefore, an explicit characterization of Ω_ρ is not needed for the LEMPC implementation.

The proposed two-layer LEMPC structure, formulated with the cost function and reactant material constraint of Eqs. (28)–(29), respectively, is applied to the reactor–separator chemical process network. To numerically integrate the dynamic model, the explicit Euler method is used with an integration step of 1×10^{-3} h. The prediction horizon and sampling period of the auxiliary LMPC and upper LEMPC are $\hat{N} = 10$ and $\hat{\Delta} = 0.1$ h, respectively, while, the lower layer LEMPC is formulated with a prediction horizon of $N = 2$ and sampling period $\Delta = 0.05$ h. To solve the optimization problems, Ipopt (Wächter & Biegler, 2006) was used and the simulations were completed on a desktop PC with an Intel® Core™ 2 Quad 2.66 GHz processor and a Linux operating system.

4.1. Two-layer LEMPC structure performance

In the first set of simulations, the proposed two layer LEMPC structure with performance-based constraints computed from an auxiliary LMPC is applied to the reactor–separator process network. Eight closed-loop simulations over a 4.0 h length of operation were completed. The closed-loop state and input trajectories of one of the simulations are shown in Figs. 2–3, respectively and demonstrate time-varying operation. The economic performance (integral of the economic cost over the length of operation) is compared to the economic performance with the auxiliary LMPC for each of these simulations. From this comparison, an average of about 10% benefit with the proposed two-layer LEMPC structure was realized over operation under the auxiliary LMPC. Additionally, a comparison between the computational time required to solve the two-layer LEMPC system and that of a one-layer LEMPC system was completed. The one-layer LEMPC system consists of the auxiliary LMPC and upper layer LEMPC where the upper layer LEMPC computes the control actions for the manipulated inputs of the system. To make the comparison consistent, the one layer LEMPC is implemented with a prediction horizon of $\hat{N} = 20$ and a sampling period of $\hat{\Delta} = 0.05$ h. Also, since the auxiliary LMPC and upper layer LEMPC are sequentially computed, the computational time at the beginning of each operating window is measured as the sum of the computational time to solve the auxiliary LMPC, the upper layer LEMPC, and the lower layer LEMPC for the proposed two-layer LEMPC system and the sum of the time to solve the auxiliary LMPC and the LEMPC for the one-layer LEMPC system. The one-layer LEMPC achieves slightly better closed-loop economic performance (less than a 1% improvement) owing to a smaller sampling period than the upper layer LEMPC in the two-layer LEMPC structure. However, the computational time required to solve the one-layer LEMPC structure is significantly greater, and the proposed two-layer LEMPC structure is able to reduce the computational time by 75% on average.

4.2. Handling disturbances

While the two-layer EMPC has been designed for nominal operation to merge guaranteed finite-time and infinite-time closed-loop performance over conventional MPC, it may be applied to the process model in the presence of disturbances, plant/model mismatch, and other uncertainties with some modifications to improve recursive feasibility of the optimization problems and to ensure greater robustness of the controller to these uncertainties. For instance, if the disturbances are relatively small, it may be sufficient to relax the performance-based constraints or treat the performance constraints as soft constraints. If one were to simply relax the performance-based constraints (e.g., use a terminal region instead of a point-wise terminal constraint), it is difficult to guarantee recursive feasibility of the optimization problem as is the case with any type of MPC formulated with terminal constraints. Another methodology is to use the performance-based constraints in the cost function, that is use a cost of the form

$$\begin{aligned} l_e(x, u) = \frac{-\alpha}{N} \left(\int_{\tau_k}^{\tau_{k+N}} l_e(\tilde{x}(t), u(t))\, dt \right) \\ + \beta \left| \tilde{x}(\tau_{k+N}) - \hat{x}^*(\tau_{k+N}) \right|_Q \end{aligned} \tag{30}$$

which the lower layer LEMPC works to minimize, where α and β are tuning parameters. This cost function works to optimize the economic performance while ensuring the predicted evolution is near the terminal state through the quadratic terminal cost. The resulting lower layer LEMPC has all the same stability and robustness to bounded disturbances properties as the LEMPC of Heidarinejad et al. (2012) (i.e., recursive feasibility and boundedness of the closed-loop state for all initial states starting in Ω_ρ). In the presence of disturbances, we can only guarantee closed-loop stability in the presence of disturbances and no provable guarantees can be made on closed-loop performance. However, the closed-loop performance benefit can be evaluated through simulations.

The two-layer LEMPC with the lower layer LEMPC designed with the cost described above in Eq. (30) and no performance-based constraints was applied to the example with significant process noise added. The noise was modeled as bounded Gaussian white noise and was introduced additively to each model state over each sampling period. The closed-loop state and input trajectories are shown in Figs. 4 and 5, respectively. We compare the

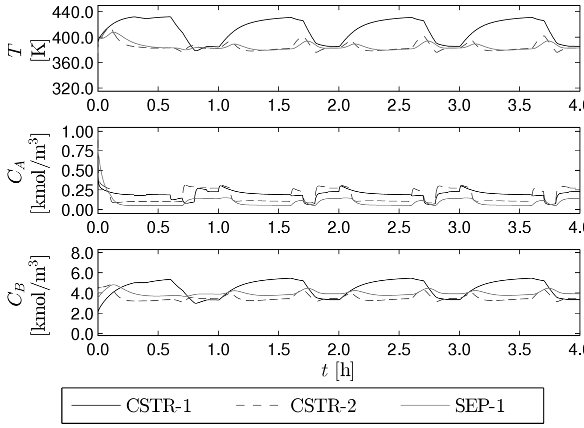

Fig. 2. Closed-loop state trajectories of the reactor–separator process network with the proposed two-layer LEMPC structure.

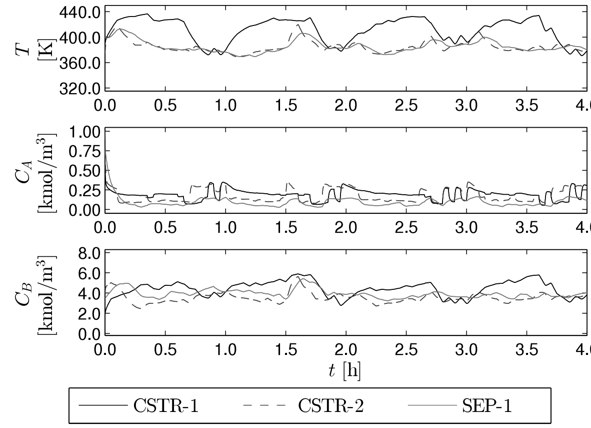

Fig. 4. Closed-loop state trajectories of the reactor–separator process network with process noise added with the proposed two-layer LEMPC structure.

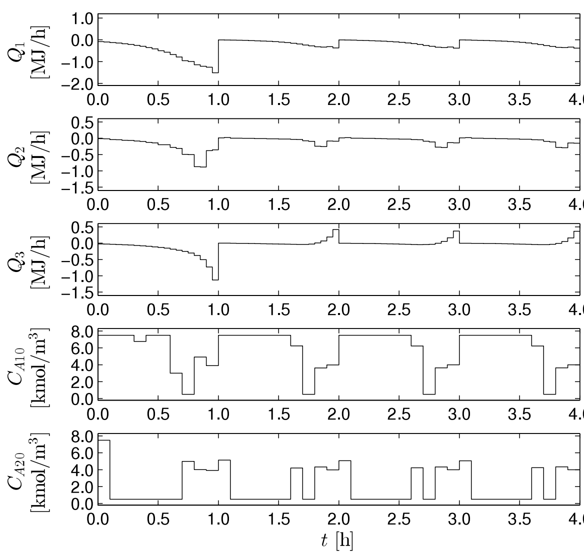

Fig. 3. Input trajectories of the reactor–separator process network computed by the two-layer LEMPC structure.

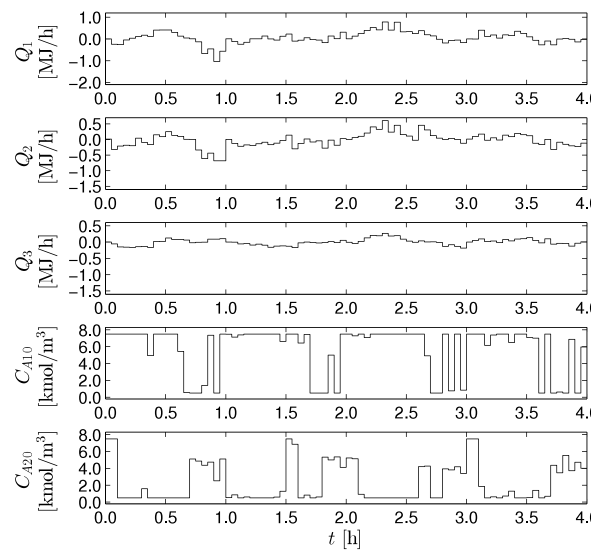

Fig. 5. Input trajectories of the reactor–separator process network with process noise added computed by the two-layer LEMPC structure.

closed-loop system performance under the two-layer LEMPC to the system with the same realization of the process noise under conventional MPC. Specifically, an LMPC is formulated with a prediction horizon of $N = 2$ and $\Delta = 0.05$ h which is the same as the lower layer LEMPC. The closed-loop performance under the two-layer LEMPC is 2.6% better than that under the LMPC for this particular realization of the process noise. More simulation results can be found in Ellis et al. (2014).

5. Conclusion

In this work, guaranteed closed-loop performance under economic model predictive control (EMPC) over finite-time and infinite-time operation of a nonlinear system was considered. Owing to the dependence of prediction horizon length on closed-loop economic performance with EMPC, a two-layer Lyapunov-based EMPC was proposed to effectively divide dynamic optimization and feedback control tasks and thus, ease the computational burden of the lower (feedback) layer LEMPC responsible for process control (stability and robustness). In the proposed two-layer LEMPC structure, performance and terminal constraints are generated by an auxiliary LMPC and then, imposed on the LEMPC optimization problems leading to guaranteed closed-loop economic performance improvement under LEMPC over the auxiliary LMPC. The two-layer LEMPC structure was applied to a chemical process network to demonstrate the closed-loop stability, performance, robustness, and computational efficiency properties of the proposed two-layer EMPC structure.

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