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Actuator and controller scheduling in nonlinear transport-reaction processes

N.H. El-Farra[a,*], M.A. Demetriou[b], P.D. Christofides[c]

[a]Department of Chemical Engineering and Materials Science, University of California, Davis, CA 95616-5294, USA
[b]Department of Mechanical Engineering, Worcester Polytechnic Institute, Worcester, MA 01609-2280, USA
[c]Department of Chemical and Biomolecular Engineering, University of California, Los Angeles, CA 90095, USA

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ABSTRACT

The objective of the present manuscript is to examine the performance improvement of a class of nonlinear transport processes subject to spatiotemporally varying disturbances through the employment of a comprehensive and systematic actuator activation and controller scheduling policy. To attain this objective, it is assumed that multiple groups of actuators are available with only one such actuator group being active over a time interval of fixed length while the remaining actuator groups are kept dormant. Using well-established enhanced controllability and performance improvement measures, the candidate actuator groups are first placed at locations that individually provide certain robustness with respect to an appropriately defined "worst'' spatial distribution of disturbances. Once the multiple actuator groups are in place, a switching scheme that dictates the switching of a different actuator group at different time intervals, as well as the corresponding control signal supplied to it while being active, is developed. Embedded in the decision policy is the activation of actuators that lie spatially closer to the spatiotemporal disturbances, thereby improving the control authority of the actuators and enhancing the ability of the system to minimize the effects of this class of disturbances. Indeed, entering disturbances that excite different system modes at different time intervals are best handled by employing the actuators that have increased spatial controllability for the above disturbance modes while simultaneously satisfy certain performance measures. In this manner, the control system designed engages the actuators that are better suited to efficiently perform spatiotemporal disturbance compensation over the duration of certain time intervals. The proposed method is demonstrated through simulations using two typical quasi-linear highly dissipative partial differential equation examples.

1. Introduction

Transport-reaction processes are characterized by highly nonlinear behavior owing to complex reaction mechanisms and strong spatial variations due to the underlying diffusion and convection phenomena. Examples include the chemical vapor deposition of thin films, the Czochralski crystallization of high-purity crystals as well as PEM fuel-cell systems and biological processes. The dynamic models of transport-reaction processes over finite spatial domains typically consist of linear/nonlinear parabolic partial differential equation (PDE) systems whose spatial differential operators are characterized by a spectrum that can be partitioned into a finite (possibly unstable) slow part and an infinite stable fast complement (Curtain and Pritchard, 1978). This allows the construction of finite-dimensional controllers based on reduced order models that can guarantee stabilization of the infinite-dimensional PDE system. Specifically, the traditional approach for control of linear/quasi-linear parabolic PDEs involves the application of spatial discretization techniques to the PDE system to derive systems of ordinary differential equations (ODEs) that accurately describe the dynamics of the dominant (slow) modes of the PDE system. The finite-dimensional systems are subsequently used as the basis for the synthesis of finite-dimensional controllers (e.g., see Balas, 1979; Curtain, 1982). While this approach works well for linear parabolic PDEs, a potential drawback of this approach, in the context of quasi-linear parabolic PDEs, is that the number of modes that should be retained to derive an ODE system that yields the desired degree of approximation may be very large, leading to complex controller design and high dimensionality of the resulting controllers. Motivated by these considerations, significant recent work has focused on the development of a general framework for the synthesis of low-order controllers for quasi-linear parabolic PDE systems (and other highly dissipative PDE systems that arise

* Corresponding author.
E-mail addresses: nhelfarra@ucdavis.edu (N.H. El-Farra), mdemetri@wpi.edu (M.A. Demetriou), pdc@seas.ucla.edu (P.D. Christofides).

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in the modeling of spatially distributed systems) on the basis of low-order nonlinear ODE models derived through a combination of the Galerkin method (using analytical or empirical basis functions) with the concept of inertial manifolds (Christofides and Daoutidis, 1997; Armaou and Christofides, 2000; Baker and Christofides, 2000; Christofides, 2001). The approaches proposed in the above works, however, do not address the issue of actuator and controller scheduling to improve closed-loop performance.

In the context of optimal actuator placement and scheduling for parabolic PDEs, previous work (Antoniades and Christofides, 2001) has led to the development of a methodology for the computation of optimal locations of point control actuators for nonlinear feedback controllers in transport-reaction processes described by a broad class of quasi-linear parabolic PDEs. Initially, Galerkin's method was employed to derive finite-dimensional approximations of the PDE system which were used for the synthesis of stabilizing nonlinear state feedback controllers via geometric techniques. Then, the optimal location problem was formulated as the one of minimizing a meaningful cost functional that includes penalty on the response of the closed-loop system and the control action and is solved by using standard unconstrained optimization techniques. This work established that the solution to this problem, which is obtained on the basis of the closed-loop finite-dimensional system, is near-optimal for the closed-loop infinite-dimensional system. Beyond the actuator placement problem, research efforts have also studied the optimal sensor placement problem for certain classes of distributed parameter systems (see, for example, Alonso et al., 2004a,b). In addition to these works, the problem of control actuator scheduling for parabolic PDEs has been addressed in El-Farra and Christofides (2004) and El-Farra and Ghantasala (2007) within a hybrid control framework, where the control system consists of a family of controller configurations (e.g., a family of feedback laws and/or a family of actuator/sensor spatial arrangements) together with a higher-level supervisor that uses logic to orchestrate switching between these control configurations. An example where consideration of hybrid control is necessary is the problem of control actuator failure. In this case, upon detection of a fault in the operating actuator configuration, it is often necessary to switch to an alternative well-functioning actuator configuration with a different spatial placement of the actuators in order to preserve closed-loop stability. Switching between spatially distributed actuators provides a means for fault-tolerant control of spatially distributed systems.

At around the same time, considerable work on processes that are modeled by parabolic linear PDEs with switching both actuating devices and control signals (Demetriou and Kazantzis, 2004c) examined the performance improvement by actuator and controller switching. Using a suboptimal method, the optimal value of a linear quadratic cost-to-go, parameterized by the admissible actuators, was examined at the beginning of a given time interval. Both the actuator and the associated controller were switched in according to the static optimization scheme which activated the actuating device that produced the smallest optimal value of the associated cost-to-go. Using a collocated actuator/sensor configuration, the above method was extended to the case of output feedback control, where a static output feedback controller was implemented (Demetriou and Kazantzis, 2005). An additional aspect of the above actuator-plus-switching scheme was to account for unknown disturbances, and more specifically, unknown spatiotemporally varying disturbances, i.e., disturbances whose spatial distribution varies with time and which is not accounted for in the controller design. Indirectly addressing the spatial distribution of disturbances was the correct placement of the candidate actuators, wherein locations were chosen based on their ability to minimize the effects of the "worst'' spatial distribution of disturbances. If the spatial distribution of disturbances was known, then the actuator placement scheme considered such distributions in the optimal locations. When the spatial distribution of disturbances was not known, then either a "worst'' spatial distribution was assumed or through another level of optimization, the "worst'' spatial distribution of disturbances was found and used in the placement of the actuating devices. Such a location scheme rendered the actuator locations *spatially robust*. In fact, the preliminary work in Demetriou and Kazantzis (2004b) addressed this issue for the Kuramoto–Sivashinsky equation, by proposing various actuator placement schemes in order to account for such disturbances, and a performance-based actuator switching was presented in order to address spatially "moving'' disturbances. The approach was verified experimentally in welding processes with a single actuating device (plasma arc welding) capable at moving at given spatial locations and injecting thermal control signal at these locations (Demetriou et al., 2003). An optimal controller-plus-actuator scanning scheme was considered in Iftime and Demetriou (2005) for a general class of linear infinite-dimensional systems. A linear quadratic cost was considered on a finite horizon which resulted in the solution of many operator differential Riccati equations. An early attempt to address the nonlinear case was examined in Demetriou and Kazantzis (2004a). Controllers based on the linear part of a nonlinear PDE were switched in and out in order to account for the nonlinear dynamics of the process and at the same time minimize the effects of spatiotemporally varying disturbances. Similarly, actuating devices were switching in and out to provide fault tolerance for both temporary and permanent actuator failures in linear distributed parameter systems (Demetriou, 2003). Parallel to diffusion processes, considerable work on flexible structures (both beams and plates) considered the actuator switching for performance enhancement and robustness against spatiotemporally varying disturbances (Demetriou, 2004; Demetriou and Murugavel, 2004).

The objective of the present manuscript is to examine the performance improvement of a class of nonlinear transport-reaction processes subject to spatiotemporally varying disturbances through the employment of a comprehensive and systematic actuator activation and controller scheduling policy. To attain the above objective, it is assumed that multiple groups of actuators are available with only one such actuator group being active over a time interval of fixed length while the remaining actuator groups are kept dormant. Using well-established enhanced controllability and performance improvement measures, the candidate actuator groups are first placed at locations that individually provide certain robustness with respect to an appropriately defined "worst'' spatial distribution of disturbances. Once the multiple actuator groups are in place, a switching scheme is developed that dictates the switching of a different actuator group at different time intervals, as well as the corresponding control signal supplied to it while being active. Embedded in the decision policy is the activation of actuators that lie spatially closer to the spatiotemporal disturbances, thereby improving the control authority of the actuators and enhancing the ability of the system to minimize the effects of the above class of disturbances. Indeed, entering disturbances that excite different system modes at different time intervals are best addressed by employing the actuators that have increased spatial controllability for the above disturbance modes while simultaneously satisfy certain performance measures. In this manner, the control system designed employs/engages the actuators that are better suited to efficiently perform spatiotemporal disturbance compensation over the duration of certain time intervals. We apply the proposed method to typical examples of quasi-linear highly dissipative PDEs.

2. Mathematical formulation

The focus of the present study is on quasi-linear highly dissipative infinite-dimensional systems. Representative examples of such systems include many diffusion-reaction processes as well as models of various fluid dynamic systems like Burgers' equation and Kuramoto–Sivashinsky equation. The class of nonlinear infinite-dimensional systems we consider here excludes those systems that are *singular* (Lions, 1985), which in a sense is the class of systems that do not admit a unique solution; hence we will be concerned with *well-posed* problems (i.e., systems that do not have instabilities, explosion phenomena, multiple solutions or bifurcation phenomena). A representative example of such a well-posed problem is the PDE $(\partial/\partial t)x + \Delta x + \lambda x^3 = u$ with $\lambda \geqslant 0$; the case with $\lambda < 0$ case results in a singular system (Lions, 1985).

2.1. The 1-D Kuramoto–Sivashinsky equation

To motivate the present study, we first describe the 1-D controlled Kuramoto–Sivashinsky equation where it is then viewed as an abstract evolution equation in an appropriate Hilbert space. Burgers' equation is subsequently presented and written once again as an abstract nonlinear evolution equation in an appropriate Hilbert space. Once the abstract framework is formulated, one may utilize the proposed control and optimization scheme in any other nonlinear transport-reaction process that can abstractly be written in the same manner. Specifically, the 1-D controlled Kuramoto–Sivashinsky equation in the bounded interval $\Omega = [-\pi, \pi]$ is given by

$$\frac{\partial x(\xi,t)}{\partial t} + v\frac{\partial^4 x(\xi,t)}{\partial \xi^4} + \frac{\partial^2 x(\xi,t)}{\partial \xi^2} + x(\xi,t)\frac{\partial x(\xi,t)}{\partial \xi} = \sum_{i=1}^{m} b_i(\xi)u_i(t) + d(\xi)w(t), \quad (1)$$

along with periodic boundary conditions

$$\frac{\partial^j x}{\partial \xi^j}(-\pi,t) = \frac{\partial^j x}{\partial \xi^j}(\pi,t), \quad j: 0,1,2,3, \quad (2)$$

and the initial condition

$$x(\xi,0) = x_0(\xi), \quad (3)$$

where $x(\xi,t)$ denotes the state of system (1), $\xi \in \Omega = [-\pi,\pi] \subset \mathbb{R}$ is the spatial coordinate and v denotes the instability parameter. The control signals $u_i \in \mathbb{R}$, $i=1,\ldots,m$ describe the temporal components of the external excitation and the functions $b_i(\xi)$ are the actuator distribution functions, describing the spatial influence of the actuating devices. The spatial function $d(\xi)$ denotes the distribution of process disturbances and $w(t)$ the associated temporal component.

In order to write the above system in a form that is conducive to controller synthesis, actuator placement, actuator switching, and derivation of an accurate finite-dimensional approximation, an abstract formulation is employed, under which the system is mathematically represented through an evolution equation in the appropriate Hilbert space. Following the formulation adopted in Robinson (2001) and Temam (1998), we consider as an appropriate state space the space of square integrable periodic functions with zero mean, defined via

$$H = \dot{L}^2(\Omega) = \left\{\phi \in L^2(\Omega), \int_{-\pi}^{\pi} \phi(\xi)\,\mathrm{d}\xi = 0\right\}, \quad (4)$$

with the standard L^2 inner product and induced norm denoted by $\langle\cdot,\cdot\rangle$ and $|\cdot|$, respectively. Associated with the above, are the two interpolating spaces V and V' given by (Dautray and Lions, 2000)

$$V = \dot{H}_p^2(\Omega), \quad V' = H^{-2}(\Omega), \quad (5)$$

where $\dot{H}_p^2$ is a Sobolev space of order 2, defined as $\dot{H}_p^2(\Omega) = \{\phi \in H : (\sum_{k=0}^{2} |\phi^{(k)}|^2)^{1/2} < \infty\}$, where $\phi^{(k)} = \mathrm{d}^k\phi/\mathrm{d}\xi^k$ is the distributional derivative of ϕ of order k. A Gelf'and triple space naturally emerges:

$$V \hookrightarrow H \hookrightarrow V', \quad (6)$$

with both embeddings being dense and continuous (Adams, 1975; Showalter, 1977). Here V is a reflexive Banach space with norm denoted by $\|\cdot\|_V$, and V' denotes the conjugate dual of V (i.e., the space of continuous conjugate linear functionals on V). Let $\|\cdot\|_*$ denote the usual norm on V'. In particular, it is assumed that $|\varphi| \leqslant c_V\|\varphi\|_V$ for some positive constant c_V. The notation $\langle\cdot,\cdot\rangle$ will also be used to denote the duality pairing between V' and V induced by the continuous and dense embeddings given in (6).

Define the operator $A: V \to V'$ by

$$\langle A\phi, \psi\rangle = \int_{-\pi}^{\pi} \phi^{(2)}(\xi)\psi^{(2)}(\xi)\,\mathrm{d}\xi, \quad \phi,\psi \in V, \quad (7)$$

with domain $D(A) = \dot{H}_p^4(\Omega)$, where $\phi^{(2)} = \mathrm{d}^2\phi/\mathrm{d}\xi^2$. One can easily show that A^{-1} is compact and symmetric operator (Robinson, 2001), and as a consequence, it has a set of orthonormal eigenfunctions that form a basis in $\dot{L}^2(\Omega)$ (Dunford and Schwartz, 1988). Furthermore, we define the linear (Laplacian) operator $L: V \to V'$

$$\langle L\phi, \psi\rangle = -\int_{-\pi}^{\pi} \phi^{(1)}(\xi)\psi^{(1)}(\xi)\,\mathrm{d}\xi \quad (8)$$

as well as the bilinear and trilinear forms, $\Gamma: V \times V \to V'$ and $\gamma: V \times V \times V \to \mathbb{R}$, respectively, via the following expression:

$$\langle \Gamma(\phi,\psi),\chi\rangle = \gamma(\phi,\psi,\chi) = \int_{-\pi}^{\pi} \phi(\xi)\psi^{(1)}\chi(\xi)\,\mathrm{d}\xi, \ \phi,\psi,\chi \in V. \quad (9)$$

Finally, *location-parameterized* input operators $B_i(\xi_a): \mathbb{R} \to V'$, $i = 1,\ldots,m$ are parameterized as follows:

$$\langle B_i(\xi_a)u_i, \phi\rangle = \left[\int_{-\pi}^{\pi} b_i(\xi)\phi(\xi)\,\mathrm{d}\xi\right]u_i(t), \quad \phi \in V, \quad (10)$$

and similarly, the disturbance operator below

$$\langle \mathscr{D}w(t), \phi\rangle = \left[\int_{-\pi}^{\pi} d(\xi)\phi(\xi)\,\mathrm{d}\xi\right]w(t), \quad \phi \in V. \quad (11)$$

Within the above framework, the original spatially distributed dynamics (1) can then be represented as an evolution equation of the following form:

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = -vAx(t) - Lx(t) + \Gamma(x(t),x(t)) + \sum_{i=1}^{m} B_i(\xi_a)u_i(t) + \mathscr{D}w(t) \quad \text{in } V'. \quad (12)$$

Notice that by setting

$$\mathscr{A}x = -vAx - Lx, \quad \mathscr{B}(\Xi) = [B_1(\xi_a) \ \ \ldots \ \ B_m(\xi_a)],$$
$$\langle \mathscr{F}(\phi),\psi\rangle = \langle \Gamma(\phi,\phi),\psi\rangle,$$

and

$$u(t) = [u_1(t) \ \ \ldots \ \ u_m(t)]^{\mathrm{T}}$$

one can re-write the above equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}x(t) = \mathscr{A}x(t) + \mathscr{F}(x(t)) + \mathscr{B}(\Xi)u(t) + \mathscr{D}w(t) \quad \text{in } V'. \quad (13)$$

2.2. Burgers' equation

Another example that belongs in the model of Eq. (1) is Burgers' equation. Specifically, the 1-D forced Burgers' equation (Burgers, 1974; Burns and Kang, 1991; Temam, 1998) with Dirichlet boundary conditions has the form

$$\frac{\partial}{\partial t}x(t,\xi)=\kappa\frac{\partial^2 x(t,\xi)}{\partial\xi^2}+\alpha x(t,\xi)-x(t,\xi)\frac{\partial}{\partial\xi}x(t,\xi)+\sum_{i=1}^{m}b_i(\xi)u_i(t)+d(\xi)w(t),$$

$$x(t,0)=0=x(t,\ell),$$
$$x(0,\xi)\in L^2(\Omega), \tag{14}$$

where $x(t,\xi)$ denotes the state, $\xi\in\Omega=[0,\ell]\subset\mathbb{R}$ is the spatial coordinate, $t\in[t_0,\infty)$ is the time variable, $u_i(t)$ denotes the control signal supplied to the ith actuating device, $b_i(\xi)$ denotes the spatial distribution of the ith actuating device, $w(t)$ the unknown exogenous input signal (disturbances) and $d(\xi)$ the spatial distribution of the disturbance. The state space in this case is $H=L^2(\Omega)$ endowed with the standard L^2 inner product and norm denoted by $\langle\cdot,\cdot\rangle$ and $|\cdot|$, respectively. Similarly, the interpolating spaces V and V' are given by

$$V=H_0^1(\Omega),\quad V'=H^{-1}(\Omega),$$

where $H_0^1(\Omega)=\{\phi\in L^2(\Omega):(\sum_{k=0}^{1}|\phi^{(k)}|^2)^{1/2}<\infty,\phi(0)=0=\phi(\ell)\}$. Define the elliptic operator $A:V\to V'$ by

$$\langle A\phi,\psi\rangle=\int_0^\ell\phi^{(1)}(\xi)\psi^{(1)}(\xi)\,\mathrm{d}\xi,\quad\phi,\psi\in V, \tag{15}$$

with domain $D(A)=H^2(\Omega)\cap H_0^1(\Omega)$, where H^2 is a Sobolev space of order 2, and the *location-parameterized* input operators $B_i(\xi_a):\mathbb{R}\to V'$, $i=1,\ldots,m$ as follows:

$$\langle B_i(\xi_a)u_i,\phi\rangle=\int_0^\ell b_i(\xi)\phi(\xi)\,\mathrm{d}\xi\,u_i(t),\quad\phi\in V. \tag{16}$$

In a similar fashion as in the Kuramoto–Sivashinsky equation, the nonlinear term can be expressed in terms of a trilinear form and subsequently as a nonlinear operator. The nonlinear operator (bilinear form) $\mathscr{F}:H^2(\Omega)\times H^2(\Omega)\to H^{-2}(\Omega)$ is given in variational form by

$$\langle\mathscr{F}(\phi),\psi\rangle=-\int_0^\ell\phi(\xi)\phi^{(1)}(\xi)\psi(\xi)\,\mathrm{d}\xi=-\frac{1}{2}\int_0^\ell\frac{\mathrm{d}}{\mathrm{d}\xi}(\phi^2(\xi))\psi(\xi)\,\mathrm{d}\xi,$$

and the disturbance operator $\mathscr{D}$ via

$$\langle\mathscr{D}w(t),\phi\rangle=\int_0^\ell d(\xi)\phi(\xi)\,\mathrm{d}\xi\,w(t),\quad\phi\in V. \tag{17}$$

Similar to Eq. (13), by setting $\mathscr{A}x=-\kappa Ax+\alpha x$, the 1-D controlled Burgers' equation can be placed in an abstract setting written as an evolution system (Dautray and Lions, 2000)

$$\dot{x}(t)=\mathscr{A}x(t)+\mathscr{F}(x(t))+\mathscr{B}u(t)+\mathscr{D}w(t)\quad\text{in } V'. \tag{18}$$

It is assumed that the spatial distribution of the actuating devices is spanned over a portion of the process spatial domain centered at a location ξ_a and is given by

$$b(\xi)=\begin{cases}\dfrac{1}{2\varepsilon} & \text{if } \xi_a-\varepsilon\leqslant\xi\leqslant\xi_a+\varepsilon,\\ 0 & \text{otherwise.}\end{cases}$$

Remark 2.1. Notice, that the above approximation for $b(\xi)$ avoids any regularity problems that may arise due to the unbounded nature of a point-wise (in space) actuator distribution (i.e., a spatial delta function) and also guarantees that $b\in L^2(\Omega)$ with norm $1/\sqrt{2\varepsilon}$. Furthermore, one must ensure that the location ξ_a of the actuator is such that *approximate controllability* of the linearized part of Eqs. (1) and (14) is attained (Curtain and Zwart, 1995). This essentially amounts to choosing each ξ_a in the orthogonal complement of the set of zeros of the eigenfunctions of the linear operator $-\nu A-L$ in Eq. (13) and $-\kappa A+\alpha I$ in Eq. (15).

Remark 2.2. The nonlinear operator satisfies $\langle\mathscr{F}(\phi),\phi\rangle=0$, $\phi\in V=H_0^1(\Omega)$, since

$$-\int_0^\ell\phi(\xi)\frac{\mathrm{d}\phi(\xi)}{\mathrm{d}\xi}\phi(\xi)\,\mathrm{d}\xi=-\frac{1}{3}\int_0^\ell\frac{\mathrm{d}}{\mathrm{d}\xi}(\phi^3(\xi))\,\mathrm{d}\xi=-\frac{1}{3}(\phi^3(\ell)-\phi^3(0))=0,$$

and hence the open-loop system with $w\equiv 0$ satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}|x(t,\cdot)|^2_{L^2(\Omega)}=-2\int_0^\ell\kappa\left(\frac{\partial}{\partial\xi}x(t,\xi)\right)^2\mathrm{d}\xi\leqslant-\alpha|x(t,\cdot)|^2_{L^2(\Omega)},$$

where α is a function of both the lower bound for κ and of the embedding constant in $H^2(\Omega)\hookrightarrow L_2(\Omega)$ (Adams, 1975).

2.3. Model reduction

A finite-dimensional approximation of the evolution systems (13) and (18) realized through the slow eigenmodes of the differential operator $\mathscr{A}$ can be derived, as it naturally emerges from the time-scale decomposition of the operator's eigenspectrum (Hyman and Nikolaenko, 1986; Jolly et al., 1990; Temam, 1998). In the present study, we adhere to the exposition presented in earlier work by Christofides and Armaou (2000). In summary, we consider the decomposition $H=H_s\oplus H_f$ in which H_s denotes the finite-dimensional space spanned by the unstable/slow part of $\mathscr{A}$'s eigenspectrum $H_s=\mathrm{span}\{\varphi_1,\ldots,\varphi_n\}$, with $H_f=\mathrm{span}\{\varphi_{n+1},\varphi_{n+2},\ldots\}$ being the infinite-dimensional one spanned by the stable/fast eigenfunctions. Under the above decomposition, one defines the orthogonal projection operators $\mathscr{P}_s$ and $\mathscr{P}_f$ that yield the following decomposition for the state

$$x=\mathscr{P}_sx+\mathscr{P}_fx=x_s+x_f.$$

Application of the projection operators $\mathscr{P}_s$ and $\mathscr{P}_f$ to the above system yields the following equivalent form of the process dynamics

$$\frac{\mathrm{d}x_s}{\mathrm{d}t}=\mathscr{A}_sx_s+\mathscr{F}_s(x)+\sum_{i=1}^{m}\mathscr{P}_sB_i(\xi)u_i(t)+\mathscr{P}_s\mathscr{D}w(t),$$
$$\frac{\mathrm{d}x_f}{\mathrm{d}t}=\mathscr{A}_fx_f+\mathscr{F}_f(x)+\sum_{i=1}^{m}\mathscr{P}_fB_i(\xi)u_i(t)+\mathscr{P}_f\mathscr{D}w(t),$$
$$x_s(0)=\mathscr{P}_sx(0),\quad x_f(0)=\mathscr{P}_fx(0), \tag{19}$$

where $\mathscr{A}_s$ is an n-dimensional matrix with diagonal structure $\mathscr{A}_s=\mathrm{diag}\{\lambda_i\}$, where $\{\lambda_i,\ldots,\lambda_n\}$ are the slow eigenvalues associated with H_s. Furthermore, under standard assumptions, one can show that

$$\mathscr{F}_s(x)=\mathscr{P}_s\mathscr{F}(x)=\mathscr{F}_s(x_s+x_f),$$
$$\mathscr{F}_f(x)=\mathscr{P}_f\mathscr{F}(x)=\mathscr{F}_f(x_s+x_f) \tag{20}$$

are Lipschitz continuous vector functions and the unbounded operator $\mathscr{A}_f$ the infinitesimal generator of an exponentially stable C_0 semigroup. Furthermore, the eigenvalues and eigenfunctions associated with the slow/unstable subsystem are given by $\lambda_j=-\nu j^4+j^2$, $\phi_j(\xi)=(1/\sqrt{\pi})\sin(j\xi)$, $j=1,2,\ldots,n$ for the Kuramoto–Sivashinsky equation and by $\lambda_j=-\kappa j^2\pi^2/\ell^2+\alpha$, $\phi_j(\xi)=\sqrt{2/\ell}\sin(j\pi\xi/\ell)$, $j=1,2,\ldots,n$ for Burgers' equation.

By neglecting the fast and stable infinite-dimensional subsystem one considers the state $\tilde{x}_s$ associated with the resulting finite-dimensional system given by

$$\frac{\mathrm{d}\tilde{x}_s}{\mathrm{d}t}=\mathscr{A}_s\tilde{x}_s+\mathscr{F}_s(\tilde{x}_s)+\sum_{i=1}^{m}\mathscr{P}_sB_i(\xi)u_i(t)+\mathscr{D}_sw(t),$$
$$\tilde{x}_s(0)=\mathscr{P}_sx(0). \tag{21}$$

On the basis of the above decomposition and model-reduction, one may now define meaningful control and optimal actuator placement objectives. All of them, however, should ensure the development of an integrated and systematic control and optimal actuator placement policy that employs actuator location-parameterized well-performing controllers synthesized for the approximate reduced-order system, while preserving stability, guaranteeing robustness and eventually enhancing the performance of the controlled system under consideration.

3. Integrated nonlinear controller synthesis and actuator placement

In the present section, the primary goal is to synthesize, for a fixed actuator location, a nonlinear state feedback control law that provides stability, induces the desired dynamic response characteristics for the controlled process, eventually allowing the attainment of a set of performance requirements. According to the proposed methodology, the controller is parameterized with respect to the actuator locations, and using a set of optimization criteria, a certain performance index is statically minimized with respect to the candidate actuator locations. The solution to the aforementioned optimization problem determines the set of actuator locations that induce the "best'' system performance. With that in mind, we now embark on the development of a fixed-actuator location-parameterized nonlinear controller synthesis.

3.1. Robust nonlinear controller design

To simplify the notation one sets

$$B(\xi_i) \triangleq B_i(\xi), \quad i = 1, 2, \ldots, m$$

to denote the input operator at location $\xi_i \in \Omega$. Therefore, Eq. (21) is now re-written as follows:

$$\frac{\mathrm{d}\tilde{x}_s}{\mathrm{d}t} = \mathscr{A}_s\tilde{x}_s + \mathscr{F}_s(\tilde{x}_s) + \sum_{i=1}^{m} \mathscr{P}_s B(\xi_i) u_i(t) + \mathscr{D}_s w(t),$$

$$\tilde{x}_s(0) = \mathscr{P}_s x(0).$$

Implicitly it is assumed that all actuating devices have identical specifications differing only at the location ξ that they are placed at.

Proposition 3.1. *Consider the above finite-dimensional system with fixed actuator locations $\Xi_0 = [\xi_1^0 \ \ldots \ \xi_m^0] \in \mathbb{R}^m$. The proposed controller takes the form*

$$u = -\mathscr{B}_s^{-1}(\Xi_0)\mathscr{F}_s(\tilde{x}_s) - \mathscr{K}\tilde{x}_s, \tag{22}$$

where $\mathscr{B}_s(\Xi_0) = \mathscr{P}_s\mathscr{B}(\Xi_0) = [\mathscr{P}_s B(\xi_1^0) \ \ldots \ \mathscr{P}_s B(\xi_m^0)]$, $u = [u_1 \ \ldots \ u_m]^{\mathrm{T}}$ and $\mathscr{K}$ being a suitably chosen $m \times n$ feedback gain matrix.

With the above control law, we have the following stability results.

Lemma 3.1. *Consider system (1) or (14) for which the number of unstable/slow modes is equal to the number of actuating devices (i.e., $n = m$). If matrix $\mathscr{B}_s$ is invertible, then the control law $u = -\mathscr{B}_s^{-1}(\Xi_0)\mathscr{F}_s(\tilde{x}_s) - \mathscr{K}\tilde{x}_s$ ensures that the solution $x(\xi, t)$ of the resulting finite-dimensional closed-loop system is exponentially stable for any initial condition $x_0 \in H$ in the absence of disturbances.*

Proof. If the choice of the feedback gain $\mathscr{K}$ is such that $\mathscr{A}_s - \mathscr{B}_s(\Xi_0)\mathscr{K}$ is a Hurwitz matrix, then the stability of the finite-dimensional closed-loop system readily follows by employing standard technical arguments found in Christofides and Armaou (2000). □

Remark 3.1. The proposed controller (22) results in partial feedback linearization. The rationale for such a choice is dictated by the fact that the actuator locations appear now explicitly (via $\mathscr{B}_s(\Xi_0)$) in the closed-loop system equations.

In the absence of disturbances, the above control law results in the following closed-loop system dynamics:

$$\frac{\mathrm{d}\tilde{x}_s}{\mathrm{d}t} = (\mathscr{A}_s - \mathscr{B}_s(\Xi_0)\mathscr{K})\tilde{x}_s. \tag{23}$$

Once the m actuator locations Ξ_0 are decided, one computes the gain $\mathscr{K}$ in order to meet a pre-specified set of closed-loop performance and robustness specifications at the controller design stage. Within the proposed context, however, one assumes that process disturbances enter explicitly the unstable subsystem's dynamic equations and the closed-loop system becomes

$$\frac{\mathrm{d}\tilde{x}_s}{\mathrm{d}t} = (\mathscr{A}_s - \mathscr{B}_s(\Xi_0)\mathscr{K})\tilde{x}_s + \mathscr{D}_s w(t), \tag{24}$$

where $\mathscr{D}_s$ denotes the spatial component of the disturbance ($=\mathscr{P}_s\mathscr{D}$) and $w(t)$ its temporal square integrable component. To further enhance the robustness of the controller gain $\mathscr{K}$ in Eq. (22), $\mathscr{D}_s$ is assumed to be the "worst'' spatial disturbance and therefore it is expressed as the sum of the first n modes, which equivalently translates to a class of disturbances whose spatial component "excites'' the first n unstable/slow modes.

Remark 3.2. A possible worst spatial distribution of process disturbances can be represented by the unit spatial step function with $d(\xi) = 1$. This describes disturbances entering uniformly at every single point in the spatial domain; hence there is no spatial bias. Alternatively, one may consider a disturbance that excites all the modes of the system and thus given by the sum $d(\xi) = \sum_{i=1}^{\infty} \phi_i(\xi)$. Thus, its projection $\mathscr{P}_s d(\xi) = \mathscr{P}_s \sum_{i=1}^{\infty} \phi_i(\xi)$ becomes the truncated sum $\sum_{i=1}^{n} \phi_i(\xi)$. Furthermore, its vector representation is explicitly given by

$$\mathscr{D}_s = \begin{bmatrix} \int_\Omega \left(\sum_{i=1}^{n} \phi_i(\xi) \right) \phi_1(\xi)\, \mathrm{d}\xi \\ \int_\Omega \left(\sum_{i=1}^{n} \phi_i(\xi) \right) \phi_2(\xi)\, \mathrm{d}\xi \\ \vdots \\ \int_\Omega \left(\sum_{i=1}^{n} \phi_i(\xi) \right) \phi_n(\xi)\, \mathrm{d}\xi \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}. \tag{25}$$

However, one may consider a spatial distribution of disturbances that affect specific modes, say the first n ones, differently, in the sense of $d(\xi) \approx \sum_{i=1}^{n} \alpha_i \phi_i(\xi)$, resulting in a vector representation similar to that in Eq. (25), but given by $\mathscr{D}_s = [\alpha_1 \ \alpha_2 \ \ldots \ \alpha_n]^{\mathrm{T}}$.

For fixed actuator locations $\Xi_0 \in \mathbb{R}^m$, one considers an associated $\mathscr{H}^\infty$ Riccati equation, and in that manner the control design reduces to that of minimizing the (RMS) L^2 gain γ of the closed-loop transfer function from $w(t)$ to $\tilde{x}_s(t)$. In particular, this requires the solution of

$$\mathscr{A}_s^{\mathrm{T}} P_\infty(\Xi_0) + P_\infty(\Xi_0)\mathscr{A}_s - P_\infty(\Xi_0)\left(\mathscr{B}_s(\Xi_0)\mathscr{B}_s^{\mathrm{T}}(\Xi_0) - \frac{1}{\gamma^2}\mathscr{D}_s\mathscr{D}_s^{\mathrm{T}} \right) P_\infty(\Xi_0) + Q = 0, \tag{26}$$

for the smallest possible $\gamma > 0$ where $Q = Q^{\mathrm{T}} > 0$. The resulting gain is then given by

$$\mathscr{K} = \mathscr{B}_s^{\mathrm{T}}(\Xi_0) P_\infty(\Xi_0). \tag{27}$$

Summarizing the main aspects of this subsection, one first decides on the "best'' actuator locations Ξ_0 using certain criteria (primarily controllability-related as seen in the sequel). Once this is achieved, then one computes the robust gain $\mathscr{K}$ from Eq. (27) using Eq. (26) wherein robustness is meant with respect to the worst possible class of disturbances entering in the slow/unstable subsystem dynamic description considered earlier. This then provides *both* a spatial and a temporal robustness of the control signal. Finally, the control law is provided by Eq. (22).

3.2. Optimal actuator placement

By parameterizing the above controller (22) by the admissible actuator locations (cf. Remark 3.1), one may proceed to the location optimization stage. First, let us define $\Omega_c \subset \Omega$ as the set of *admissible candidate actuator locations* given by

$$\Omega_c = \left\{ \xi \in \Omega : \int_\Omega b_i(\xi)\phi_j(\xi)\,\mathrm{d}\xi \neq 0, \right. \\ \left. \text{for at least all } j \geqslant n \text{ and all } i = 1, \ldots, m \right\}, \tag{28}$$

where $\phi_j(\xi)$ are the eigenmodes associated with the slow/unstable dynamics. Associated with the above, we define

$$\Theta_c = \{ \Xi = [\xi_1 \ \ldots \ \xi_m] : \xi_i \in \Omega_c \text{ and} \\ \mathscr{B}_s(\Xi) = [\mathscr{P}_s B(\xi_1) \ \ldots \ \mathscr{P}_s B(\xi_m)] \text{ is invertible} \},$$

which denotes the set of admissible locations for a given group of actuators whose actuating devices are placed at locations Ξ that yield approximate controllability and which ensure that $\mathscr{B}_s(\Xi)$ is invertible.

Remark 3.3. The search for optimal actuator locations $\Xi = [\xi_1 \ \ldots \ \xi_m]$ will be restricted to the candidate locations in the set Θ_c which guarantees approximate controllability, i.e., all m-dimensional Ξ whose elements are in Ω_c. This condition would ensure that (at least) the first n modes are controllable by each of the m actuating devices. Therefore, the candidate actuator locations would guarantee that the slow finite-dimensional subsystem is a controllable one and would also provide the necessary conditions for the invertibility of $\mathscr{B}_s(\Xi_0)$ as required for the implementation of the control law (22). A related scheme, presented in Armaou and Demetriou (2006), considered the effects of the actuator location on specific modes, using the notion of spatial norms. An issue that was also considered there was that of spillover, and in that case the spectrum was divided into three different subsystems, the slow, the fast and a middle-range. The actuator locations, via the use of spatial norms, allowed for minimization of spillover effects by incorporating information about the mid-range modes. This was subsequently capitalized in the design of fault detection schemes for nonlinear distributed processes (Demetriou and Armaou, 2007).

Remark 3.4. With regard to the system of interest and considering periodic odd functions and point-wise actuator distributions (i.e., $b_i(\xi) = b(\xi_i) = \delta(\xi - \xi_i)$), the set of admissible actuator locations (i.e., locations that yield approximate controllability) is given by

$$\Omega_c = \{ \xi \in \Omega : \sin(j\xi_i) \neq 0, \text{ for at least all } j \geqslant n \text{ and all} \\ i = 1, \ldots, m \}.$$

In essence this is the set of all locations that do not coincide with the zeros of the first m eigenfunctions. In a similar fashion, one can easily define Ω_c for Burgers' equation in Eq. (14).

Remark 3.5. It should be noted that the characterization given for Ω_c in Remark 3.4 is applicable only in the absence of constraints on the manipulated input. In the presence of control constraints, the set of admissible actuator locations (i.e., locations that guarantee constrained controllability or stabilizability) becomes dependent on the initial condition due to the limitations imposed by the constraints on the set of stabilizing initial conditions that can be steered to the origin under a fixed actuator location. Therefore, for a given initial condition, the set of admissible locations under control constraints will in general be only a subset of Ω_c. In addition to further restricting the set of admissible actuator locations, the presence of constraints on the manipulated input requires that the feedback control law be re-designed (e.g., using bounded control techniques) to account for the constraints and provide an explicit characterization of the constrained stability region in terms of the magnitude of the constraints, the initial condition and the locations of the control actuators. The reader is referred to El-Farra et al. (2003) for results on bounded control of systems described by nonlinear PDEs, where a Lyapunov-based approach is developed to synthesize stabilizing bounded nonlinear controllers and simultaneously characterize the set of stabilizing actuator locations and initial conditions under control constraints.

Two different methods are summarized here that provide an integrated controller synthesis and actuator placement framework. The first one is an "open-loop'' procedure in which the actuator placement problem is decoupled from the controller design one. In this case, one *first* selects the actuator locations so that the system is "more'' controllable in the above sense, and *then* designs the feedback gain based on some performance and/or robustness criteria (e.g., $\mathscr{H}^2/\mathscr{H}^\infty$ as in Eq. (26)). The second method introduces a coupling between the actuator placement and the controller synthesis one by considering sets of matrix inequalities that provide both a feedback gain and optimal actuator locations that enhance the stability of the closed-loop system.

Method 1a (*Open-loop—all modes*). According to the basic principles of this method, one may simply use controllability criteria to first identify the optimal actuator locations that would yield a "more'' controllable system. Therefore, one must choose the "optimal'' actuator locations Ξ from the set Θ_c that maximize the bound $\alpha = \alpha(\Xi)$ in

$$\langle \mathscr{W}_C(\Xi)\phi, \phi \rangle \geqslant \alpha \|\phi\|^2, \quad \phi \in H, \tag{29}$$

where $\mathscr{W}_C(\Xi)$ denotes the Ξ-parameterized controllability Gramian operator of the linearized system, defined via

$$\langle \mathscr{A}\mathscr{W}_C(\Xi)\phi_1, \phi_2 \rangle + \langle \mathscr{W}_C(\Xi)\mathscr{A}^*\phi_1, \phi_2 \rangle \\ = -\langle \mathscr{B}(\Xi)\mathscr{B}^*(\Xi)\phi_1, \phi_2 \rangle, \quad \Xi \in \Theta_c, \tag{30}$$

for $\phi_1, \phi_2 \in \mathscr{D}(\mathscr{A}^*)$. In other words, the "best'' actuator locations are given via

$$\Xi^{\mathrm{opt}} = \arg \sup_{\Xi \in \Theta_c} \alpha(\Xi), \tag{31}$$

where $\alpha(\Xi)$ is given in Eq. (29), and which makes the Gramian operator "more'' coercive. Once the actuator locations Ξ^{opt} via Eq. (31) are found, one may proceed to the design of the (location-decoupled) controller gain $\mathscr{K}$ as summarized by Eqs. (26), (27) in Remark 3.2. Using $\mathscr{H}^2/\mathscr{H}^\infty$ cost functionals, one may then find an optimal gain that corresponds to these "most'' controllable actuator locations.

The above procedure finds the best actuator locations that make "more'' controllable *all* the available modes. Since we are interested in the controllability of the unstable/slow subsystem, we introduce a certain "bias'' for the controllability of the slow subsystem being more controllable, and therefore we arrive at the following modification:

Method 1b (*Open-loop—slow modes*). Choose the "optimal" actuator locations Ξ from the set Θ_c that maximize the bound $\alpha(\Xi)$ in

$$z^{\mathrm{T}} W_c(\Xi) z \geqslant \alpha(\Xi) z^{\mathrm{T}} z, \quad z \in \mathbb{R}^n, \tag{32}$$

where $W_c(\Xi)$ denotes the Ξ-parameterized controllability Gramian of the finite-dimensional subsystem defined via

$$\mathscr{A}_s W_c(\Xi) + W_c(\Xi)\mathscr{A}_s^{\mathrm{T}} = -\mathscr{B}_s(\Xi)\mathscr{B}_s^{\mathrm{T}}(\Xi), \quad \Xi \in \Theta_c. \tag{33}$$

In other words, the "best" actuator locations are now given via

$$\Xi^{\mathrm{opt}} = \arg \sup_{\Xi \in \Theta_c} \alpha(\Xi) = \arg \sup_{\Xi \in \Omega_c} \lambda_{\min}(W_c(\Xi))$$

or

$$\Xi^{\mathrm{opt}} = \arg \sup_{\Xi \in \Theta_c} \mathrm{trace}\,[W_c(\Xi)]. \tag{34}$$

The above would make the unstable system "more" controllable, where the measure of controllability is naturally realized through the positiveness of the associated Gramian. Once these actuator locations are found, one may then proceed to the design of the feedback gain which exhibits robustness with respect to the worst spatial distribution of the disturbances, as presented in the previous subsection via Eq. (26). Notice that one may consider "closed-loop" techniques for the actuator placement, in which performance and/or stability considerations are utilized instead of only enhanced controllability.

Method 2a (*Locations yielding robust stability*). First, using solely stability criteria, the optimal location set $\Xi \in \mathbb{R}^m$ is found as the one that provides quadratic stability to the system under consideration.

Remark 3.6. Using solely stability criteria, one searches in the set Θ_c to find the actuator group locations Ξ_j that yield quadratic stability. This translates to finding both the locations Ξ_j (if more than one) within Θ_c and the feedback gain $\mathscr{K}$ that render the following inequalities:

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K})^{\mathrm{T}} P + P(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}) < 0 \tag{35}$$

feasible. However, this procedure may generate quite a conservative controller gain $\mathscr{K}$.

Remark 3.7. In the above optimization problem, where the feedback gain $\mathscr{K}$ is computed, the above LMI can be convexified by setting $Y = \mathscr{K}Q$. Hence the system is quadratically stabilizable if and only if there exist $Q > 0$ and Y such that the LMI

$$\mathscr{A}_s Q + Q\mathscr{A}_s + \mathscr{B}_s(\Xi_j) Y + Y^{\mathrm{T}} \mathscr{B}_s^{\mathrm{T}}(\Xi_j) < 0 \tag{36}$$

is feasible. If this is achieved, then the quadratic function $V = \tilde{x}^{\mathrm{T}} Q^{-1} \tilde{x}$ establishes quadratic stability under the linear state feedback control law: $u = YQ^{-1}\tilde{x}_s$. If more than one set Ξ_j of actuator locations renders the above LMIs feasible, then one may adopt the approach presented in Demetriou and Kazantzis (2004b) and extend it to the case of placing a group of actuators. In this case the feedback gain $\mathscr{K}$ and the actuator group locations Ξ_j are found so that in addition to quadratic stabilizability and enhanced controllability, they also provide a certain robustness with respect to disturbances. In particular, one chooses the feedback gain $\mathscr{K}$ so that the (RMS) L^2 gain γ in the following polytopic LMIs is minimized:

$$\begin{bmatrix} (\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K})Q + Q(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K})^{\mathrm{T}} + \mathscr{D}_s\mathscr{D}_s^{\mathrm{T}} & QC_z^{\mathrm{T}} \\ C_z Q & -\gamma^2 I \end{bmatrix} \leqslant 0,$$
$$\Xi_j \in \Theta_c, \tag{37}$$

where $\mathscr{D}_s, C_z$ denote the matrix distributions of the process disturbance and controlled output, respectively, in the closed-loop representation

$$\dot{\tilde{x}}_s(t) = \mathscr{A}_s x(t) + \mathscr{B}_s(\Xi_j) u + \mathscr{D}_s w(t),$$
$$z(t) = C_z \tilde{x}(t). \tag{38}$$

In essence, one minimizes the effects of the disturbance on the controlled output, i.e., minimize the H^∞ norm of the closed-loop transfer function $T_{zw}(s, \Xi_j) \triangleq C_z(Is - (\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}))^{-1}\mathscr{D}_s$. Following Remark 3.2, to further enhance this robustness property of the feedback gain, one may choose $C_z \equiv I_n$ which ensures that the effects of the disturbance on every single component of $\tilde{x}_s$ are minimal. Furthermore, the "worst" case of spatial distribution of the disturbance vector may be assumed, which translates to setting $d(\xi) = 1$ so that the disturbance affects each of the states (or modes) of the slow subsystem. In summary, the LMIs above with D_s given by Eq. (25) in Remark 3.2 and $C_z = I_n$ will ensure that the effects of the disturbance on every single state are minimized and that the "worst" possible disturbance distribution function is considered. In relation to the original PDE, it translates to having the disturbance enter at every single point in the spatial domain Ω, with the property that it affects at least the slow eigenmodes, (i.e., $d(\xi) = \mathscr{P}_s \sum_{j=1}^{\infty} \phi_j(\xi)$).

By coupling together the actuator location with the robustness of the feedback controller requirements, one then simply considers

$$\Xi^{\mathrm{opt}} = \arg \inf_{\Xi \in \Theta_c} \gamma(\Xi), \tag{39}$$

where now Eq. (26) (or equivalently Eq. (37)) becomes

$$\mathscr{A}_s^{\mathrm{T}} P_\infty(\Xi) + P_\infty(\Xi)\mathscr{A}_s - P_\infty(\Xi)\left(\mathscr{B}_s(\Xi)\mathscr{B}_s^{\mathrm{T}}(\Xi) - \frac{1}{\gamma^2(\Xi)}\mathscr{D}_s\mathscr{D}_s^{\mathrm{T}}\right)P_\infty(\Xi) + I = 0. \tag{40}$$

Finally, in this case, the associated *truly* robust controller gain (worst spatial disturbance, smallest RMS gain) is given by

$$\mathscr{K} = \mathscr{B}_s(\Xi^{\mathrm{opt}}) P_\infty(\Xi^{\mathrm{opt}}). \tag{41}$$

Method 2b (*Locations yielding optimal performance*). While the above method provided both the optimal actuator location and the feedback gain by considering robustness bounds on the $\mathscr{H}^\infty$ norm of the closed-loop system, the method below considers an approach in which the feedback gain and the optimal locations are found by minimizing the $\mathscr{H}^2$ norm of the closed-loop transfer function. Motivated by the results derived in de Oliveira and Geromel (2000), we consider the infinite horizon $\mathscr{H}^2$ cost functional

$$J = \int_{t_0}^{\infty} [\tilde{x}_s^{\mathrm{T}}(\tau) Q \tilde{x}_s(\tau) + u^{\mathrm{T}}(\tau) R u(\tau)]\, \mathrm{d}\tau. \tag{42}$$

Its optimal value for a given actuator location Ξ_0 is given by

$$J^{\mathrm{opt}} = \tilde{x}_s^{\mathrm{T}}(t_0) P_2 \tilde{x}_s(t_0),$$

where P_2 is the solution to the associated $\mathscr{H}^2$ algebraic Riccati equation. When one parameterizes the Riccati solution by the actuator locations Ξ and minimizes this location-parameterized optimal cost, one would arrive at the optimal location. The optimal locations are therefore given via

$$\Xi^{\mathrm{opt}} = \arg \min_{\Xi \in \Theta_c} \tilde{x}_s^{\mathrm{T}}(t_0) P_2(\Xi) \tilde{x}_s(t_0), \tag{43}$$

where $P_2(\Xi)$ is the solution to the Ξ-parameterized Riccati equation

$$\mathscr{A}_s^{\mathrm{T}} P_2(\Xi) + P_2(\Xi)\mathscr{A}_s - P_2(\Xi)\mathscr{B}_s(\Xi) R^{-1} \mathscr{B}_s^{\mathrm{T}}(\Xi) P_2(\Xi) + Q = 0, \tag{44}$$

and the corresponding optimal gain is thus given by $\mathscr{K}(\Xi^{\rm opt}) = -R^{-1}\mathscr{B}_s^{\rm T}(\Xi^{\rm opt})P_2(\Xi^{\rm opt})$.

The dependence of Eq. (43) on initial conditions $\tilde{x}_s(t_0)$ can be circumvented by assuming that $\tilde{x}_s(t_0)$ is a random vector uniformly distributed on the unit sphere, and hence the measure is simply given by the trace of the matrix $P_2(\Xi)$:

$$\Xi^{\rm opt} = \arg\min_{\Xi\in\Theta_c} {\rm trace}[P_2(\Xi)].$$

Remark 3.8. Similar to the extended LQR problem considered in Eq. (43), one may employ an $\mathscr{H}^2$ formulation, in which the $\mathscr{H}^2$ norm of the closed-loop transfer function

$$T_{zw}(s,\Xi) = I(sI - (\mathscr{A}_s - \mathscr{B}_s(\Xi)\mathscr{K}(\Xi)))^{-1}\mathscr{D}_s$$

is minimized, and in this case the criterion (43) becomes

$$\Xi^{\rm opt} = \arg\min_{\Xi\in\Theta_c} {\rm trace}[\mathscr{D}_s^{\rm T}P_2(\Xi)\mathscr{D}_s].$$

In this case, the scheme takes into consideration the spatial distribution of disturbances, as in the first method via Eqs. (39)–(41).

4. An integrated framework for control and actuator switching policies

The procedure described by Eq. (35) or (37) provides a method for *simultaneously* finding the *optimal* actuator locations and the *global* feedback gain $\mathscr{K}$ common to possibly more than one candidate actuator group. More precisely, it finds those N group locations Ξ within a subset $\Theta_c^{\rm lmi} \subset \Theta_c$ that render (37) feasible, where $\Theta_c^{\rm lmi}$ is defined via

$$\Theta_c^{\rm lmi} = \{\Xi \in \Theta_c : \text{LMIs in (37) are feasible for } \textit{some } \gamma\}.$$

Even further, the proposed method identifies those locations within $\Theta_c^{\rm lmi}$ that render the LMIs (37) feasible *and* simultaneously provide robustness with respect to the worst spatial disturbance distribution. The resulting nonlinear control law is then given by

$$u = -\mathscr{B}_s^{-1}(\Xi^{\rm opt})\mathscr{F}_s(\tilde{x}_s) - \mathscr{K}\tilde{x}_s. \tag{45}$$

The reason for considering many candidate group locations Ξ within the subset $\Theta_c^{\rm lmi}$ that all have a *common* feedback gain will become apparent later in the present section, where one can further simplify the proposed switching actuator policy. In summary, one keeps the *same* linear part of the above control signal (i.e., $-\mathscr{K}\tilde{x}_s$ in Eq. (45)) that is common to *all* (say N) candidate actuator groups (regardless of actuators used) within $\Theta_c^{\rm lmi}$. Every Δt time units, one switches to a different group of actuating devices (each of which is optimal in its own right if that were to be used) using a certain switching policy, in order to further enhance the performance of the closed-loop system. The issues at hand are twofold:

(i) *The actuator switching policy*: How does one decide at the beginning of a given time interval $[t_k, t_k+\Delta t)$ which actuator group to activate, and which $(N-1)$ ones to keep dormant for the duration of that time interval (i.e., for Δt time units)?
(ii) *The control policy*: Once a given set of actuators is chosen to be active over the interval $[t_k, t_k+\Delta t)$, what is the controller signal that must be supplied to this group of actuators?

Both issues will be simultaneously addressed in the sequel. In the first subsection, both a *common* feedback gain $\mathscr{K}$ and its associated *common* Lyapunov function ($\tilde{x}_s^{\rm T}P\tilde{x}_s$ in Eq. (35) or $\tilde{x}_s^{\rm T}Q^{-1}\tilde{x}_s$ in Eq. (37)) are assumed to exist thus simplifying the stability-under-switching arguments. In the next subsection, it is assumed that each candidate group of actuator locations (that makes the inversion of $\mathscr{B}_s(\Xi)$ in Eq. (22) feasible) has its own (and not necessarily common) feedback gain derived from the solution of an associated optimal H^2/H^∞ problem with a common Lyapunov function. Lastly, the assumption of the existence of a common Lyapunov function is relaxed, and thus each actuator group is allowed to have its own associated feedback gain, but the decision to switch is now based on multiple Lyapunov function (MLF) arguments.

4.1. Common feedback with common Lyapunov function

It is assumed that N different groups of actuators exist, each of which satisfy Eq. (35) or (37). In other words, each of the $n\times m$ matrices $\mathscr{B}_s(\Xi_j)$ representing a given group of actuators, satisfies the LMIs for $j = 1, 2, \ldots, N$ with the same common feedback gain $\mathscr{K}$. Thus, the control law $u = -\mathscr{B}_s^{-1}(\Xi_j)\mathscr{F}_s(\tilde{x}_s) - \mathscr{K}\tilde{x}_s$, would result in a stable closed-loop system regardless which of the N actuator groups is utilized. Since a common feedback gain exists, stability under switching is guaranteed (Liberzon, 2003). What is needed now, is a procedure/policy for switching to a specific actuator group based on performance enhancement arguments.

Using a *closed-loop* performance measure at the beginning of each time subinterval, one typically considers a *cost-to-go* quadratic cost

$$J(t_k,\Xi) = \int_{t_k}^{\infty} \tilde{x}_s^{\rm T}(\tau)Q_{cl}\tilde{x}_s(\tau)\,{\rm d}\tau \tag{46}$$

corresponding to the actuator location-parameterized closed-loop system

$$\dot{\tilde{x}}_s = (\mathscr{A}_s - \mathscr{B}_s(\Xi)\mathscr{K})\tilde{x}_s, \quad \Xi\in\Theta_c^{\rm lmi}. \tag{47}$$

The optimal value of $J(t_k,\xi)$, for a given choice of actuator locations Ξ, is given by

$$J^{\rm opt}(t_k,\Xi) = \tilde{x}_s^{\rm T}(t_k)\Sigma_1(\Xi)\tilde{x}_s(t_k), \quad \Xi\in\Theta_c^{\rm lmi}, \tag{48}$$

where $\Sigma_1(\Xi)$ is the location-parameterized solution to the *location-parameterized Lyapunov equation*

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi)\mathscr{K})^{\rm T}\Sigma_1(\Xi) + \Sigma_1(\Xi)(\mathscr{A}_s - \mathscr{B}_s(\Xi)\mathscr{K}) = -Q_{cl}, \quad \Xi\in\Theta_c^{\rm lmi}. \tag{49}$$

The choice of the actuator group to be activated in the time interval $[t_k, t_k+\Delta t)$ would then be dictated by the minimum of the optimal cost $J^{\rm opt}(t_k,\Xi)$ with respect to all candidate actuator locations in $\Theta_c^{\rm lmi}$. This observation is captured by the following:

Proposition 4.1. *Consider the slow subsystem and assume that N actuator groups can be found from the set* Θ_c *that render the LMIs feasible (i.e., from* $\Theta_c^{\rm lmi}$*) and result in a common gain* $\mathscr{K}$ *satisfying Eq.* (35) *or* (37). *For each of these N actuator groups, solve the corresponding location-parameterized Lyapunov equations* (49). *Then the actuator group to be activated in the interval* $[t_k, t_k+\Delta t)$ *is given by*

$$\Xi^{\rm opt} = \arg\min_{\Xi\in\Theta_c^{\rm lmi}} \tilde{x}_s^{\rm T}(t_k)\Sigma_1(\Xi)\tilde{x}_s(t_k). \tag{50}$$

Furthermore, the associated control signal that must be supplied to the active actuator group is given by

$$u^{\rm opt}(t) = -\mathscr{B}_s^{-1}(\Xi^{\rm opt})\mathscr{F}_s(\tilde{x}_s) - \mathscr{K}\tilde{x}_s(t), \quad t\in[t_k, t_k+\Delta t). \tag{51}$$

Finally, stability under switching is guaranteed by the common Lyapunov function in the LMIs in Eq. (35) *or* (37).

Under the above considerations, the proposed algorithm for actuator switching is presented below.

Switching Algorithm 1.

1. Find the N actuator groups that satisfy either (35) or (37).
2. For each of these locations Ξ_i, $i=1,2,\ldots,N$, find the N solutions to the associated location-parameterized Lyapunov equations

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi)\mathscr{K})^{\mathrm{T}}\Sigma_1(\Xi) + \Sigma_1(\Xi)(\mathscr{A}_s - \mathscr{B}_s(\Xi)\mathscr{K}) = -Q_{cl}, \Xi \in \Theta_c^{\mathrm{lmi}}.$$

3. At the beginning of each time interval $[t_k, t_k+\Delta t)$, form the N inner products $\widetilde{x}_s^{\mathrm{T}}(t_k)\Sigma_1(\Xi)\widetilde{x}_s(t_k)$ and choose the actuator group to be activated over $[t_k, t_k+\Delta t)$ via

$$\arg\min_{\Xi\in\Theta_c^{\mathrm{lmi}}} \widetilde{x}_s^{\mathrm{T}}(t_k)\Sigma_1(\Xi)\widetilde{x}_s(t_k).$$

4. Repeat step 3 for the next time interval $[t_{k+1}, t_{k+1}+\Delta t)$.

Remark 4.1. The idea of using a closed-loop performance measure to choose the appropriate control configuration (from a pre-selected set of possible candidates) has also been pursued in the context of reconfiguration-based fault-tolerant control of both lumped (Mhaskar et al., 2006) and distributed (El-Farra and Ghantasala, 2007) parameter nonlinear systems. The basic idea in these works is to evaluate (and compare) on-line, at the time of fault detection, a certain cost-to-go for each configuration to determine (and subsequently activate) the one with the smallest cost. In these works, however, switching between control configurations takes place only when a fault occurs, whereas switching in the current work is introduced actively at fixed time-intervals in order to enhance the robustness of the closed-loop system to spatiotemporally varying disturbances. Also, due to the nonlinear structure of the closed-loop systems obtained in Mhaskar et al. (2006) and El-Farra and Ghantasala (2007), the costs-to-go there cannot be determined explicitly in terms of the closed-loop state (unlike the case in Eqs. (46)–(48)) and are therefore evaluated through on-line simulations instead. Another important difference to note is that in the lumped parameter system case considered in Mhaskar et al. (2006), each control configuration refers to a distinct manipulated input, whereas in the spatially distributed case considered here (and in El-Farra and Ghantasala, 2007), the different configurations share the same manipulated input and differ only in the spatial placement of the actuating devices.

4.2. Common Lyapunov function with different feedback gains

Considering Eq. (35) to be valid for all N actuator groups might appear rather restrictive. Furthermore, the resulting controller gain $\mathscr{K}$ might be too conservative. Therefore, we now consider N actuator groups that yield an invertible $\mathscr{B}_s(\Xi)$ and satisfy some improved controllability properties as described by Eq. (34), i.e., $\Xi \in \Theta_c$. For each of these actuator groups, one designs an associated optimal feedback gain $\mathscr{K}(\Xi)$. If in addition to the improved controllability properties, one imposes a condition similar to Eq. (35) for each of the N actuator groups

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j))^{\mathrm{T}}P + P(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j)) < 0,\quad k=1,2,\ldots,N, \tag{52}$$

then stability under switching can also be guaranteed. Now one has to find a method for switching to a specific actuator group for the duration of the time interval $[t_k, t_k+\Delta t)$.

Proposition 4.2. *Consider the slow subsystem and assume that there are N actuator groups that in addition to the enhanced controllability measures given by Eq.* (34), *they also satisfy Eq.* (52), *where each $\mathscr{K}(\Xi_j)$ (associated with each actuator group) is designed to satisfy certain performance and robustness measures unique to its corresponding actuator group. Then the actuator group to be activated in the interval $[t_k, t_k+\Delta t)$ is given by*

$$\Xi^{\mathrm{opt}} = \arg\min_{\Xi\in\Theta_c} \widetilde{x}_s^{\mathrm{T}}(t_k)\Sigma_2(\Xi)\widetilde{x}_s(t_k), \tag{53}$$

where $\Sigma_2(\Xi_j)$ is the positive definite solution to the Lyapunov equation associated with each of the actuator groups (*cf. Eq.* (49))

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j))^{\mathrm{T}}\Sigma_2(\Xi_j) + \Sigma_2(\Xi_j)(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j)) = -Q_{cl},\quad j=1,2,\ldots,N. \tag{54}$$

The associated control signal that must be supplied to the active actuator group (*cf. Eq.* (51)) *is given by*

$$u^{\mathrm{opt}}(t) = -\mathscr{B}_s^{-1}(\Xi^{\mathrm{opt}})\mathscr{F}_s(\widetilde{x}_s) - \mathscr{K}(\Xi^{\mathrm{opt}})\widetilde{x}_s(t), t \in [t_k, t_k+\Delta t). \tag{55}$$

Finally, stability under switching is guaranteed by the common Lyapunov function.

In this case, the proposed algorithmic procedure for an actuator activation/switching policy is the following one:

Switching Algorithm 2.

1. Find the N actuator groups and the N feedback gains $\mathscr{K}(\Xi)$ associated with them that satisfy

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j))^{\mathrm{T}}P + P(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j)) < 0,\quad j=1,2,\ldots,N,$$

2. For each of these locations Ξ_j, $j=1,2,\ldots,N$, find the N solutions to the location-parameterized Lyapunov equations

$$(\mathscr{A}_s - \mathscr{B}_s(\Xi_j)\mathscr{K}(\Xi_j))^{\mathrm{T}}\Sigma_2(\Xi_j) + \Sigma_2(\Xi_j)(\mathscr{A}_s - \mathscr{B}_s(\Xi_j) \times \mathscr{K}(\Xi_j)) = -Q_{cl},\quad j=1,2,\ldots,N.$$

3. At the beginning of each time interval $[t_k, t_k+\Delta t)$, form the N inner products $\widetilde{x}_s^{\mathrm{T}}(t_k)\Sigma_2(\Xi)\widetilde{x}_s(t_k)$ and choose the actuator group to be activated over $[t_k, t_k+\Delta t)$ via

$$\arg\min_{\Xi\in\Theta_c^{\mathrm{lmi}}} \widetilde{x}_s^{\mathrm{T}}(t_k)\Sigma_2(\Xi)\widetilde{x}_s(t_k).$$

4. Repeat step 3 for the next time interval $[t_{k+1}, t_{k+1}+\Delta t)$.

4.3. MLFs and different feedback gains

In this case, it is assumed that no common Lyapunov function can be found that satisfies Eq. (52). Therefore, the decision to switch to a specific actuator group will be based on MLFs stability criteria. Towards that, we rewrite the system with its N actuator groups as follows:

$$\frac{\mathrm{d}\widetilde{x}_s}{\mathrm{d}t} = \mathscr{A}_s\widetilde{x}_s + \mathscr{F}_s(\widetilde{x}_s) + [\mathscr{B}_s(\Xi_1)\ \ \ldots\ \ \mathscr{B}_s(\Xi_N)]\mathbf{u}(t) + \mathscr{D}_s w(t),\quad \widetilde{x}_s(0) = \mathscr{P}_s x(0), \tag{56}$$

where the $Nm \times 1$ control vector $\mathbf{u}(t)$ has the specific form

$$\mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_N(t) \end{bmatrix}. \tag{57}$$

The individual m-dimensional controllers u_i, $i=1,\ldots,N$ are the control signals that are fed to each actuator group $\mathscr{B}_s(\Xi_i)$. To bring it to a form that is familiar to a switched control setting, we define the *N basic controllers* (Savkin and Evans, 2002)

$$\mathbf{u}_1(t)=\begin{bmatrix}u_1(t)\\ \mathbf{0}_{n\times 1}\\ \vdots\\ \mathbf{0}_{n\times 1}\\ \mathbf{0}_{n\times 1}\end{bmatrix},\quad \mathbf{u}_2(t)=\begin{bmatrix}\mathbf{0}_{n\times 1}\\ u_2(t)\\ \vdots\\ \mathbf{0}_{n\times 1}\\ \mathbf{0}_{n\times 1}\end{bmatrix},$$
$$\mathbf{u}_{N-1}(t)=\begin{bmatrix}\mathbf{0}_{n\times 1}\\ \mathbf{0}_{n\times 1}\\ \vdots\\ u_{N-1}(t)\\ \mathbf{0}_{n\times 1}\end{bmatrix},\quad \mathbf{u}_N(t)=\begin{bmatrix}\mathbf{0}_{n\times 1}\\ \mathbf{0}_{n\times 1}\\ \vdots\\ \mathbf{0}_{n\times 1}\\ u_N(t)\end{bmatrix}. \tag{58}$$

When the ith basic controller $\mathbf{u}_i$ is switched in, it results in

$$\begin{aligned}&[\mathscr{B}_s(\Xi_1)\ \ldots\ \mathscr{B}_s(\Xi_i)\ \ldots\ \mathscr{B}_s(\Xi_N)]\mathbf{u}_i(t)\\ &\quad=[\mathscr{B}_s(\Xi_1)\ \ldots\ \mathscr{B}_s(\Xi_i)\ \ldots\ \mathscr{B}_s(\Xi_N)]\begin{bmatrix}\mathbf{0}_{n\times 1}\\ \vdots\\ \mathbf{0}_{n\times 1}\\ u_i(t)\\ \mathbf{0}_{n\times 1}\\ \vdots\\ \mathbf{0}_{n\times 1}\end{bmatrix}\\ &\quad=\mathscr{B}_s(\Xi_i)u_i(t),\end{aligned} \tag{59}$$

which designates that the ith actuator group is active and is the one that receives the control signal $u_i(t)$. The remaining $(N-1)$ actuators receive the 0 control signal, designating that they are dormant. Following Eq. (59), the problem of actuator and controller switching of Eq. (21) now reduces to that of controller switching for the system (56) in which the structure of the N basic controllers is given by Eq. (58).

In order to address the issue of stability under switching, we consider a quadratic criterion for construction of the feedback gain $\mathscr{K}(\Xi)$ associated with each actuator group $\mathscr{B}_s(\Xi)$. In other words, for each actuator group $\mathscr{B}_s(\Xi)$ with $\Xi\in\Theta_c$, we consider the infinite horizon quadratic cost function

$$J(t_0,\Xi)=\int_{t_0}^{\infty}[\tilde{x}_s^{\mathrm{T}}(t)Q\tilde{x}_s(t)+u^{\mathrm{T}}(t)Ru]\,\mathrm{d}t. \tag{60}$$

Its optimal value is given by

$$J^*(t_0,\Xi)=\tilde{x}_s^{\mathrm{T}}(t_0)P(\Xi)\tilde{x}_s(t_0), \tag{61}$$

where $P(\xi)$ solves the Ξ-parameterized algebraic Riccati equation

$$\begin{aligned}&\mathscr{A}_s^{\mathrm{T}}\Sigma_3(\Xi)+\Sigma_3(\Xi)\mathscr{A}_s-\Sigma_3(\Xi)\left(\mathscr{B}_s(\Xi)R^{-1}\mathscr{B}_s^{\mathrm{T}}(\Xi)\right.\\ &\quad\left.-\frac{1}{\gamma^2}\mathscr{D}_s\mathscr{D}_s^{\mathrm{T}}\right)\Sigma_3(\Xi)+Q=0.\end{aligned} \tag{62}$$

The reason that the quadratic cost is chosen, is because the optimal value of Eq. (60) can serve as a Lyapunov function for the linearized system

$$\begin{aligned}&\dot{\tilde{x}}_s(t)=\mathscr{A}_s\tilde{x}_s(t)+\mathscr{B}_s(\Xi)u(t)+\mathscr{D}_sw(t),\\ &u(t)=-R^{-1}\mathscr{B}_s^{\mathrm{T}}(\Xi)\Sigma_3(\Xi)\tilde{x}_s(t).\end{aligned} \tag{63}$$

We thus define the following sequence of Lyapunov functions.

Definition 4.1. We define the Lyapunov function corresponding to a system with the ith actuator on and with the ith robust controller $u_i(t)=-R^{-1}\mathscr{B}_s^{\mathrm{T}}(\Xi_i)\Sigma_3(\Xi_i)$ supplied to it via

$$\begin{aligned}&V_i(\tilde{x}_s(t_0))=J^*(t_0,\Xi_i)=\tilde{x}_s^{\mathrm{T}}(t_0)\Sigma_3(\Xi_i)\tilde{x}_s(t_0),\\ &\quad i=1,2,\ldots,N.\end{aligned} \tag{64}$$

Remark 4.2. Since the closed-loop system

$$\dot{\tilde{x}}_s(t)=(\mathscr{A}_s-\mathscr{B}_s(\Xi)R^{-1}\mathscr{B}_s^{\mathrm{T}}(\Xi)\Sigma_3(\Xi))\tilde{x}_s(t) \tag{65}$$

is stable with the matrix $\mathscr{A}_s-\mathscr{B}_s(\Xi)R^{-1}\mathscr{B}_s(\Xi)\Sigma_3(\Xi)$ being Hurwitz, we then have that

$$\begin{aligned}V_i(\tilde{x}_s(t_2))&=\tilde{x}_s^{\mathrm{T}}(t_2)\Sigma_3(\Xi_i)\tilde{x}_s(t_2)\leqslant V_i(\tilde{x}_s(t_1))\\ &=\tilde{x}_s^{\mathrm{T}}(t_1)\Sigma_3(\Xi_i)\tilde{x}_s(t_1),\end{aligned} \tag{66}$$

for all t_1,t_2 with $t_0\leqslant t_1\leqslant t_2<\infty$.

The above remark would allow us to use MLFs in order to guarantee stability under actuator/controller switching. Relevant to the stability of switched systems, we summarize a theorem from Branicky (1998) that provides conditions for stability under switching.

Theorem 4.1 (*Branicky, 1998*). *Given the N-switched system*

$$\dot{x}(t)=f_{\sigma(t)}(x(t)),\quad \sigma(t)\in\mathscr{I}=\{1,2,\ldots,N\}, \tag{67}$$

suppose that each vector field f_i has an associated Lyapunov-like function V_i in the region Ω_i, an equilibrium point $x_{\mathrm{eq}}=0$, and suppose $\bigcup_i\Omega_i=\mathbb{R}^n$. Let $\sigma(t)$ be a given switching sequence such that $\sigma(t)$ can take the value of i only if $x(t)\in\Omega_i$, and in addition

$$V_i(x(t_{i_k}))\leqslant V_i(x(t_{i_{k-1}})), \tag{68}$$

where t_{i_k} denotes the kth time that the vector field f_i is switched in, i.e., $\sigma(t_{i_k}^-)\neq\sigma(t_{i_k}^+)=i$. Then, system (67) is Lyapunov stable.

Now, Eq. (21) is re-written as

$$\begin{aligned}\frac{\mathrm{d}\tilde{x}_s}{\mathrm{d}t}&=\mathscr{A}_s\tilde{x}_s+\mathscr{F}_s(\tilde{x}_s)+\mathscr{B}_s(\Xi_i)u_i+\mathscr{D}_sw\\ &=\mathscr{A}_s\tilde{x}_s+\mathscr{F}_s(\tilde{x}_s)-\mathscr{B}_s(\Xi_i)(\mathscr{B}_s^{-1}(\Xi_i)\mathscr{F}_s(,\tilde{x}_s)+\mathscr{K}(\Xi_i)\tilde{x}_s)+\mathscr{D}_sw\\ &=(\mathscr{A}_s-\mathscr{B}_s(\Xi_i)R^{-1}\mathscr{B}_s^{\mathrm{T}}(\Xi_i)\Sigma_3(\Xi_i))\tilde{x}_s+\mathscr{D}_sw\\ &=\mathscr{A}_{s_i}\tilde{x}_s+\mathscr{D}_sw,\end{aligned} \tag{69}$$

where $\mathscr{A}_{s_i}\triangleq\mathscr{A}_s-\mathscr{B}_s(\Xi_i)R^{-1}\mathscr{B}_s^{\mathrm{T}}(\Xi_i)\Sigma_3(\Xi_i)$. Alternatively, it may be written as

$$\begin{aligned}\frac{\mathrm{d}\tilde{x}_s}{\mathrm{d}t}&=\mathscr{A}_s\tilde{x}_s+\Gamma_s(\tilde{x}_s,\tilde{x}_s)+\mathscr{B}_s(\Xi_{\sigma(t)})u_{\sigma(t)}+\mathscr{D}_sw\\ &=\mathscr{A}_{s_{\sigma(t)}}\tilde{x}_s+\mathscr{D}_sw,\quad \sigma(t)\in\mathscr{I}=\{1,2,\ldots,N\},\end{aligned} \tag{70}$$

where $\sigma:[0,\infty)\to\mathscr{I}$ is the switching signal which is assumed to be a piecewise continuous function of time. Now, for each value that σ assumes in the index set $\mathscr{I}$, the slow dynamics is governed by a different set of differential equations reflecting the different actuator group with its associated controller used, as presented in Eq. (64). We adopt the terminology and notation used in El-Farra and Christofides (2003) (see also Christofides and El-Farra, 2005) and therefore we let t_{i_k} and t_{i_k+1} to denote the kth time that the ith actuator (and consequently the ith subsystem) is switched in and out; thus $\sigma(t_{i_k}^+)=\sigma(t_{i_k+1}^-)=i$. Relating to Eq. (70), we have that $\mathrm{d}\tilde{x}_s/\mathrm{d}t=\mathscr{A}_{s_i}\tilde{x}_s+\mathscr{D}_sw$, for $t_{i_k}\leqslant t\leqslant t_{i_k+1}$. A condition for closed-loop stability under switching can be established as follows: If at any time T the following holds:

$$V_j(\tilde{x}_s(T))<V_j(\tilde{x}_s(t_{j_*+1})), \tag{71}$$

for some $j\in\mathscr{I}$, $j\neq i$, where t_{j_*+1} denotes the time that the jth actuator (equivalently jth subsystem) was last utilized, i.e., $\sigma(t_{j_*+1}^+)\neq\sigma(t_{j_*+1}^-)=j$, then choosing $\sigma(T^+)=j$ guarantees that the closed-loop

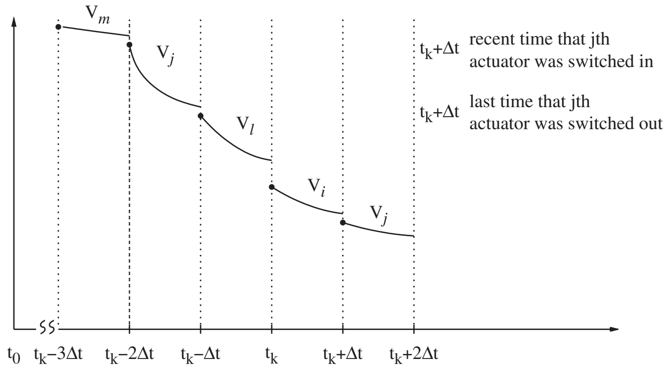

Fig. 1. Stability analysis using MLF's corresponding to different actuator groups.

system is asymptotically stable. In summary, the above stability requirement says that the jth subsystem is only switched on if its corresponding value function at the current time is less than its corresponding value function evaluated at the last time it was utilized (switched out). If the actuator groups $\mathscr{B}_s(\Xi_i)$ are switched on and off every Δt time units with $\Delta t \geqslant \tau_d$, where τ_d denotes the average dwell time, then using the criterion

$$\begin{aligned} \Xi_{j_*} &= \arg \min_{\Xi_j \in \Theta_c} V_j(\widetilde{x}_s(t_k)) \\ &= \arg \min_{\Xi_j \in \Theta_c} \widetilde{x}_s(t_k) \Sigma_3(\Xi_j) \widetilde{x}_s(t_k), \quad t_k \leqslant t \leqslant t_k + \Delta t, \end{aligned} \tag{72}$$

subject to the condition of Eq. (71) would provide both stability under switching and optimal actuator switching. The following analysis provides a glimpse on the stability under switching in which the jth actuator group is used in the current subinterval $[t_k + \Delta t, t_k + 2\Delta t)$ and was last switched on two time subintervals prior, i.e., at $[t_k - 2\Delta t, t_k - \Delta t)$, see Fig. 1:

$$\begin{aligned} & V_j(\widetilde{x}_s(t_k + \Delta t)) \\ & \quad < V_i(\widetilde{x}_s(t_k + \Delta t)) \text{from (72)}, \ \ i \neq j, \ \ \text{use } j\text{thin } [t_k + \Delta t, t_k + 2\Delta t) \\ & \quad \leqslant V_i(\widetilde{x}_s(t_k)) \quad \text{from (66)} \\ & \quad < V_\ell(\widetilde{x}_s(t_k)) \text{from (72)}, \ \ \ell \neq j, i, \ \ \text{use } i\text{th in } [t_k, t_k + \Delta t) \\ & \quad \leqslant V_\ell(\widetilde{x}_s(t_k - \Delta t)) \quad \text{from (66)} \\ & \quad < V_j(\widetilde{x}_s(t_k - \Delta t)) \text{from (72) use } \ell\text{th in } [t_k - \Delta t, t_k) \\ & \quad \leqslant V_j(\widetilde{x}_s(t_k - 2\Delta t)) \quad \text{from (66)} \\ & \quad < V_m(\widetilde{x}_s(t_k - 2\Delta t)) \text{from (72) use } j\text{th in } [t_k - 2\Delta t, t_k - \Delta t). \end{aligned} \tag{73}$$

By setting $t_{j_k} = t_k + \Delta t$ (most recent time that jth was switched in) and $t_{j_{k-1}} = t_k - \Delta t$ (previous time that jth was switched out), we then have

$$V_j(\widetilde{x}_s(t_{j_k})) < V_j(\widetilde{x}_s(t_{j_{k-1}})) \tag{74}$$

since $V_j(\widetilde{x}_s(t_k + \Delta t)) < V_j(\widetilde{x}_s(t_k - \Delta t))$ follows from Eq. (73). Note that Eq. (74) is condition (68) of Theorem 4.1, and therefore stability under switching easily follows. If, of course, i is replaced by j (i.e., the previous time that the jth actuator was switched out was at $t_k^- + \Delta t$ and the most recent time that the jth actuator was switched in was $t_k + \Delta t$), then stability under switching easily follows from Eq. (66).

In this particular case, the proposed actuator switching algorithm assumes the following structure:

Switching Algorithm 3.

1. Find the N actuator groups using any of the methods presented in Section 3.
2. For each of these actuator groups, find the N feedback gains $\mathscr{K}(\Xi)$ via the solution of

$$\begin{aligned} & \mathscr{A}_s^{\mathrm{T}} \Sigma_3(\Xi_i) + \Sigma_3(\Xi_i) \mathscr{A}_s - \Sigma_3(\Xi_i) \\ & \quad \times \left(\mathscr{B}_s(\Xi_i) R^{-1} \mathscr{B}_s^{\mathrm{T}}(\Xi_i) - \frac{1}{\gamma^2} \mathscr{D}_s \mathscr{D}_s^{\mathrm{T}} \right) \Sigma_3(\Xi_i) + Q = 0, \\ & \quad i = 1, 2, \ldots, N. \end{aligned}$$

3. At the beginning of each time interval $[t_k, t_k + \Delta t)$, form the N inner products $\widetilde{x}_s^{\mathrm{T}}(t_k) \Sigma_3(\Xi) \widetilde{x}_s(t_k)$ and choose the actuator group to be activated over $[t_k, t_k + \Delta t)$ via

$$\arg \min_{\Xi \in \Theta_c} \widetilde{x}_s^{\mathrm{T}}(t_k) \Sigma_3(\Xi) \widetilde{x}_s(t_k).$$

4. If the chosen actuator group has never been activated before, implement it for $[t_k, t_k + \Delta t)$, else check first if Eq. (71) holds. If this condition holds, implement the chosen group for $[t_k, t_k + \Delta t)$, else exclude the chosen group from the set of candidate actuators and select another one according to

$$\arg \min_{\Xi \in \Theta_c'} \widetilde{x}_s^{\mathrm{T}}(t_k) \Sigma_3(\Xi) \widetilde{x}_s(t_k),$$

where Θ_c' is Θ_c minus the excluded actuator group.
5. Repeat steps 3 and 4 for the next time interval $[t_{k+1}, t_{k+1} + \Delta t)$.

Remark 4.3. We note that the control and switching algorithms presented in this work—which are designed on the basis of the approximate finite-dimensional slow subsystem of Eq. (21)—continue to enforce the desired closed-loop stability and performance properties when implemented on the infinite-dimensional closed-loop system of Eq. (19) provided that the separation between the slow and fast eigenvalues of the differential operator is sufficiently large. This can be justified using singular perturbation arguments similar to the ones used in Christofides and Daoutidis (1997).

5. Simulation studies

5.1. Application to the Kuramoto–Sivashinsky equation

We considered Eq. (1) with the instability parameter $v = 0.2$ which results in two unstable modes $\lambda_i = 0.8$, $i = 1, 2$, and thus we considered $n = 2$ as the dimension of the slow/unstable subsystem. The infinite-dimensional system was approximated by 100 modes and thus the resulting 100-dimensional system was assumed to capture the salient features of the system in Eq. (1). The computations were carried out via codes written in Matlab® run on a dual processor DELL® workstation (Xeon 2.8 GHz, 2 × 2 GB). The resulting finite-dimensional system of ODEs was integrated using the stiff ODE solver from the Matlab® ODE library, routine ode15s based on a fourth-order Runge–Kutta scheme. The high-order system of differential equations was simulated for the time interval $[t_0, t_f] = [0, 2]$ and the actuator switching was implemented every $\Delta t = 0.1$.

The initial condition of the state was $x_0(\xi) = \sin(\xi) + \xi^3 - \pi^2 \xi$. The spatial distribution of the actuating devices was considered to be of the following form

$$b_i(\xi) = \begin{cases} \dfrac{1}{\varepsilon} & \text{if } \xi_i - \dfrac{\varepsilon}{2} \leqslant \xi \leqslant \xi_i + \dfrac{\varepsilon}{2}, \\ 0 & \text{otherwise.} \end{cases}$$

A total of $N = 5$ actuator groups were assumed to be available for switching in the time interval $[t_0, t_f] = [0, 1]$, and placed at

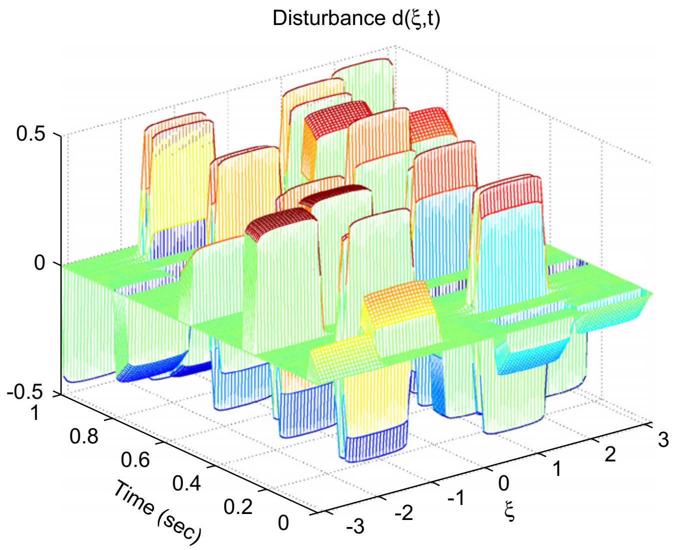

Fig. 2. Evolution of spatiotemporal disturbance $d(\xi, t)w(t)$.

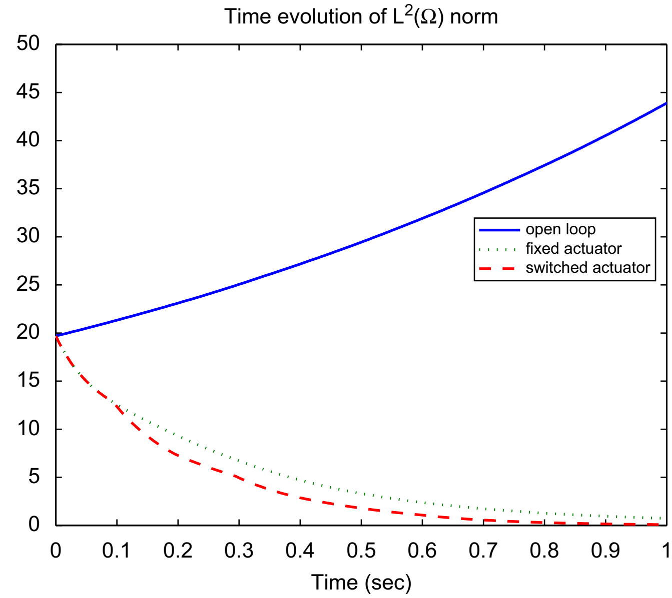

Fig. 3. Evolution of system norm $\|x(t,\cdot)\|_2$ for open-loop (solid), closed-loop with a fixed actuator (dotted) and closed-loop with a switching actuator (dashed).

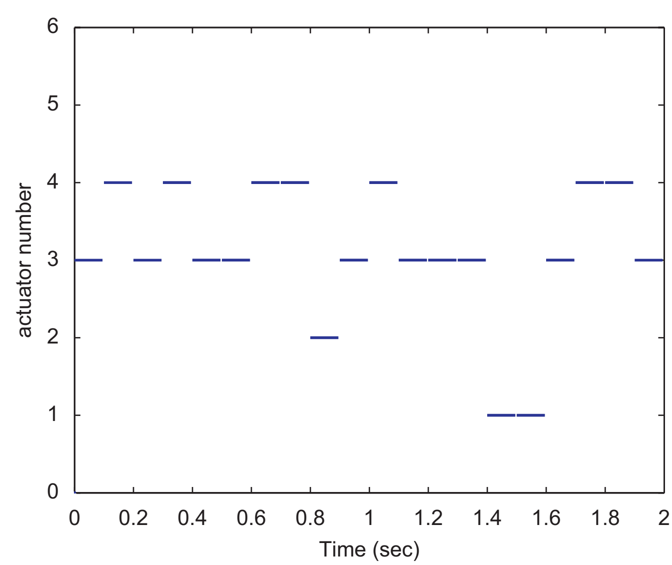

Fig. 4. Actuator activation sequence for the controlled Kuramoto–Sivashinsky equation.

locations

$\mathscr{B}_s(\Xi_1) = B(-1.3509)\ \ B(0.5969)]$,
$\mathscr{B}_s(\Xi_2) = [B(-1.6022)\ \ B(-1.2881)]$,
$\mathscr{B}_s(\Xi_3) = [B(1.2881)\ \ B(1.6022)]$,
$\mathscr{B}_s(\Xi_4) = [B(-2.2934)\ \ B(1.7907)]$,
$\mathscr{B}_s(\Xi_5) = [B(-1.9792)\ \ B(2.6075)]$,

where the length ε was taken to be equal to $2\pi/100$. These locations were found using Method 1b, which in essence considers locations that render the matrix $\mathscr{B}_i(\Xi)$ invertible and enhanced the controllability of the first n modes. The disturbance term $d(\xi)w(t)$ was chosen to represent a moving spatiotemporal disturbance whose spatial distribution $d(\xi)$ changes within the spatial domain Ω. It is better expressed as $d(\xi, t)w(t)$ (depicted in Fig. 2), while being analytically represented by

$$\sum_{k=1}^{10}\left[H(\xi-\xi_k(t)) - H\left(\xi-\xi_k(t)-\frac{2\pi}{5}\right)\right] \times \left[H(t-t_k) - H\left(t-t_k-\frac{t_f-t_0}{10}\right)\right]\sin(2\pi t f_k),$$

where for $k = 1, 2, \ldots, 10$

$$t_k = t_0 + (k-1)\frac{t_f - t_0}{20}, \quad \xi_k(t) = -\pi + j_k\frac{2\pi}{5},$$
$$j_k = \mathrm{mod}\left(\mathrm{round}\left(\frac{20t}{t_f}\right), 5\right).$$

It should be noted that j_k takes values in the integer set $\{0, 1, 2, 3, 4\}$ and thus the left endpoint of the spatially "moving" characteristic function (sequentially) takes on the values $\xi_k(t) = -\pi$, $-\pi + 2\pi/5$, $-\pi+4\pi/5$, $-\pi+6\pi/5$ and $-\pi+8\pi/5$. Similarly, the forcing frequency f_k sequentially takes values from the first five eigenfrequencies of the open-loop system.

The evolution of the system norm for the open-loop case, the closed-loop with a fixed actuator and the closed-loop with a switching actuator is depicted in Fig. 3. The feedback gain for the fixed actuator case was in fact found as the LQR gain that provided the smallest value of the optimal cost $\tilde{x}_s^{\mathrm{T}}(t_0)\Sigma_3(\Xi)\tilde{x}_s(t_0)$ with $Q = I_s$, $R = 0.01$ and $\gamma = \infty$. In other words, for each of the five actuator groups above, the solution to the corresponding ARE (52) was found and the group that resulted in the smallest value of the optimal cost was the one chosen. In this case, the optimal actuator was $\mathscr{B}_s(\Xi_3)$. Similarly, the same five feedback gains were used in the case of a switching actuator group. The switching policy used was the one given in Switching Algorithm 3. The associated actuator activation sequence is depicted in Fig. 4, where it is observed that $\mathscr{B}_3(\Xi)$ and $\mathscr{B}_4(\Xi)$ are used for most of the time, with $\mathscr{B}_1(\Xi)$ and $\mathscr{B}_2(\Xi)$ used occasionally and $\mathscr{B}_5(\Xi)$ not used at all. Finally, the cumulative norm

$$\|x(t,\xi)\|_{L^2(t_0,t_f;L^2(\Omega))} = \left(\int_0^1\int_{-\pi}^{\pi} x^2(t,\xi)\,\mathrm{d}\xi\,\mathrm{d}t\right)^{1/2}$$

is tabulated in Table 1 for all three cases. As can be observed from Fig. 3, the performance improvement for the switching actuator case is substantial, which suggests that the switching actuator case can better address spatiotemporally varying disturbances.

Table 1
$\sqrt{\int_0^1 \|x(t,\xi)\|^2_{L^2(\Omega)}\,\mathrm{d}t}$ norm

Open loop	31.01
Fixed actuator	6.98
Switched actuator	6.26

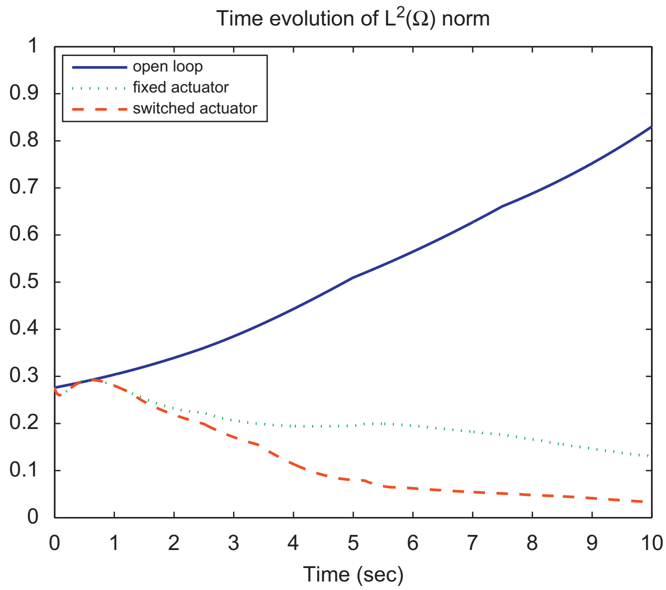

Fig. 5. Evolution of system norm $\|x(t,\cdot)\|_2$ for open-loop (solid), closed-loop with a fixed actuator (dotted) and closed-loop with a switching actuator (dashed).

5.2. Application to Burgers' equation

Similarly, we considered Eq. (14) in the spatial interval $[0,\ell]=[0,1]$ with $\kappa=0.001$ and $\alpha=10\kappa(\pi/\ell)^2$ leading to three unstable modes $\lambda_1=0.0888$, $\lambda_2=0.0592$ and $\lambda_3=0.0099$ and in this case we considered $n=3$ as the dimension of the slow/unstable subsystem. The infinite dimensional system was once again approximated by 100 modes. The high-order system of differential equations was simulated for the time interval $[t_0,t_f]=[0,8]$ and the actuator switching was implemented every $\Delta t=0.1$.

The initial condition of the state was $x_0(\xi)=-\xi^3+\ell^2\xi^2$. The same spatial distribution of the actuating devices presented in the previous example was also used here. A total of $N=5$ actuator groups were assumed to be available for switching in the time interval $[t_0,t_f]=[0,8]$, and placed at locations

$$\mathscr{B}_s(\Xi_1)=[B(0.335)\ \ B(0.6250)\ \ B(0.805)],$$
$$\mathscr{B}_s(\Xi_2)=[B(0.145)\ \ B(0.295)\ \ B(0.645)],$$
$$\mathscr{B}_s(\Xi_3)=[B(0.105)\ \ B(0.405)\ \ B0.925)],$$
$$\mathscr{B}_s(\Xi_4)=[B(0.135)\ \ B(0.585)\ \ B(0.865)],$$
$$\mathscr{B}_s(\Xi_5)=[B(0.185)\ \ B(0.505)\ \ B(0.915)],$$

where the length ε was taken to be equal to $\ell/100$. These locations were found using Method 1b, which in essence considers locations that render the matrix $\mathscr{B}_s(\Xi_i)$ invertible and enhanced the controllability of the first n modes.

The evolution of the system norm for the open-loop case, the closed-loop with a fixed actuator and the closed-loop with a switching actuator is depicted in Fig. 5. The feedback gain for the fixed actuator case was in fact found as the LQR gain that provided the smallest value of the optimal cost $\tilde{x}_s^{\mathrm{T}}(t_0)\Sigma_3(\Xi)\tilde{x}_s(t_0)$ with $Q=0.01I_s$, $R=0.01$ and $\gamma=\infty$. The optimal actuator was $\mathscr{B}_s(\Xi_1)$. Similarly, the same five feedback gains were used in the case of a switching actuator group. The switching policy used was the one given in Switching Algorithm 3. The associated actuator activation sequence is depicted in Fig. 6. The switching scheme utilized mostly actuator #5 and #2 with occasional use of actuator #1 and #3 with no use of actuator #4 at all. Such an actuator employment can naturally change if the spatially moving disturbance is altered in such a way that actuator #4 would have more control authority over a specific time interval. Finally, the cumulative norm

$$\|x(t,\xi)\|_{L^2(t_0,t_f;L^2(\Omega))}=\left(\int_0^8\int_0^\ell x^2(t,\xi)\,\mathrm{d}\xi\,\mathrm{d}t\right)^{1/2}$$

is tabulated in Table 2 for all three cases. Once again, it is observed that actuator switching results in enhanced convergence of the L^2 state norm.

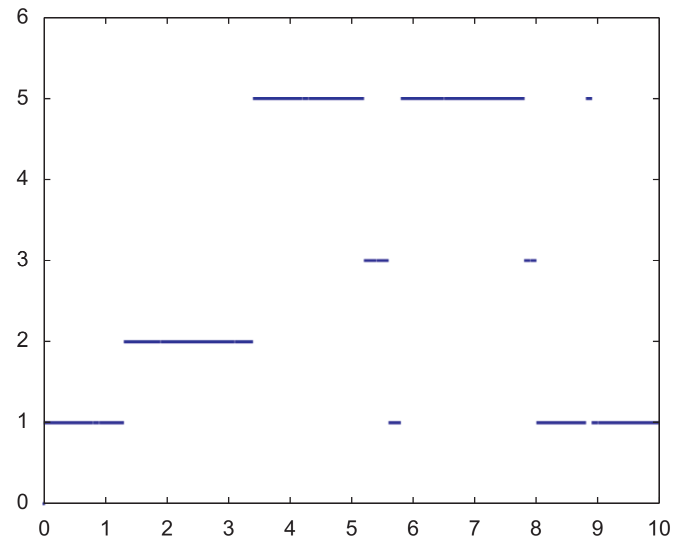

Fig. 6. Actuator activation sequence for the controlled Burgers' equation.

Table 2
$\sqrt{\int_0^8 \|x(t,\xi)\|^2_{L^2(\Omega)}\,\mathrm{d}t}$ norm

Open loop	1.344
Fixed actuator	0.626
Switched actuator	0.474

6. Conclusions

In this work, a control activation scheme for a class of nonlinear transport processes modeled by highly dissipative PDEs was presented. Using performance measures for the linearized process, a switching actuator-plus-controller scheme that accounts for process nonlinearities and spatiotemporally varying disturbances was developed on the basis of an appropriate reduced-order model that captures the dominant dynamic characteristics of the PDE. A set of candidate actuators, optimally placed within the spatial domain, was used as the set of available devices to switch from, and a performance metric was used to switch from one actuator to another at the beginning of a given time interval. The key idea of the switching policy is the activation of actuators that lie spatially closer to the spatiotemporal disturbances, thereby improving the control authority of the actuators and enhancing the ability of the system to minimize the effects of this class of disturbances. Extensive simulation studies for two representative examples were presented and demonstrated the performance enhancement that can be achieved using the proposed actuator scheduling scheme.

An immediate extension of the current work will be on the use of output feedback for the case when the full state is inaccessible and only partial measurements from sensing devices placed at certain locations in the spatial domain are available. The issue of sensor placement and the subsequent compensator design with now actuator-plus-sensor and controller switching will be examined in a forthcoming publication.

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