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Algorithms for improved fixed-time performance of Lyapunov-based economic model predictive control of nonlinear systems

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ABSTRACT

This work presents algorithms for improved fixed-time performance of Lyapunov-based economic model predictive control (LEMPC) of nonlinear systems. Unlike conventional Lyapunov-based model predictive control (LMPC) schemes which typically utilize a quadratic cost function and regulate a process at a steady-state, LEMPC designs very often dictate time-varying operation to optimize an economic (typically non-quadratic) cost function. The LEMPC algorithms proposed here utilize a shrinking prediction horizon with respect to fixed (but potentially large) operation period to ensure improved performance, measured by the desired economic cost, over conventional LMPC by solving auxiliary LMPC problems and incorporating appropriate constraints, based on the LMPC solution, in their formulations at various sampling times. The proposed LEMPC schemes also take advantage of a predefined Lyapunov-based explicit feedback law to characterize their stability region while maintaining the closed-loop system state in an invariant set subject to bounded process disturbances. The LEMPC algorithms are demonstrated through a nonlinear chemical process example.

1. Introduction

The development of optimal operation and control policies for chemical process systems aiming at optimizing process economics has always been an important research subject with major practical implications. Generally, economic considerations are addressed via a two-layer approach in which the upper layer carries steady-state process optimization to obtain economically optimal process operation set-points (steady-states) while the lower layer employs appropriate feedback control laws to steer the process state to the economically optimal steady-state operating point computed by the upper layer. Model predictive control (MPC) is widely utilized in the process control layer to provide optimal manipulated input values by minimizing a (typically) quadratic cost function which usually penalizes the deviation of the system state and manipulated inputs from their economically-optimal steady-state values subject to input and state constraints [4]. This two-layer approach typically restricts process operation to a steady-state. In order to account for general economic optimization considerations, the quadratic cost function used in conventional MPC should be replaced by an economics-based cost function which may be non-convex and may result in a time-varying process operation in order to be optimized over a fixed time window. Consequently, the conventional MPC scheme, where an economics-based cost function is used, should be re-formulated in an appropriate way to guarantee closed-loop stability. With respect to recent results on conventional MPC, efforts have focused on combination of steady-state optimization and linear MPC [6], stability of economic MPC of nonlinear systems through employing a terminal constraint which requires that the closed-loop system state settles to a steady-state at the end of the prediction horizon [5] and economic MPC of cyclic processes (including closed-loop stability analysis using a suitable terminal constraint) [10]. The work in [8] formulated a linear robust economic MPC by taking advantage of second-order cone programming. The work in [11] considered a cooperative distributed linear economic MPC scheme subject to convex economic objectives. Simultaneous consideration of economic and control performance in a single layer architecture has been studied in [14]. The use of economic MPC for the reduction of energy related costs using an economic cost accounting for time-of-use energy charge and demand charge has been considered in [13]. Economic performance of MPC for a simulated electric arc furnace has been studied in [19]. Furthermore, economic considerations of MPC were addressed in [16] through the addition of a nonlinear term related to the economic objective in a conventional quadratic cost. Robust stability of economically oriented nonlinear MPC with infinite prediction horizon for cyclic processes has been considered in [9]. In a previous work [7], we presented a two-mode Lyapunov-based economic

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MPC (LEMPC) design for nonlinear systems which is also capable of handling asynchronous and delayed measurements and extended it in the context of distributed MPC [2]. Despite the above recent progress, at this point, there is limited work to ensure improvement of closed-loop performance through time-varying operation via economic MPC with respect to operation under conventional MPC in the context of fixed-time operation. An important recent work has established improved economic MPC performance over steady-state operation for infinite-time operation [1].

Motivated by the lack of available methodologies to guarantee performance of economic MPC for fixed-time (in a sense to be made precise below) operation, this work presents two Lyapunov-based economic model predictive control (LEMPC) algorithms for nonlinear systems which are capable of optimizing closed-loop performance with respect to a general objective function that directly addresses economic considerations. The LEMPC algorithms proposed in this work utilize a shrinking prediction horizon with respect to a fixed operation period to ensure improved performance, measured by the desired economic cost, over conventional LMPC by solving auxiliary LMPC problems and incorporating appropriate constraints in their formulations at various sampling times. The proposed LEMPC schemes may dictate time-varying process operation, take advantage of a predefined Lyapunov-based explicit feedback law to characterize their stability region while maintaining the closed-loop system state in an invariant set subject to bounded process disturbances. The LEMPC algorithms are applied to a chemical process example and improved average economic performance, both in the nominal case and under disturbances, with respect to conventional LMPC schemes is demonstrated for a broad set of initial conditions.

2. Preliminaries

2.1. Notation

The notation $|\cdot|$ is used to denote the Euclidean norm of a vector. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. The symbol Ω_r is used to denote the set $\Omega_r := \{x \in R^{n_x} : V(x) \leq r\}$ where V is a continuously differentiable, positive definite scalar function and $r > 0$, and the operator '\' denotes set subtraction, that is, $A \setminus B := \{x \in R^{n_x} : x \in A, x \notin B\}$. The symbol $diag(v)$ denotes a matrix whose diagonal elements are the elements of vector v and all the other elements are zeros.

2.2. Class of nonlinear systems

We consider a class of nonlinear systems which can be described by the following state-space model:

$$\dot{x}(t) = f(x(t), u(t), w(t)) \tag{1}$$

where $x \in R^{n_x}$ denotes the vector of state variables of the system, and $u \in R$ and $w \in R^{n_w}$ denote the control (manipulated) input and the disturbance vector, respectively. The control input is restricted to be in a nonempty convex set $U \subseteq R$, which is defined as $U := \{u \in R : |u| \leq u^{\max}\}$ where $u^{\max}$ is the magnitude of the input constraint. The disturbance $w \in R^{n_w}$ is bounded, i.e., $w \in W$ where $W := \{w \in R^{n_w} : |w| \leq \theta, \theta > 0\}$. We assume that f is a locally Lipschitz vector function and that the origin is an equilibrium point of the unforced nominal system (i.e., the system of Eq. (1) with $u(t) \equiv 0$ and $w(t) \equiv 0$ for all times) which implies that $f(0, 0, 0) = 0$. We assume that the state x of the system is sampled synchronously and the time instants at which we have state measurements are indicated by the time sequence $\{t_{k\geq 0}\}$ with $t_k = t_0 + k\Delta$, $k = 0, a, \ldots$ where t_0 is the initial time and Δ is the sampling time.

2.3. Stabilizability assumption

We assume that there exists a Lyapunov-based controller $u = h(x)$ locally Lipschitz in x which satisfies the input constraints for all the states x inside a given stability region and makes, under continuous, state-feedback implementation, the origin of the closed-loop system asymptotically stable. Using converse Lyapunov theorems [3,12], this assumption implies that there exist class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ and a continuously differentiable Lyapunov function $V(x)$, that satisfy the following inequalities:

$$\begin{aligned} &\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \\ &\frac{\partial V(x)}{\partial x} f(x, h(x), 0) \leq -\alpha_3(|x|) \\ &\left|\frac{\partial V(x)}{\partial x}\right| \leq \alpha_4(|x|) \\ &h(x) \in U \end{aligned} \tag{2}$$

for all $x \in D \subseteq R^{n_x}$ where D is an open neighborhood of the origin. We represent the region $\Omega_\rho \subseteq D$ as the stability region of the closed-loop system under the controller $h(x)$. Using the Lipschitz property assumed for f, and taking into account that the manipulated input u and the disturbance vector w are bounded, there exists a positive constant M such that

$$|f(x, u, w)| \leq M \tag{3}$$

for all $x \in \Omega_\rho$, $u \in U$ and $w \in W$. In addition, by the continuous differentiability property of the Lyapunov function $V(x)$ and the Lipschitz property assumed for the vector field f, there exist positive constants L_x, L_w, L'_x and L'_w such that

$$\begin{aligned} &\left|f(x, u, w) - f(x', u, 0)\right| \leq L_x \left|x - x'\right| + L_w |w| \\ &\left|\frac{\partial V(x)}{\partial x} f(x, u, w) - \frac{\partial V(x')}{\partial x} f(x', u, 0)\right| \leq L'_x \left|x - x'\right| + L'_w |w| \end{aligned} \tag{4}$$

for all $x, x' \in \Omega_\rho$, $u \in U$ and $w \in W$.

Remark 1. It should be emphasized that in the current work, we take advantage of the level set Ω_ρ of the Lyapunov function $V(x)$ to estimate the stability region (i.e., domain of attraction) of the closed-loop system under the feedback controller $h(x)$. Specifically, computation of an estimation of the domain of attraction of the closed-loop system proceeds as follows: first, a controller (e.g., $h(x)$) is designed that makes the time-derivative of a Lyapunov function, $V(x)$, along the closed-loop system trajectory negative definite in an open neighborhood around the equilibrium point; then, an estimate of the set where $\dot{V}$ is negative is computed, and finally, a level set (ideally the largest) of V (denoted by Ω_ρ in the present work) embedded in the set where $\dot{V}$ is negative, is computed; see Section 5 for an application of this approach.

2.4. Lyapunov-based MPC

The Lyapunov-based MPC (LMPC) design [15] inherits the closed-loop stability properties of the Lyapunov-based controller $h(\cdot)$ when it is applied in a sample and hold fashion. Specifically, using a conventional quadratic cost function

$$L_s(x(t), u(t)) = x^T(t)Qx(t) + u^T(t)Ru(t) \tag{5}$$

and the following sampled state trajectory $x_h(t)$ when the Lyapunov-based controller $h(\cdot)$ is applied in a sample-and-hold fashion

$$\begin{aligned} &\dot{x}_h(t) = f(x_h(t), h(x_h(t_k + l\Delta)), 0),\ t_k + l\Delta \leq t < t_k + (l+1)\Delta, \\ &\quad l = 0, \ldots, N-1 \end{aligned} \tag{6}$$

the LMPC at sampling time t_k is formulated as follows

$$\min_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} L_s(\tilde{x}(\tau), u(\tau)) d\tau \tag{7a}$$

$$\text{s.t.}\ \dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), u(\tau), 0) \tag{7b}$$

$$u(\tau) \in U,\ \tau \in [t_k, t_{k+N}) \tag{7c}$$

$$\tilde{x}(t_k) = x(t_k) \tag{7d}$$

$$\frac{\partial V(x(t_j))}{\partial x} f(x(t_j), u(t_j), 0) \le \frac{\partial V(x_h(t_j))}{\partial x_h} f(x_h(t_j), h(x_h(t_j)), 0),\quad j = k, \ldots, k+N-1 \tag{7e}$$

where $\tilde{x}$ is the predicted state trajectory of the system with control input calculated by this LMPC, $S(\Delta)$ is the set of piecewise constant functions with period Δ, N is the finite prediction horizon and Q and R are positive definite weighting matrices. Eq. (7b) utilizes a nominal system model to predict the future evolution of the system state with initialization by sampled state feedback at time t_k (Eq. (7d)) and Eq. (7c) denotes the constraint on the manipulated input. The Lyapunov-based constraint of Eq. (7e) guarantees that the amount of reduction in the Lyapunov function when we apply the input computed by the LMPC is at least at the level when the Lyapunov-based controller $h(\cdot)$ is applied in a sample and hold fashion. It should be emphasized that in traditional LMPC [15], Eq. (7e) holds only for the sampling time t_k; however, in the present work, for reasons that will become clear in Section 3 below, this constraint is enforced over N sampling times. The LMPC inherits the closed-stability properties of $h(\cdot)$. For a detailed closed-loop stability analysis, in the context of receding horizon control applied on an unbounded time interval, please refer to [4,15].

2.5. Lyapunov-based economic MPC

Lyapunov-based economic MPC (LEMPC) includes an economic cost function $L_e(x(t), u(t))$ in its formulation which may directly address economic considerations and it does not necessarily take its optimum value at the steady-state point corresponding to the LMPC formulation of Eq. (7) (taken to be the origin in this work for simplicity). This LEMPC characteristic requires reformulation of the conventional LMPC to address possibly time-varying operation instead of steady-state operation achieved by the LMPC of Eq. (7); however, the closed-loop stability region needs to be precisely characterized. Recently, in [7], we proposed an LEMPC scheme through taking advantage of the properties of the Lyapunov-based controller $h(x)$. Specifically, the LEMPC was formulated as follows:

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_{k+N}} L_e(\tilde{x}(\tau), u(\tau)) d\tau \tag{8a}$$

$$\text{s.t.}\ \dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), u(\tau), 0) \tag{8b}$$

$$u(\tau) \in U,\ \tau \in [t_k, t_{k+N}) \tag{8c}$$

$$\tilde{x}(t_k) = x(t_k) \tag{8d}$$

$$V(\tilde{x}(t)) \le \rho,\quad \forall t \in [t_k, t_{k+N}) \tag{8e}$$

The constraint of Eq. (8e) maintains the predicted system state along the prediction horizon in the invariant set Ω_ρ where Ω_ρ has been defined in Section 2.3, and within this set, the LEMPC addresses economic considerations by optimizing the economic cost function of Eq. (8a). It should be emphasized that the control design in [7] deals with the general concept of economic MPC which achieve boundedness of the closed-loop system state as well as steady-state operation through utilizing a two-mode operation and through simulations we demonstrated that the economic MPC of [7] may outperform steady-state operation; however, the current work deals with guaranteed performance improvement from a theoretical point of view with respect to the LMPC (with quadratic cost) operation. Refer to [7] for a detailed description and analysis of the LEMPC formulation of Eq. (8) in the context of receding horizon control applied on an unbounded time interval.

3. LEMPC Algorithm I: nominal operation

In this section, we consider the design of Lyapunov-based economic MPC (LEMPC) for nonlinear systems under nominal operation (i.e., $w(t) \equiv 0$). We propose a finite prediction horizon LEMPC formulation which leads to improvement in economic closed-loop performance compared to a conventional steady-state LMPC operation for a given initial condition and a fixed-time interval of operation. The proposed scheme at the first stage solves an auxiliary LMPC problem and then through different sampling times, it incorporates the solution of this LMPC problem to the LEMPC formulation. Specifically, we define the manipulated input of the LMPC design of Eq. (7) which is only evaluated at sampling time t_0 as follows:

$$u_{LMPC}(t) = u^*(t|t_0),\quad \forall t \in [t_0, t_N). \tag{9}$$

For simplicity, we assume that $\Delta = (t_N - t_0)/N$. Subsequently, let the state trajectory $x_{LMPC}(t)$ be defined as follows:

$$\dot{x}_{LMPC}(t) = f(x_{LMPC}(t), u_{LMPC}(t), 0),\quad \forall t \in [t_0, t_N) \tag{10}$$

which is the system state trajectory if the manipulated input obtained through the LMPC of Eq. (7) is applied. Also, we define

$$\bar{u}_{LMPC} = \int_{t_0}^{t_N} u_{LMPC}(\tau) d\tau \tag{11}$$

as the overall amount of control action utilized by the LMPC of Eq. (7) over a finite prediction horizon N. Subsequently, let us define

$$c_{LMPC} = \int_{t_0}^{t_N} L_e(x_{LMPC}(\tau), u_{LMPC}(\tau)) d\tau \tag{12}$$

as the overall value of the corresponding economic cost function when we apply the LMPC solution obtained at sampling time t_0 over the period $[t_0, t_N)$. The purpose of the LEMPC design discussed below is to obtain an optimal manipulated input trajectory which uses the same amount of control action obtained by the LMPC of Eq. (7), while improving the economic cost function value over the process operation period $[t_0, t_N)$; this allows a consistent comparison of the economic cost under LEMPC and LMPC for the case where the control action is not penalized in the economic cost as in the example in Section 5.

3.1. Implementation strategy

At time t_0, first the LMPC obtains its manipulated input trajectory over $[t_0, t_N)$ based on state feedback $x(t_0)$. After computing the amount of control action and economic cost over time $[t_0, t_N)$ induced by the LMPC state and manipulated input trajectories using Eqs. (11) and (12), respectively, the LEMPC obtains its optimal manipulated input trajectory using a shrinking horizon approach. At each sampling time t_k, based on $x(t_k)$, the LEMPC takes advantage of the nominal system model to predict the future state of the system over a finite prediction horizon while maximizing an economic cost function. A schematic diagram of the proposed LEMPC design is depicted in Fig. 1. The implementation strategy of the proposed LEMPC can be summarized as follows:

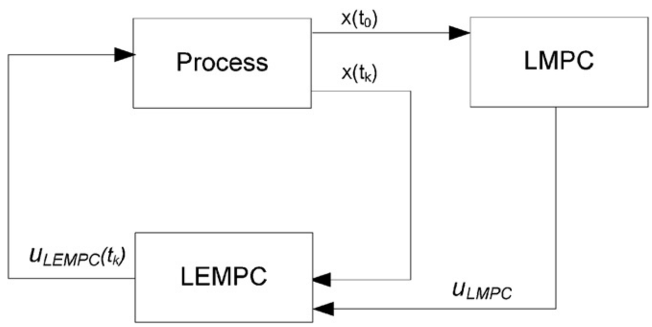

Fig. 1. LEMPC architecture under nominal operation.

1. At time t_0, an auxiliary LMPC optimization problem of Eq. (7) is solved based on state measurement $x(t_0)$.
2. The amount of control action used by the LMPC and its corresponding economic cost over $[t_0, t_N)$ are computed using Eqs. (11) and (12), respectively.
3. The LEMPC receives state feedback $x(t_k)$ $(k = 0, \ldots, N-1)$ and solves an optimization problem with prediction horizon $N_k = N - k$.
4. The LEMPC sends the first step of its optimal solution to the control actuators.
5. Go to Step 3 $(k \longleftarrow k+1)$.

3.2. LEMPC formulation

The LEMPC is evaluated to obtain the future input trajectories based on state feedback $x(t_k)$ at sampling time t_k. Specifically, the optimization problem of the proposed LEMPC under nominal operation is as follows:

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_N} L_e(\tilde{x}(\tau), u(\tau))d\tau \tag{13a}$$

$$\text{s.t. } \dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), u(\tau), 0) \tag{13b}$$

$$u(\tau) \in U,\ \tau \in [t_k, t_N) \tag{13c}$$

$$\tilde{x}(t_k) = x(t_k) \tag{13d}$$

$$V(\tilde{x}(t)) \leq \rho, \quad \forall t \in [t_k, t_N) \tag{13e}$$

$$\int_{t_k}^{t_N} u(\tau)d\tau = \bar{u}_{LMPC} - \bar{u}_k \tag{13f}$$

$$\int_{t_k}^{t_N} L_e(\tilde{x}(\tau), u(\tau))d\tau \geq c_{LMPC} - c_k \tag{13g}$$

The constraint of Eq. (13e) restricts the predicted system state to be in the set Ω_ρ. The constraint of Eq. (13f) ensures that the same amount of control action is used by both the LMPC of Eq. (7) and the LEMPC of Eq. (13) in the time interval $[t_0, t_N)$. The constraint of Eq. (13g) guarantees that the economic cost function value over the time interval $[t_0, t_N)$ is at least at the level achieved when we use the state and manipulated input trajectory obtained through the LMPC of Eq. (7) considering the optimal manipulated inputs obtained by the LEMPC at previous sampling times (see also Eqs. (15) and (16) below). It should be mentioned that through this implementation strategy, LEMPC utilizes a decreasing sequence of finite prediction horizons $N_k = N - k$ $(k = 0, \ldots, N-1)$ where N is the horizon of LMPC optimization problem which is solved at sampling time t_0 and its solution is incorporated at the LEMPC formulation. The optimal solution to this optimization problem is denoted by $u^*(t|t_k)$, which is defined for $t \in [t_k, t_N)$. The manipulated input of the LEMPC of Eq. (13) is defined as follows:

$$u_{LEMPC}(t) = u^*(t|t_k), \quad \forall t \in [t_k, t_N) \tag{14}$$

Based on the LEMPC solution, we have

$$\bar{u}_k = \int_{t_0}^{t_k} u_{LEMPC}(\tau)d\tau \tag{15}$$

and

$$c_k = \int_{t_0}^{t_k} L_e(\tilde{x}(\tau), u_{LEMPC}(\tau))d\tau \tag{16}$$

which are incorporated in the LEMPC of Eq. (13) to account for the optimal manipulated input obtained at previous sampling times t_j where $j = 0, 1, \ldots, k-1$.

Remark 2. We note that we can formulate the LEMPC in a slightly different manner by removing the equality constraint of Eq. (13f) and allowing the LEMPC to use more (or even less) control energy than the LMPC in order to optimize the economic cost further; however, this may lead to an inconsistent comparison between the two approaches, particularly, in the case where the control action is not penalized in the economic cost as it is the case in the chemical process example considered in Section 5. It should be emphasized that without the constraint of Eq. (13f), we can still prove closed-loop stability as well as performance results. Furthermore, the constraint of Eq. (13g) guarantees improvement in the economic cost function compared to the solution provided by LMPC. This constraint guarantees feasibility of the LEMPC optimization problem as well as improvement in economic cost function value. Without this constraint it is not guaranteed to improve the economic cost function via LEMPC over LMPC, given the non-convexity of the cost function.

3.3. Closed-loop stability and performance

Theorem 1 below provides sufficient conditions under which the LEMPC of Eq. (13) guarantees that the state of the closed-loop system of Eq. (1) is always bounded in an invariant set and it yields a closed-loop economic cost that is as good or superior to the one of the LMPC for finite-time operation over $[t_0, t_N)$.

Theorem 1. *Consider the system of Eq. (1) in closed-loop under the LEMPC of Eq. (13) based on a controller $h(x)$ that satisfies the conditions of Eq. (2). Let $\epsilon_w > 0$, $\Delta > 0$ and $\rho > \rho_s > 0$ satisfy*

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + L'_x M \Delta \leq -\frac{\epsilon_w}{\Delta}. \tag{17}$$

If $x(t_0) \in \Omega_\rho$, then the state $x(t)$ of the closed-loop system is bounded in $\Omega_\rho\ \forall t \in [t_0, t_N)$ and

$$\int_{t_0}^{t_N} L_e(x_{LEMPC}(\tau), u_{LEMPC}(\tau))d\tau \geq \int_{t_0}^{t_N} L_e(x_{LMPC}(\tau), u_{LMPC}(\tau))d\tau \tag{18}$$

Proof. For the optimization problem of the LMPC of Eq. (7) when $x(t_0) \in \Omega_\rho$, $u(\tau) = h(\tilde{x}(\tau))$, $\forall \tau \in [t_0, t_N)$ is a feasible solution. For the LEMPC optimization problem of Eq. (13), at sampling time t_0, the solution of the LMPC of Eq. (7) $(u_{LMPC}(\tau), \forall \tau \in [t_0, t_N))$ is also a feasible solution. This solution satisfies the input constraint of Eq. (8c) through the stabilizability assumption of Eq. (2) and the Lyapunov-based stability constraint of Eq. (7e). At sampling time t_k where $k > 0$, the last $N_{k-1} - 1$ steps of the optimal solution of the LEMPC at sampling time t_{k-1} (i.e., $u^*(t|t_{k-1}), \forall t \in [t_k, t_N)$) is a feasible solution to the LEMPC of Eq. (13) at t_k due to the fact that there is no process disturbance. Furthermore, through the enforcement of the constraint of Eq. (13e) and also satisfaction of the Eq. (17), it has been proved in [7] that the closed-loop system state is bounded within Ω_ρ.

To be consistent in the comparison from an economic closed-loop performance point of view between the LMPC of Eq. (7) and the LEMPC of Eq. (13), we choose a decreasing sequence of prediction horizons to ensure that both formulations have been evaluated over time interval $[t_0, t_N)$. Considering the constraint of Eq. (13g) at sampling time t_{N-1}, we can write

$$\int_{t_{N-1}}^{t_N} L_e(\tilde{x}(\tau), u(\tau))d\tau \geq c_{LMPC} - c_{N-1} \tag{19}$$

Replacing c_{LMPC} and c_{N-1} using Eqs. (12) and (16) in Eq. (19), we can obtain Eq. (18) where the state trajectory $x_{LEMPC}(t)$ is defined as follows:

$$\dot{x}_{LEMPC}(t) = f(x_{LEMPC}(t), u_{LEMPC}(t), 0), \quad \forall t \in [t_0, t_N) \tag{20}$$

□

Remark 3. Theorem 1 essentially says that solving the 'economic MPC' problem over a fixed time interval produces an economically equal or superior solution to solving the 'LMPC stabilization' problem under the same initial conditions. However, this does not mean that there is no need to solve the LEMPC problem at each t_k during nominal operation. The reason for resolving this problem at each sampling time (albeit with a shrinking horizon) is the incorporation of *feedback* through the use of the state measurement in the controller; this incorporation of feedback ensures the robustness of the control solution with respect to infinitesimally small but unmodeled plant/model mismatch (that goes beyond the specific disturbance model included in Section 4 below) as well as the stabilization of open-loop unstable systems that can not be accomplished with an open-loop implementation (when the LEMPC problem is solved only in the beginning of the interval and it is implemented thereafter) of the LEMPC solution.

Remark 4. It should be emphasized that the LEMPC architecture proposed in [7] employs a two-mode operation where at the first mode it deals with addressing economic considerations by maintaining the system state in an invariant set and at the second mode it focuses on convergence to a small invariant set around a steady-state by incorporating an appropriate Lyapunov-based constraint. Since in this work, we focus on economic closed-loop performance, we only described the proposed LEMPC designs in the context of mode one operation in [7] by evaluating both the LMPC and the LEMPC over a finite-time operation interval $[t_0, t_N)$. Note that, depending on the application and certain specifications, the LEMPC of Eq. (13) can operate at mode 2 after time t_N to achieve practical closed-loop stability (i.e., ultimate convergence of the closed-loop state to a small invariant set including the origin). We deal with applying the LEMPC in the interval $[t_N, t_f)$ where t_f denotes the final time of LEMPC evaluation and the time the closed-loop system state enters a small invariant set around the origin. In a similar manner, first the LMPC is evaluated at sampling time t_N and the corresponding input and economic cost based constraints (Eqs. (13f) and (13g)) are obtained and are incorporated in the formulation of the LEMPC at mode 2 at subsequent sampling times t_k where $k \geq N$. In terms of the LEMPC formulation at mode 2, it includes the Lyapunov-based constraint of Eq. (7e) to address closed-loop stability instead of the constraint of Eq. (13e) which deals with maintaining the closed-loop system state in an invariant set for economic optimization purposes. Specifically, at sampling time t_N the auxiliary LMPC problem of Eq. (7) is solved with $t_0 = t_N$ and $t_N = t_f$. From the solution of this problem, we can obtain $u_{LMPC}(t) = u^*(t|t_N)$, $\dot{x}_{LMPC}(t) = f(x_{LMPC}(t), u_{LMPC}(t), 0)$, $\forall t \in [t_N, t_f)$, $\bar{u}_{LMPC} = \int_{t_N}^{t_f} u_{LMPC}(\tau)d\tau$ and $c_{LMPC} = \int_{t_N}^{t_f} L_e(x_{LMPC}(\tau), u_{LMPC}(\tau))d\tau$. Using a decreasing sequence of finite prediction horizons $N_k = (t_f - t_k)/\Delta$, the LEMPC at mode 2 at sampling time t_k where $k \geq N$ is formulated as follows

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_f} L_e(\tilde{x}(\tau), u(\tau))d\tau \tag{21a}$$

$$\text{s.t.}\ \dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), u(\tau), 0) \tag{21b}$$

$$u(\tau) \in U,\ \tau \in [t_k, t_f) \tag{21c}$$

$$\tilde{x}(t_k) = x(t_k) \tag{21d}$$

$$\int_{t_k}^{t_f} u(\tau)d\tau = \bar{u}_{LMPC} - \bar{u}_k \tag{21e}$$

$$\int_{t_k}^{t_f} L_e(\tilde{x}(\tau), u(\tau))d\tau \geq c_{LMPC} - c_k \tag{21f}$$

$$\frac{\partial V(x(t_k))}{\partial x} f(x(t_k), u(t_k), 0) \leq \frac{\partial V(x(t_k))}{\partial x} f(x(t_k), h(x(t_k)), 0) \tag{21g}$$

where

$$\bar{u}_k = \int_{t_N}^{t_k} u_{LEMPC}(\tau)d\tau \tag{22}$$

and

$$c_k = \int_{t_N}^{t_k} L_e(\tilde{x}(\tau), u_{LEMPC}(\tau))d\tau \tag{23}$$

4. LEMPC Algorithm II: operation under disturbances

In this section, we consider the design of an LEMPC scheme for nonlinear systems subject to bounded process disturbances (i.e., $w(t) \neq 0$). The proposed scheme at each sampling time solves the auxiliary LMPC problem and then incorporates the LMPC solution to the LEMPC formulation as constraints. The manipulated input of the LMPC design of Eq. (7) from time t_k to t_N is defined as follows:

$$u_{LMPC}(t) = u^*(t|t_k), \quad \forall t \in [t_k, t_N). \tag{24}$$

Let the state trajectory $x_{LMPC}(t)$ be defined as follows:

$$\dot{x}_{LMPC}(t) = f(x_{LMPC}(t), u_{LMPC}(t), 0), \quad \forall t \in [t_k, t_N) \tag{25}$$

which is the system state trajectory when the manipulated input is obtained through the LMPC of Eq. (7). Also, we define $\bar{u}_{LMPC} = \int_{t_k}^{t_N} u_{LMPC}(\tau)d\tau$ as the amount of control action utilized by the LMPC of Eq. (7) over a finite prediction horizon N_k where $N_k = N - k$ and $k = 0, 1, \ldots, N-1$. Subsequently, let us define $c^1_{LMPC} = \int_{t_k}^{t_{k+1}} L_e(x_{LMPC}(\tau), u_{LMPC}(\tau))d\tau$ and $c^2_{LMPC} = \int_{t_k}^{t_N} L_e(x_{LMPC}(\tau), u_{LMPC}(\tau))d\tau$ as the value of the economic cost function induced by the LMPC over the first prediction step and the entire prediction horizon, respectively. The purpose of the LEMPC design is to obtain an optimal manipulated input trajectory which uses the same amount of control action as the LMPC of Eq. (7) at each sampling time while it ensures that at each sampling time t_k, it achieves closed-loop state boundedness and a better economic cost value compared to the LMPC over the operation periods $[t_k, t_{k+1})$ and $[t_k, t_N)$ for the nominal, i.e.,($w(t) \equiv 0$) case, respectively.

4.1. Implementation strategy

The proposed control scheme solves an auxiliary LMPC optimization problem of Eq. (7) at each sampling time t_k. In order to account for the bounded process disturbance effect, we consider another region Ω_{ρ_e} with $\rho_e < \rho$. If the state measurement $x(t_k)$ is in the region Ω_{ρ_e}, the LEMPC maximizes the cost function within the region Ω_{ρ_e}; if the state measurement is in the region $\Omega_\rho / \Omega_{\rho_e}$, the LEMPC first drives the system state to the region Ω_{ρ_e} and then maximizes the cost function within Ω_{ρ_e}.

Specifically, the implementation strategy of the proposed LEMPC can be summarized as follows:

1. At time t_k with $t_k \in [t_0, t_N)$, an auxiliary LMPC optimization problem of Eq. (7) is solved based on state measurement $x(t_k)$ for $t \in [t_k, t_N)$.
2. The amount of control action used by the LMPC and its corresponding economic cost are computed.
3. The LEMPC receives $x(t_k)$ and solves its optimization problem with prediction horizon $N - k$.
4. If $x(t_k) \in \Omega_{\rho_e}$, go to Step 4.1. Else, go to Step 4.2.
   4.1. The controller maximizes the economic cost function within Ω_{ρ_e}. Go to Step 5.
   4.2. The controller drives the system state to the region Ω_{ρ_e} and then maximizes the economic cost function within Ω_{ρ_e}. Go to Step 5.
5. Go to Step 1 ($k \longleftarrow k+1$).

4.2. LEMPC formulation

The optimization problem of the proposed LEMPC at sampling time t_k is as follows:

$$\max_{u \in S(\Delta)} \int_{t_k}^{t_N} L_e(\tilde{x}(\tau), u(\tau))d\tau \tag{26a}$$

$$\text{s.t.}\ \dot{\tilde{x}}(\tau) = f(\tilde{x}(\tau), u(\tau), 0) \tag{26b}$$

$$u(\tau) \in U,\ \tau \in [t_k, t_N) \tag{26c}$$

$$\tilde{x}(t_k) = x(t_k) \tag{26d}$$

$$V(\tilde{x}(t)) \leq \rho_e, \quad \forall t \in [t_k, t_N),\ \text{if} V(x(t_k)) \leq \rho_e \tag{26e}$$

$$\int_{t_k}^{t_N} u(\tau)d\tau = \bar{u}_{LMPC} \tag{26f}$$

$$\int_{t_k}^{t_{k+1}} L_e(x(\tau), u(\tau))d\tau \geq c^1_{LMPC} \tag{26g}$$

$$\int_{t_k}^{t_N} L_e(x(\tau), u(\tau))d\tau \geq c^2_{LMPC} \tag{26h}$$

$$\frac{\partial V(x(t_k))}{\partial x} f(x(t_k), u(t_k), 0) \leq \frac{\partial V(x(t_k))}{\partial x} f(x(t_k), h(x(t_k)), 0),$$
$$\text{if} \rho_e < V(x(t_k)) \leq \rho \tag{26i}$$

The constraint of Eq. (26e) restricts the predicted system state to be in the set Ω_{ρ_e}. The constraint of Eq. (26i) guarantees that the reduction rate of the Lyapunov function value when the first step of the LEMPC input is applied is at least at the level achieved by applying the Lyapunov-based controller $h(x)$ when it is applied in a sample and hold fashion and $\rho_e < V(x(t_k)) \leq \rho$. It should be mentioned that a decreasing sequence of finite prediction horizons $N_k = N - k$ where $k = 0, \ldots, N-1$ is incorporated in the LMPC and LEMPC formulations at sampling time $t_k \in [t_0, t_N)$. The optimal solution to this optimization problem is denoted by $u^*(t|t_k)$ and is defined for $t \in [t_k, t_N)$. The manipulated input of the LEMPC of Eq. (26) is defined as $u_{LEMPC}(t) = u^*(t|t_k)$, $\forall t \in [t_k, t_N)$.

Remark 5. The main difference of the proposed LEMPC algorithms of Eqs. (13) and (26) arises from the existence of bounded process disturbance. The LEMPC of Eq. (13) takes only advantage of the solution of the auxiliary LMPC problem at time t_0 through a decreasing sequence of finite prediction horizons, while the LEMPC of Eq. (26), utilizes the solution of the LMPC at each sampling time t_k, while accounting for the influence of disturbance on the process through the state measurement feedback $x(t_k)$.

4.3. Closed-loop stability and performance

Corollary 1 below provides sufficient conditions under which the LEMPC of Eq. (26) guarantees that the state of the closed-loop system of Eq. (1) is always bounded in Ω_ρ for $w(t) \neq 0$ and at each sampling time the LEMPC yields a closed-loop economic cost that is as good or superior to the one of the LMPC over the interval $[t_k, t_{k+1})$ when $w(t) \equiv 0$.

Corollary 1. *Consider the system of Eq. (1) in closed-loop under the LEMPC design of Eq. (26) based on a controller $h(x)$ that satisfies the conditions of Eq. (2). Let $\epsilon_w > 0$, $\Delta > 0$, $\rho > \rho_e > 0$ and $\rho > \rho_s > 0$ satisfy*

$$\rho_e \leq \rho - f_V(f_W(\Delta)) \tag{27}$$

and

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + L'_x M\Delta + L'_w\theta \leq -\frac{\epsilon_w}{\Delta}. \tag{28}$$

where

$$f_V(s) = \alpha_4(\alpha_1^{-1}(\rho))s + M_v s^2 \tag{29}$$

with M_v being a positive constant and

$$f_W(\tau) = \frac{L_w\theta}{L_x}(e^{L_x\tau} - 1) \tag{30}$$

If $x(t_0) \in \Omega_\rho$ and $\rho_s \leq \rho_e$, then the state $x(t)$ of the closed-loop system is bounded in Ω_ρ $\forall t \in [t_0, t_N)$, and if $w(t) \equiv 0$ at each sampling time t_k

$$\int_{t_k}^{t_{k+1}} L_e(x_{LEMPC}(\tau), u_{LEMPC}(\tau))d\tau \geq \int_{t_k}^{t_{k+1}} L_e(x_{LMPC}(\tau), u_{LMPC}(\tau))d\tau \tag{31}$$

Proof. For a detailed proof regarding boundedness (i.e., $x(t_0) \in \Omega_\rho$ implies that $x(t) \in \Omega_\rho$ for $t \in [t_0, t_N)$) of the closed-loop system state under $h(x)$ and LMPC, please refer to [7,4]. From a feasibility point of view, since at sampling time t_k $x(t_k) \in \Omega_\rho$, the LMPC solution $u_{LMPC}(\tau)$, $\forall \tau \in [t_k, t_{k+N})$ is a feasible solution for the LEMPC of Eq. (26), and thus, the LEMPC of Eq. (4.2) also ensures that $x(t) \in \Omega_\rho$, $\forall t \in [t_0, t_N)$. Through enforcing the constraints of Eqs. (26g) and (26h) at each sampling time t_k, the LEMPC of Eq. (26) ensures that it obtains an input trajectory that optimizes the economic cost function over the first step and the entire prediction horizon for $w(t) \equiv 0$ with respect to LMPC, respectively, while, both the LMPC and LEMPC designs use the same amount of control action over the entire prediction horizon by enforcing the constraint of Eq. (26f) at each sampling time. □

Remark 6. Eqs. (13g), (26g) and (26g) guarantee improvement in the economic cost function compared to the solution provided by LMPC for $w(t) \equiv 0$. It should be emphasized that these two constraints guarantee feasibility of the LEMPC optimization problem as well as improvement in economic cost function value. Without these constraints it may not be guaranteed to improve the economic cost function via LEMPC, given the non-convexity of the cost function. Furthermore, even though we cannot make any economic performance improvement claims for the LEMPC over LMPC when $w(t) \neq 0$, our extensive closed-loop simulations in the next section indicate certain benefits of the proposed LEMPC.

Table 1
Parameter values.

$T_0 = 300$	K	$F = 5$	m^3/h
$V = 1.0$	m^3	$E = 5\times10^3$	kJ/kmol
$k_0 = 13.93$	1/h	$\Delta H = 1.15\times10^4$	kJ/kmol
$C_p = 0.231$	kJ/(kg K)	$R = 8.314$	kJ/(kmol K)
$\sigma = 1000$	kg/m^3	$C_{As} = 2$	$kmol/m^3$
$T_s = 350$	K	$C_{A0s} = 4$	$kmol/m^3$
$Q = 1.73\times10^5$	kJ/h		

5. Application to a chemical process example

Consider a well-mixed, non-isothermal continuous stirred tank reactor (CSTR) where an irreversible, second-order, endothermic reaction $A \to B$ takes place, where A is the reactant and B is the desired product. The feed to the reactor consists of pure A at flow rate F, temperature T_0 and molar concentration C_{A0}. Due to the non-isothermal nature of the reactor, a jacket is used to provide heat to the reactor. The dynamic equations describing the behavior of the reactor, obtained through material and energy balances under standard modeling assumptions, are given below:

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - k_0 e^{-E/RT} C_A^2 \quad (32a)$$

$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{-\Delta H}{\sigma C_p} k_0 e^{-E/RT} C_A^2 + \frac{Q}{\sigma C_p V} \quad (32b)$$

where C_A denotes the concentration of the reactant A, T denotes the temperature of the reactor, Q denotes the rate of heat supply to the reactor, V represents the volume of the reactor, ΔH, k_0 and E denote the enthalpy, pre-exponential constant and activation energy of the reaction, respectively, and C_p and σ denote the heat capacity and the density of the fluid in the reactor, respectively. The values of the process parameters used in the simulations are shown in Table 1. The process model of Eq. (32) is numerically simulated using an explicit Euler integration method with integration step $h_c = 10^{-4}$ h.

The process model has one stable steady-state in the operating range of interest. The control objective is to optimize the process operation in a region around the stable steady-state (C_{As}, T_s) to maximize the average production rate of B through manipulation of the concentration of A in the inlet to the reactor, C_{A0}. The steady-state input value associated with the steady-state point is denoted by C_{A0s}. The process model of Eq. (32) belongs to the following class of nonlinear systems:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t)$$

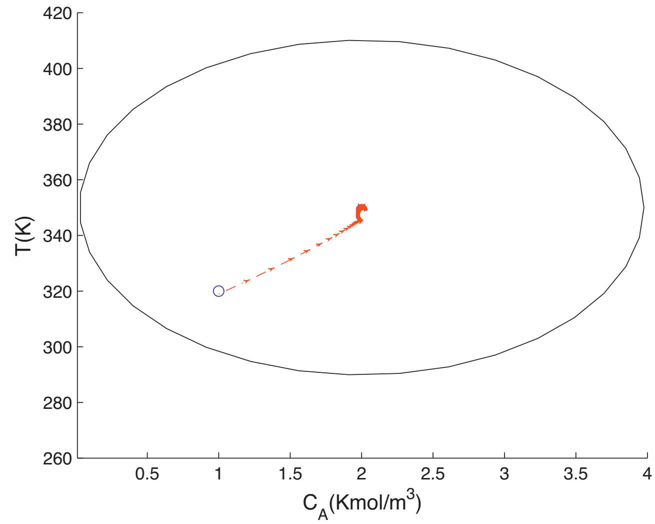

Fig. 2. Ω_ρ and state trajectories of the process under the LMPC design of Eq. (7) with initial state $(C_A(0), T(0)) = (1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation. The symbols $\circ$ and $\times$ denote the initial ($t = 0$ h) and final ($t = 1$ h) state of these closed-loop system trajectories, respectively.

where $x^T = [x_1\ x_2] = [C_A - C_{As}\ T - T_s]$ is the state, $u = C_{A0} - C_{A0s}$ is the input, and $f = [f_1\ f_2]^T$ and $g = [g_1\ g_2]^T$ are vector functions. The input is subject to constraints as follows: $|u| \le 3.5$ kmol/m^3. The economic measure considered in this example is as follows [17]:

$$L_e(x, u) = \frac{1}{t_N}\int_0^{t_N} k_0 e^{-E/RT(\tau)} C_A^2(\tau) d\tau \quad (33)$$

where $t_N = 1$ h is the time duration of the reactor operation. This economic objective function highlights the maximization of the average production rate over process operation for $t_N = 1$ h (different, yet finite, values of t_N can be chosen.). According to the proposed LEMPC schemes, under mode one operation, auxiliary LMPC problems are solved to obtain constraints on the amount of reactant material (control action) which can be used by the LEMPC control schemes. For the sake of simplicity, we will refer to this type of constraint as the material constraint. We consider both nominal and subject to bounded disturbance operations.

In terms of the Lyapunov-based controller, we use a proportional controller (P controller) in the form $u = -\gamma_1 x_1 - \gamma_2 x_2$ subject to input constraints and the quadratic Lyapunov function

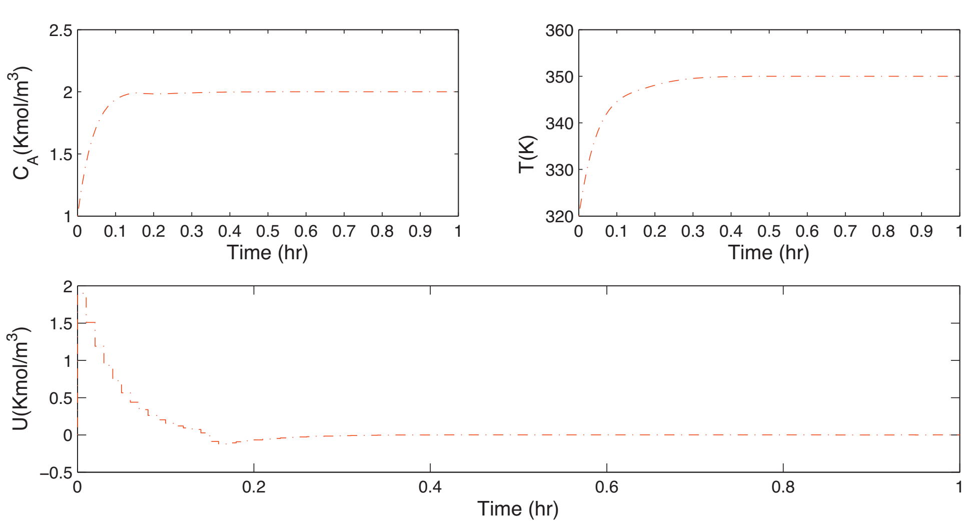

Fig. 3. State and manipulated input trajectories of the process under the LMPC design of Eq. (7) with initial state $(C_A(0), T(0)) = (1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation.

Table 2
Economic closed-loop performance comparison based on the economic cost of Eq. (34) and the initial states in Fig. 6.

	LEMPC	LMPC		LEMPC	LMPC		LEMPC	LMPC
1	10.43	9.53	11	11.31	10.35	21	10.85	9.94
2	11.35	10.37	12	10.60	9.67	22	10.55	9.57
3	11.12	10.17	13	10.57	9.67	23	10.46	9.56
4	10.53	9.58	14	10.99	10.05	24	11.61	10.61
5	10.94	10	15	10.59	9.64	25	10.47	9.55
6	10.36	9.44	16	10.64	9.68	26	11.53	10.54
7	11.55	10.59	17	11.50	10.53	27	10.52	9.56
8	10.70	9.73	18	10.79	9.81	28	11.06	10.11
9	11.09	10.14	19	10.68	9.77	29	11.48	10.49
10	10.84	9.94	20	10.61	9.71	30	11.36	10.40

$V(x)=x^TPx$ where $\gamma_1=1.6$, $\gamma_2=0.01$, $P=diag([110.11\ 0.12])$ and $\rho=430$. It should be emphasized that Ω_ρ has been estimated through evaluation of $\dot{V}$ when we apply the proportional controller. Specifically, using the P controller as the Lyapunov-based controller, we apply it to the nonlinear process model as a manipulated input, and then we evaluate the time derivative of the Lyapunov function $\dot{V}$ for every point in the physically meaningful range for C_A and T. Then, within this range, we find the largest level set of the Lyapunov function while $\dot{V} < 0$. We assume that the full system state $x=[x_1\ x_2]^T$ is measured and sent to the LEMPC at synchronous time instants $t_k=k\Delta$, $k=0, 1, \ldots$, with $\Delta=0.01$ h = 36 s.

5.1. Nominal operation

An LMPC optimization problem which is only solved at sampling time $t_0=0$ is formulated with prediction horizon $N=100$ ($t_N=1$ h) and weighting matrix $Q=diag([1\ 0.01])$ and $R=1$. The optimization problems were solved using the open source interior point optimizer Ipopt [18]. Figs. 2 and 3 display the closed-loop state and manipulated input profiles for the LMPC of Eq. (7) which leads to steady-state operation. The LMPC scheme steers the closed-loop system state to the steady-state. However, this steady-state operation may not be optimal from the standpoint of the cost of Eq. (33). It should be emphasized that this LMPC formulation is only evaluated at time t_0.

Considering the material constraint which needs to be satisfied through each period of process operation, a decreasing LEMPC horizon sequence $N_0, \ldots, N_{99}$ where $N_i=100-i$ and $i=0, \ldots, 99$ is utilized at the different sampling times. Figs. 4 and 5 represent the closed-loop state and the manipulated input for the LEMPC of Eq. (13) which dictates a time-varying operation to achieve optimal economic closed-loop performance. It should be mentioned that

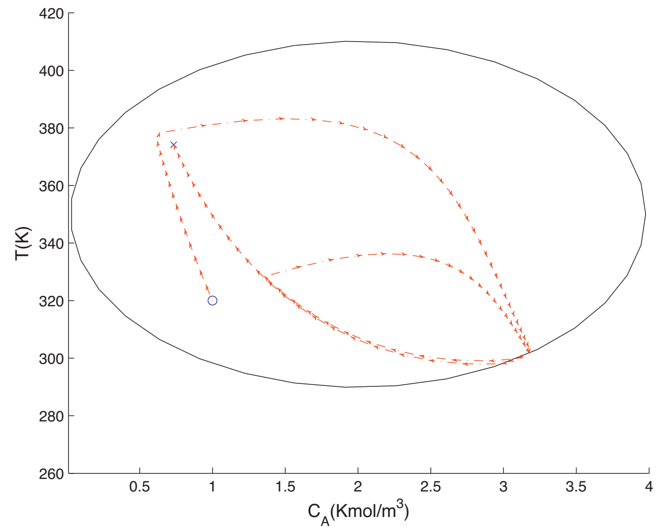

Fig. 4. Ω_ρ and state trajectories of the process under the LEMPC design of Eq. (13) with initial state $(C_A(0), T(0))=(1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation. The symbols ∘ and × denote the initial ($t=0$ h) and final ($t=1$ h) state of these closed-loop system trajectories, respectively.

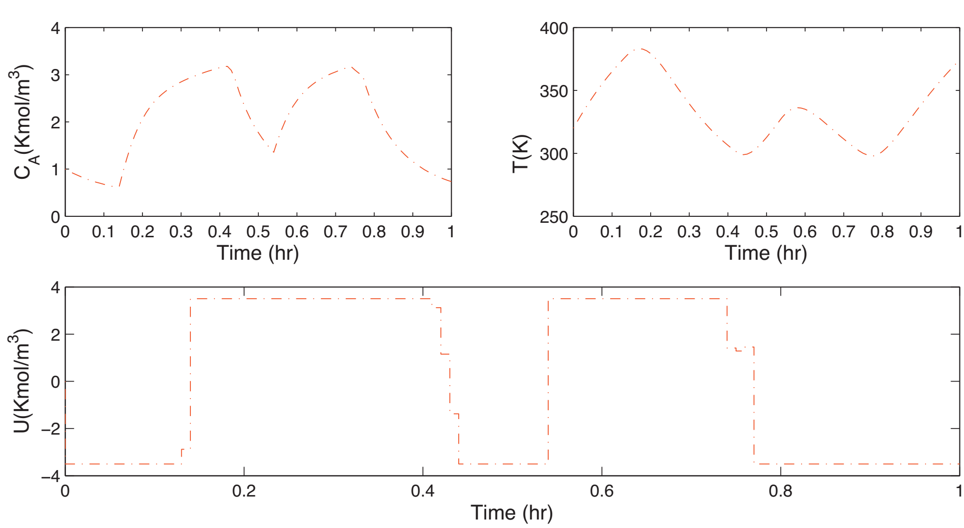

Fig. 5. State and manipulated input trajectories of the process under the LEMPC design of Eq. (13) with initial state $(C_A(0), T(0))=(1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation.

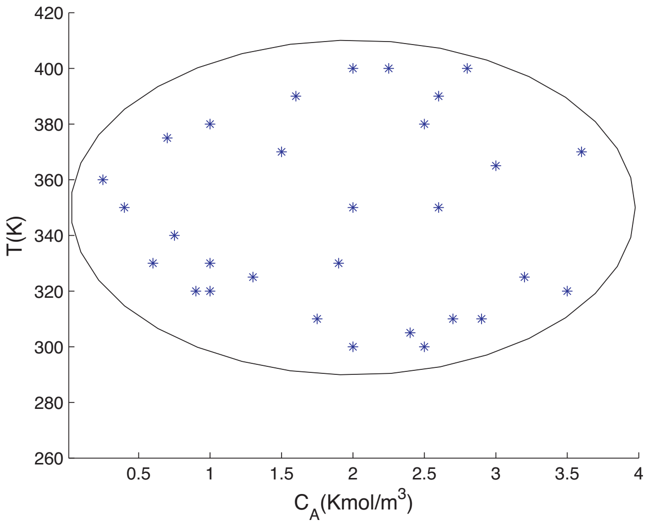

Fig. 6. 30 different initial states for evaluation of LEMPC and LMPC schemes.

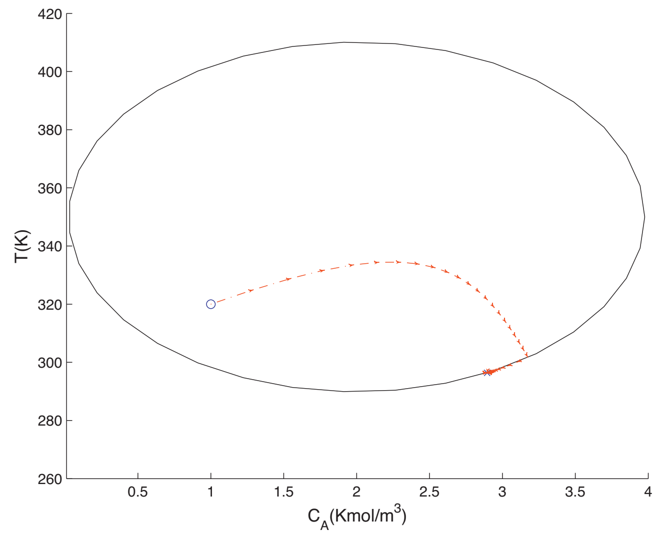

Fig. 7. Ω_ρ and state trajectories of the process under the LEMPC design of Eq. (13) (without enforcing the constraint of Eq. (13f)) with initial state ($C_A(0)$, $T(0)$) = (1 kmol/m^3, 320 K) for one period of operation. The symbols ∘ and × denote the initial (t = 0 h) and final (t = 1 h) state of these closed-loop system trajectories, respectively.

$u = C_{A0} - C_{As}$. So, negative u means that C_{A0} is less than C_{A0s} but it is always greater than zero. Since there is a limited amount of reactant material due to the need for consistency between LMPC and LEMPC (note that in this example the control action u is not penalized in the economic cost), LEMPC should find its solution in an optimal fashion to meet this constraint as well as maximizing the economic cost function.

Regarding the implementation of the LMPC of Eq. (7), we integrate the proces model using Euler's method based on the Lyapunov-based (proportional) controller as well as the LMPC computed control input, separately. To implement the constraint of Eq. (7e), we first evaluate the Lyapunov function at sampling time t_j where $j = k, \ldots, k+N-1$ and we consider the evolution of the system over only one integration step when we apply the Lyapunov-based controller and the control input computed by the LMPC for computing the gradient of the Lyapunov function; this process is repeated from $j = k, \ldots, k+N-1$. For the implementation of the constraint of Eq. (8e), this constraint should be enforced $\forall t \in [t_k, t_{k+N})$; however, for small integration step and sampling time, and to keep the computational burden manageable, we first implement this constraint at sampling times t_j where $j = k, \ldots, k+N-1$ and then we check that for the computed control input trajectory this constraint is satisfied for all times, i.e., $\forall t \in [t_k, t_{k+N})$; in particular, this constraint is satisfied for all times, for all the closed-loop simulation reported in this work. With respect to the satisfaction of the assumptions in Section 2.3 in the context of the process example model, we need to point out that the process model $f(x)$ vector - right hand side of the differential equations of the process model - is continuous differentiable and thus the assumptions on f (locally Lipschitz) as well as the bounds of Eqs. (3) and (4) hold for appropriate values of the parameters.

Table 2 shows the evaluation of LEMPC and LMPC from an economic cost function point of view for 30 different initial states within Ω_ρ as illustrated in Fig. 6. To carry out this comparison, we have computed the total cost of each operating scenario based on an index of the following form:

$$J = \frac{1}{t_{100}} \sum_{i=0}^{100} [k_0 e^{-\frac{E}{RT(t_i)}} C_A^2(t_i)] \qquad (34)$$

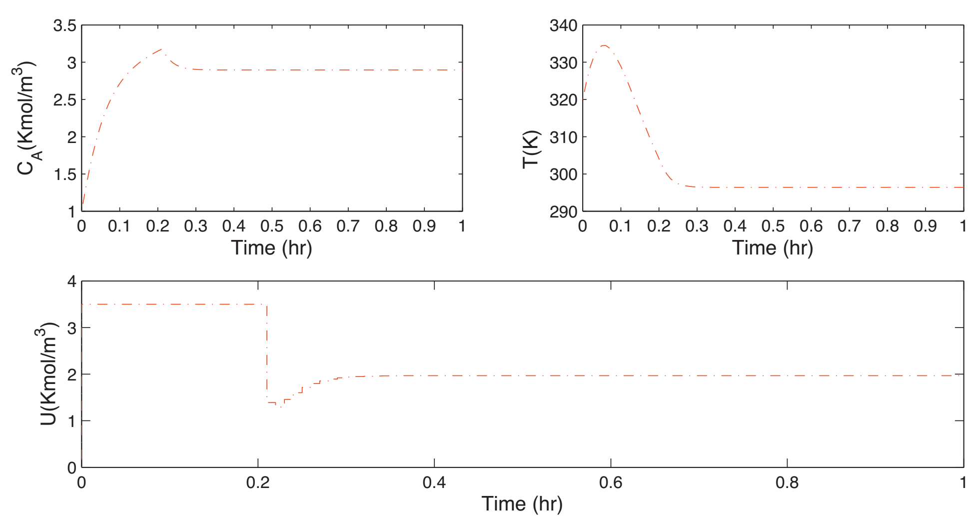

Fig. 8. State and manipulated input trajectories of the process under the LEMPC design of Eq. (13) (without enforcing the constraint of Eq. (13f)) with initial state ($C_A(0)$, $T(0)$) = (1 kmol/m^3, 320 K) for one period of operation.

where $t_0 = 0$ h and $t_{100} = 1$ h. It has been confirmed by these sets of simulations that the LEMPC through time-varying process operation improves the economic closed-loop performance by about 10% on average against steady-state operation by LMPC.

Furthermore, we performed a simulation (Figs. 7 and 8) which deals with economic MPC without enforcing the constraint of Eq. (13f) which indicates that in the absence of the constraint of Eq. (13f), the LEMPC significantly optimizes the economic cost function further (15.26 vs. 10.53) compared to the case where the constraint of Eq. (13f) is used, at the cost of using more reactant material compared to LMPC in an average sense (2.26 vs. 0.08), for the same initial condition and operating period. Also, we performed another simulation which deals with economic MPC with enforcing the constraint of Eq. (13f) as an inequality (i.e., the LEMPC can use the same or less control action than the LMPC) and we found, as expected, that the LEMPC obtains the same control input trajectory solution and cost compared to the case that we enforce this constraint as an equality one.

5.2. Operation subject to bounded process disturbances

Considering the material constraint which needs to be satisfied through each period of process operation, a decreasing finite prediction horizon sequence $N_0, \ldots, N_{99}$ where $N_i = 100 - i$ and $i = 0, \ldots, 99$ is utilized at different sampling times. At each sampling time t_k, after solving an auxiliary LMPC problem with prediction horizon N_k, the LEMPC with prediction horizon N_k takes into account the control action and cost constraints and adjusts its finite prediction horizon to predict the future system evolution up to time $t_N = 1$ hr to maximize the cost of Eq. (34). Since the LEMPC is evaluated at discrete-time instants during the closed-loop simulation, the material constraint is enforced as follows:

$$\sum_{i=0}^{N_k-1} u_{LEMPC}(t_i) = \sum_{i=0}^{N_k-1} u_{LMPC}(t_i) \tag{35}$$

The above equation indicates that the same amount of reactant material at each sampling time is used to solve both the LMPC and the LEMPC optimization problems. For the purpose of simulations, we set $\rho_e = 400$. Also, bounded process disturbances have been added to the right hand side of the dynamic model of Eq. (32) which have been sampled from a gaussian distribution. The absolute values of process disturbances are bounded by 2 and 50 for the first and the second equation, respectively. First, the LMPC formulation of Eq. (7) at sampling time t_k for the chemical process example is first solved. Having the solution of the LMPC at sampling time t_k, the LEMPC of Eq. (26) is then solved. Figs. 9 and 10 display the closed-loop system state and the manipulated input with initial state $(C_A(0), T(0)) = (1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation subject to bounded process disturbances. Through time-varying operation, LEMPC achieves 15.12 in economic cost function of Eq. (34) while the LMPC yields 13.91. Furthermore, we evaluated LEMPC and LMPC economic closed-loop performance for 10 different process disturbance realizations and we found that there is between 8 to 9 percent improvement when LEMPC is applied over LMPC, indicating the robustness of the obtained economic benefits for different disturbance realizations.

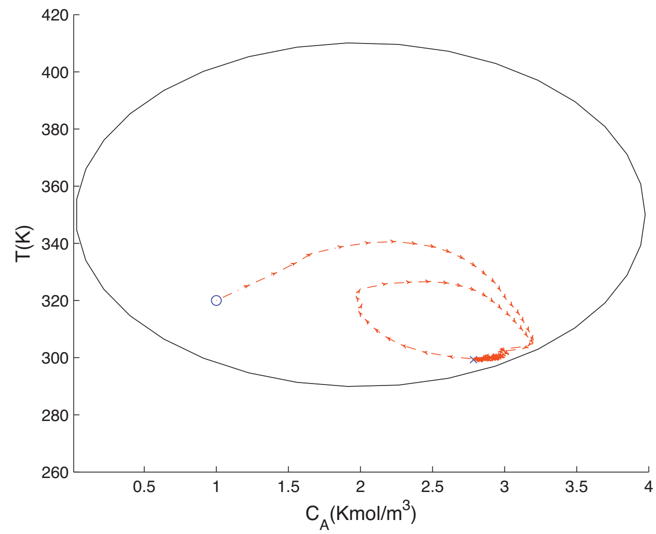

Fig. 9. Ω_ρ and state trajectories of the process under the LEMPC design of Eq. (26) with initial state $(C_A(0), T(0)) = (1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation subject to bounded process disturbance. The symbols ∘ and × denote the initial ($t = 0$ h) and final ($t = 1$ h) state of these closed-loop system trajectories, respectively.

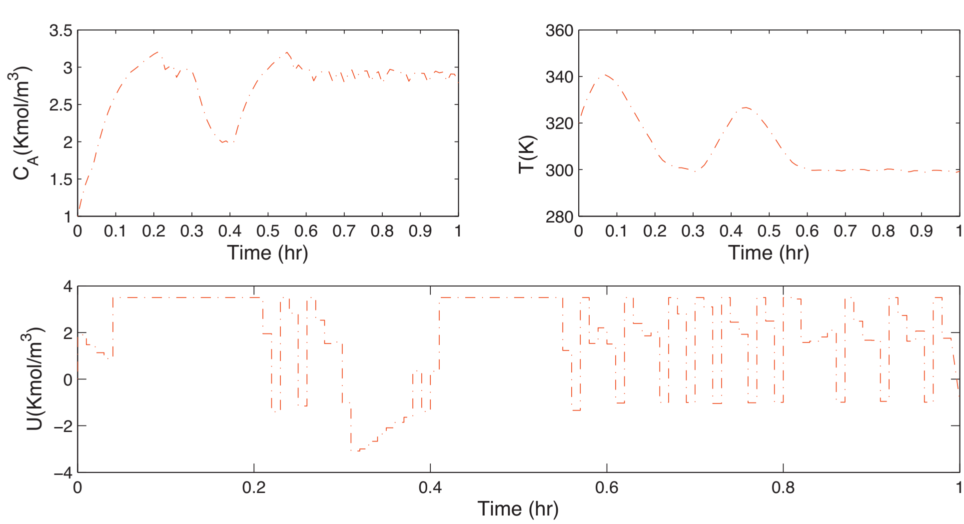

Fig. 10. State and manipulated input trajectories of the process under the LEMPC design of Eq. (26) with initial state $(C_A(0), T(0)) = (1\ \text{kmol/m}^3, 320\ \text{K})$ for one period of operation subject to bounded process disturbance.

6. Conclusions

This work focused on the design of LEMPC algorithms for a class of nonlinear systems which are capable of optimizing closed-loop performance with respect to a general objective function that directly addresses economic considerations. Under appropriate stabilizability assumptions, the proposed LEMPC designs use a shrinking horizon with respect to a fixed-time interval and very often dictate time-varying operation to optimize an economic (typically non-quadratic) cost function in contrast to conventional LMPC designs which typically include a quadratic objective function and regulate a process at a steady-state. The proposed LEMPC algorithms took advantage of the solution of auxiliary LMPC problems at different sampling times to incorporate appropriate economic cost and control action-based constraints in the LEMPC formulations and ensure improved performance, measured by the desired economic cost, with respect to conventional LMPC. Using a chemical process example, the LEMPC algorithms were demonstrated to improve average economic performance, both in the nominal case and with disturbances present.

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