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Robust optimal control and estimation of constrained nonlinear processes

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Abstract

This work proposes a robust optimal and control estimation strategy for a broad class of nonlinear processes with uncertain variables and actuator constraints. Using combination of a high-gain observer with a bounded robust optimal state feedback controller synthesized via Lyapunov's direct method and the inverse optimal approach, we construct a bounded robust near-optimal dynamic output feedback controller with well-characterized performance and stability properties. The controller enforces, in the presence of active constraints, exponential stability and robust asymptotic output tracking with arbitrary degree of attenuation of the effect of the uncertainty on the output of the closed-loop system. In addition, the controller design yields an explicit and intuitive characterization of the regions in state space where the aforementioned properties are guaranteed. The developed controller is successfully applied to an exothermic chemical reactor.

Keywords: Inverse optimality; Lyapunov's direct method; Bounded control; State estimation; Chemical processes

1. Introduction

The development of an effective process control strategy to achieve safe and profitable process operation is a central task at the interface of process control and operation. At the stage of controller design, this requires that the characteristics of the process to be controlled be explicitly integrated into the design. For example, many important industrial processes exhibit highly nonlinear behavior, involve time-varying uncertain variables, and are subject to hard actuator constraints. The presence of uncertainty and input constraints, if not appropriately accounted for in the controller design, may cause serious deterioration in the closed-loop process performance and even lead to instability. Motivated by these problems, significant research has focused on the synthesis of robust controllers for nonlinear uncertain processes (see, for example, Kravaris & Palanki, 1988; Arkun & Calvet, 1992; Christofides, Teel & Daoutidis, 1996), and the analysis and control of processes with constraints (Kothare, Campo, Morari & Nett, 1994; Chmielewski & Manousiouthakis, 1998; Kapoor & Daoutidis, 1998; Valluri & Soroush, 1998; Rao & Rawlings, 1999).

At this stage, however, existing process control methods lead to the synthesis of controllers that can deal with either model uncertainty or input constraints but not simultaneously or effectively with both. This clearly limits the achievable control quality and subsequent process performance. A unified framework for control of nonlinear systems that explicitly accounts for uncertainty and constraints is therefore needed. A natural approach to address the problem of controlling constrained uncertain nonlinear processes is the design of robust optimal controllers that expend minimal control effort to achieve robust stabilization. One way to do this is within the nonlinear H_∞ control framework (van der Schaft, 1992). However, the practical applicability of this approach is still questionable because the explicit construction of the controllers requires the analytic solution of the steady-state Hamilton–Jacobi–Isaacs (HJI) equation which is not a feasible task except for simple problems. An appealing approach to robust optimal controller design, which does not require solving the HJI equation, is the inverse optimal approach proposed by Kalman and introduced recently in the context of robust stabilization in Freeman and Kokotovic (1996). This approach has been employed for the design of robust optimal controllers in Freeman and Kokotovic 1996) and Krstic and Li (1997).

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In a recent work (El-Farra & Christofides, 2000a) (see also El-Farra & Christofides, 1999), we developed a direct robust optimal controller design method that integrates robustness, optimality, and explicit constraint handling capabilities in the controller design and provides, simultaneously, an explicit characterization of the regions of guaranteed closed-loop stability. Using a general state space Lyapunov framework, the developed controller design method led to the synthesis of bounded robust optimal state feedback controllers that enforce asymptotic stability and asymptotic robust output tracking in the presence of uncertainty and constraints. Due to their optimality, the controllers possess larger regions of closed-loop stability than those obtained under existing process control techniques and, therefore, enhance process operation by enlarging the set of feasible operating conditions and allowing the process to operate safely and reliably under conditions deemed infeasible under other control strategies.

Motivated by recent advances in output feedback controller design (Khalil, 1994; Teel & Praly, 1994; Isidori, 1999; Christofides, 2000; El-Farra & Christofides, 2000b) and the fact that the complete state often cannot be measured, we focus in this paper on the task of designing bounded robust near-optimal output feedback controllers that guarantee the aforementioned closed-loop stability and performance properties in the presence of process uncertainty and active input constraints for initial conditions and uncertainty in large compact sets whose size is limited by the size of stability region obtained under state feedback. We show that the performance properties obtained under the output feedback controller can be made arbitrarily close to those obtained under the optimal state feedback controllers when the gain of the observer is selected to be sufficiently large. The proposed control method is applied successfully to an exothermic chemical reactor example.

2. Preliminaries

We consider the class of continuous-time single-input single-output nonlinear processes with uncertain variables with the following state-space description:

$$\dot{x} = f(x) + g(x)\text{sat}(u) + \sum_{k=1}^{q} w_k(x)\theta_k(t)$$
$$y = h(x) \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the vector of state variables, $u \in \mathbb{R}$ denotes the manipulated input, $\theta_k(t) \in \mathcal{W} \subset \mathbb{R}$ denotes the kth uncertain (possibly time varying) but bounded variable taking values in a nonempty compact convex subset $\mathcal{W}$ of $\mathbb{R}$, $y \in \mathbb{R}$ denotes the output to be controlled, and sat refers to the standard saturation nonlinearity. The uncertain variable $\theta_k(t)$ may describe time-varying parametric uncertainty and/or exogenous disturbances. It is assumed that $w_k(0) = 0$ and therefore the origin is an equilibrium point of the system of Eq. (1). The vector functions $f(x)$, $w_k(x)$ and $g(x)$, and the scalar function $h(x)$ are assumed to be sufficiently smooth.

We begin by reviewing the concept of inverse optimality introduced in the context of robust stabilization in (Freeman & Kokotovic, 1996) and used as a tool for robust optimal controller design. To this end, consider the system of Eq. (1) with $q = 1$ and suppose there exists a positive definite radially unbounded C^1 scalar function V such that

$$\inf_{u \in \mathbb{R}} \sup_{\theta \in \mathcal{W}} (L_f V + L_g V u + L_w V \theta) < 0 \quad \forall\ x \neq 0 \tag{2}$$

Also, let $l(x)$ and $R(x)$ be two continuous scalar functions such that $l(x) \geq 0$ and $R(x) > 0\ \forall\ x \in \mathbb{R}^n$ and consider the cost functional

$$J = \int_0^{\infty} (l(x) + uR(x)u)\mathrm{d}t \tag{3}$$

A stabilizing control law $u(x)$ is said to be inverse optimal with respect to the cost functional of Eq. (3) if it can be expressed in the following form

$$u = -p(x) = -\frac{1}{2}R^{-1}(x)L_g V \tag{4}$$

where the negative definiteness of $\dot{V}$ is achieved with the control $u^* = 1/2p(x)$ that is

$$\sup_{\theta \in \mathcal{W}} \dot{V} = L_f V - \frac{1}{2}L_g V p(x) + |L_w V|\theta_b < 0 \tag{5}$$

where $\theta_b = \|\theta\| = \text{ess. sup.}|\theta(t)|$, $t \geq 0$. When the function $-l(x)$ is set equal to the right hand side of Eq. (5), then $V(x)$ is a solution to the following steady state HJI equation.

$$0 \equiv l(x) + L_f V - \frac{1}{4}L_g V R^{-1}(x) L_g V + |L_w V|\theta_b \tag{6}$$

and the optimal (minimal) value of J is $V(x(0))$. The approach is inverse because the functions $l(x)$ and $R(x)$ are a posteriori determined by the chosen stabilizing feedback control law, rather than a priori specified by the designer.

3. Bounded robust near-optimal output feedback controller synthesis

Our objective is to synthesize a robust nonlinear dynamic output feedback controller of the form:

$$\dot{\omega} = \mathcal{F}(\omega, y, \bar{v})$$
$$u = \mathcal{P}(\omega, y, \bar{v}, t) \tag{7}$$

where $\omega \in \mathbb{R}^s$ is a state, $\mathcal{F}(\omega, y, \bar{v})$ is a vector function, $\mathcal{P}(\omega, y, \bar{v}, t)$ is a *bounded* scalar function (i.e. $|u| \leq u_{\max}$),

$\bar{v}=[v\ v^{(1)}\cdots v^{(r)}]^{\mathrm{T}}$ is a generalized reference input ($v^{(k)}$ denotes the kth time derivative of the reference input v, which is assumed to be a sufficiently smooth function of time), that: (a) enforces, in the presence of active actuator constraints, exponential stability and asymptotic robust output tracking with arbitrary degree of attenuation of the effect of the uncertainty on the output, (b) minimizes a meaningful infinite-time cost functional that imposes penalty on the control action and tracking error, and (c) possesses an explicit characterization of the region where the aforementioned properties are guaranteed.

The design of the dynamic controller is carried out using combination of a high-gain observer and a bounded robust optimal state feedback control design proposed in El-Farra and Christofides (2000a) and inspired by the results on bounded control in Lin and Sontag (1991). In particular, the system $\dot{\omega}=\mathscr{F}(\omega,y,\bar{v})$ in Eq. (7) is synthesized to provide estimates of the system state variables, while the bounded static component $\mathscr{P}(\omega,y,\bar{v},t)$ is synthesized to enforce the requested properties in the closed-loop system and at the same time provide the necessary explicit characterization of the region of guaranteed stability. The stability analysis of the closed-loop system employs standard singular perturbation techniques (due to the high-gain nature of the observer) and utilizes the concept of input-to-state stability and nonlinear small gain theorem-type arguments. Near-optimality is established through the inverse optimal approach and using standard singular perturbation results.

In order to proceed with the design of the controllers, we need to impose the following three assumptions on the system of Eq. (1). The first assumption is motivated by the requirement of output tracking and allows transforming the system of Eq. (1) into a partially linear form.

Assumption 1. There exists an integer r and a set of coordinates:

$$\begin{bmatrix}\zeta\\ \eta\end{bmatrix}=\begin{bmatrix}\zeta_1\\ \zeta_2\\ \vdots\\ \zeta_r\\ \eta_1\\ \vdots\\ \eta_{n-r}\end{bmatrix}=\chi(x)=\begin{bmatrix}h(x)\\ L_fh(x)\\ \vdots\\ L_f^{r-1}h(x)\\ \chi_1(x)\\ \vdots\\ \chi_{n-r}(x)\end{bmatrix} \tag{8}$$

where $\chi_1(x), \ldots, \chi_{n-r}(x)$ are nonlinear scalar functions of x, such that the system of Eq. (1) takes the form:

$$\begin{aligned}\dot{\zeta}_1&=\zeta_2\\ &\vdots\\ \dot{\zeta}_{r-1}&=\zeta_r\\ \dot{\zeta}_r&=L_f^rh(\chi^{-1}(\zeta,\eta))+L_gL_f^{r-1}h(\chi^{-1}(\zeta,\eta))u\\ &\quad+\sum_{k=1}^{q}L_{\omega k}L_f^{r-1}h(\chi^{-1}(\zeta,\eta))\theta_k\\ \dot{\eta}_1&=\Psi_1(\zeta,\eta)\\ &\vdots\\ \dot{\eta}_{n-r}&=\Psi_{n-r}(\zeta,\eta)\\ y&=\zeta_1\end{aligned} \tag{9}$$

where $L_gL_f^{r-1}h(x)\neq 0\ \forall x\in\mathbb{R}^n,\ \theta\in\mathbb{R}^q$
Moreover, for each $\theta\in\mathbb{R}^q$,$(\zeta,\ \eta)\to(0,\ 0)$ if and only if $x\to 0$.

We note that the change of variables of Eq. (8) is independent of θ and invertible, since, for every x, the variables ζ, η are uniquely determined by Eq. (8). This implies that if we can estimate the values of ζ, η for all times, using appropriate state observers, then we automatically obtain estimates of x for all times. This property will be exploited later to synthesize a state estimator for the system of Eq. (1) on the basis of the system of Eq. (9). We also note that assumption 1 includes the matching condition of our robust control method. In particular, we consider systems of the form Eq. (1) for which the uncertain variables enter the system in the same equation with the manipulated input. This assumption is motivated by our requirement to eliminate the presence of θ in the η subsystem of the system of Eq. (9). This requirement and the stability requirement of assumption 2 below will allow including in the controller a replica of the η subsystem of Eq. (9) which provides estimates of the η states (see Theorem 1). Introducing the notation, $e=[e_1e_2,\ \ldots,\ e_r]^{\mathrm{T}}$, $e_i=\zeta_i-v^{(i-1)}$, $i=1,\ \ldots,\ r$, the ζ subsystem of Eq. (9) can be further transformed into the following form

$$\dot{e}=\bar{f}(e,\eta,\bar{v})+\bar{g}(e,\eta,\bar{v})u+\sum_{k=1}^{q}\bar{\omega}_k(e,\eta,\bar{v})\theta_k \tag{10}$$

where $\bar{f}(e,\eta,\bar{v})$, $\bar{g}(e,\eta,\bar{v})$, $\bar{\omega}_k(e,\eta,\bar{v})$ are $r\times 1$ vector fields. We now use the above normal form to construct the positive definite function V of Eq. (4) whose time-derivative can be rendered negative definite via feedback. One way to do this, for instance, is to use a quadratic function $V=e^{\mathrm{T}}Pe$ where the positive definite matrix P is chosen to satisfy the following Ricatti inequality:

$$A^TP+PA-Pbb^{\mathrm{T}}P<0 \tag{11}$$

where

$$A=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\ \vdots&&&&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix},\quad b=\begin{bmatrix}0\\0\\ \vdots\\ \\1\end{bmatrix} \tag{12}$$

are an $r\times r$ matrix and $r\times 1$ vector, respectively.

Assumption 2. the system:

$$\begin{aligned}\dot{\eta}_1 &= \Psi_1(\zeta,\eta)\\ &\vdots\\ \dot{\eta}_{n-r} &= \Psi_{n-r}(\zeta,\eta)\end{aligned} \tag{13}$$

is ISS with respect to ζ with$\beta_\eta(|\eta(0)|, t) = K_\eta|\eta(0)|e^{-at}$ where K_η, a are positive real numbers and $K_\eta \geq 1$.

Following (Christofides et al., 1996), the requirement of input-to-state stability of the system of Eq. (13) with respect to ζ is imposed to allow the synthesis of a robust state feedback controller that enforces the requested properties in the closed-loop system for arbitrarily large initial conditions and uncertain variables. On the other hand, the requirement that $\beta_\eta(|\eta(0)|, t) = K_\eta e^{-at}$ allows incorporating in the robust output feedback controller a dynamical system identical to the one of Eq. (13) that provides estimates of the variables η. Assumption 2 is satisfied by many chemical processes (see, (El-Farra & Christofides, 2000a) and the chemical reactor example in Section 4).

Finally, in order to attenuate the effect of the uncertain variables on the output, we need to assume the existence of known bounds that capture the size of the uncertain variables for all times.

Assumption 3. There exists known positive constants θ_{bk} such that $\|\theta_k(t)\| = \theta_{bk}$.

Theorem 1 below provides a formula for the bounded robust near-optimal output feedback controller and states precise conditions under which the proposed controller enforces the desired properties in the closed-loop system.

Theorem 1. *Consider the constrained uncertain nonlinear system of* Eq. (1), *for which assumptions* 1, 2, *and* 3 *hold, under the robust output feedback controller:*

$$\begin{aligned}\dot{\tilde{y}} &= \begin{bmatrix} -La_1 & 1 & 0 & \cdots & 0\\ -L^2a_2 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -L^{r-1}a_{r-1} & 0 & 0 & \cdots & 1\\ -L^ra_r & 0 & 0 & \cdots & 0\end{bmatrix}\tilde{y} + \begin{bmatrix} La_1\\ L^2a_2\\ \vdots\\ L^{r-1}a_{r-1}\\ L^ra_r\end{bmatrix} y\\ \dot{\omega}_1 &= \Psi_1(\mathrm{sat}(\tilde{y}),\ \omega)\\ &\vdots\\ \dot{\omega}_{n-r} &= \Psi_{n-r}(\mathrm{sat}(\tilde{y}), \omega)\\ u &= -\frac{1}{2}R^{-1}(\hat{x})L_{\hat{g}}V\end{aligned} \tag{14}$$

where

$$\frac{1}{2}R^{-1}(\hat{x}) = \frac{L_{\hat{f}}V + \left(\rho + \chi\sum_{k=1}^{q}\theta_{bk}|L_wL_f^{r-1}h(\hat{x})|\right)\left(\frac{|2b^TP\hat{e}|^2}{|2b^TP\hat{e}|+\phi}\right)}{(L_{\hat{g}}V)^2[1+\sqrt{1+(u_{\max}L_{\hat{g}}V)^2}]} + \frac{\sqrt{(L_{\hat{f}}V + \chi\sum_{k=1}^{q}\theta_{bk}|L_{\hat{w}k}V|)^2 + (u_{\max}L_{\hat{g}}V)^4}}{(L_{\hat{g}}V)^2[1+\sqrt{1+(u_{\max}L_{\hat{g}}V)^2}]} \tag{15}$$

$\hat{x} = \chi^{-1}(sat(\tilde{y}),\omega))$, $V = e^TPe$, P is a positive definite matrix that satisfies the Riccati inequality of Eq. (11) and χ, ϕ are adjustable parameters that satisfy $\chi > 2$, and $\phi > 0$. Let $\epsilon = 1/L$. Then given any pair of positive real numbers δ_x, δ_θ such that

$$L_{\tilde{f}}V + \chi\sum_{k=1}^{q}\theta_{bk}|L_{\tilde{w}k}V| \leq u_{\max}|L_{\tilde{g}}V| \tag{16}$$

for $|x| \leq \delta_x$, $\|\theta\| \leq \delta_\theta$, *there exists* $\phi^* > 0$ *and for each* $\phi \in (0, \phi^*]$, *there exists an* $\epsilon^*(\phi) > 0$, *such that if* $\phi \in (0, \phi^*]$, $\epsilon \in (0, \epsilon^*(\phi)]$, $sat(\cdot) = \min\left\{1, \frac{\zeta_{\max}}{|\cdot|}\right\}(\cdot)$ *with* $\zeta_{\max}$ *being the maximum value of the vector* $[\zeta 1, \zeta 2 \ldots \zeta r]$ *for* $|\zeta| \leq \beta_\zeta(\delta_\zeta, 0)$ *where* β_ζ *is a class KL function and* δ_ζ *is the maximum value of the vector* $[h(x)\ L_fh(x) \ldots L_f^{r-1}h(x)]$ *for* $|x| \leq \delta_x$, $|x(0)| < \delta_x$, $\|\theta\| < \delta_\theta$, $|\tilde{y}(0)| < \delta_\zeta$, $\omega(0) = \eta(0) + O(\epsilon)$ *the following holds:*

1. *the closed-loop system is exponentially stable;*
2. $\lim\sup_{t\to\infty}|y(t) - v(t)| = 0$;
3. the output feedback controller of Eq. (14) is near optimal in the sense that

$$J = \int_0^\infty (l(e) + u(\hat{x})R(x)u(\hat{x}))\mathrm{d}t \to V(e(0)) \quad \text{as } \epsilon \to 0 \tag{17}$$

where $R(x) > 0$ *and* $l(e) \geq 0$.

Remark 1. The robust output feedback controller of Eq. (14) consists of a high gain observer which provides estimates of the derivatives of the output y up to order $r-1$, denoted as $\tilde{y}_0, \tilde{y}_1, \ldots, \tilde{y}_{r-1}$, and thus estimates of the variables $\zeta_1, \ldots, \zeta_r$, an observer that simulates the inverse dynamics of the system of Eq. (9), and a staticstate feedback controller (see discussion in Remark 2 below) that attenuates the effect of the uncertain variables on the output and enforces exponential stability and reference input tracking. To eliminate the peaking phenomenon associated with the high gain observer, we use a standard saturation function, sat, to eliminate wrong estimates of the output derivatives for short times. In most practical applications, it is possible to use knowledge of process operating conditions to derive nonconservative bounds on the actual values of the output derivatives.

Remark 2. Regarding the static component of the controller of Eq. (14), we note that it is synthesized via Lyapunov's direct method and the inverse optimal approach and that it depends explicitly on both process uncertainty (i.e. θ_{bk}) and actuator constraints (i.e. u_{max}). Theorem 1 therefore provides a direct controller design method that accounts explicitly and simultaneously for process uncertainty and active input constraints. This is in contrast to the two-step approach typically employed for constrained processes which involves first the design of a controller for the unconstrained process and then accounts for the input constraints through a suitable anti-windup modification. It was shown in El-Farra and Christofides (2000a) that the static state feedback controller of Eq. (14) is optimal with respect to a meaningful cost of the form of Eq. (17) and that the minimum cost achieved in the state feedback problem is $V(e(0))$.

Remark 3. By examining the structure of the static component of Eq. (14), one can appreciate, at least qualitatively, the optimality properties of this component. In this regard, note that the static component of Eq. (14) has the ability to: (a) recognize the potential beneficial (stabilizing) effects of process nonlinearities and prevent their unnecessary cancellation, and (b) assess the extent of the effect of uncertainty on the process and prevent their cancellation if this effect is not significant. To understand this point, note that when the process contains stabilizing nonlinearities and does not suffer from significant uncertainty, the term $L_{\bar{f}}V+\chi\theta_{bk}|L_{\bar{w}k}V|$ will be negative and therefore corresponding squared term under the square root will prevent it unnecessary and wasteful cancellation. The ability of the controller to expend minimal control effort in achieving robust stabilization is a necessary ingredient that any well-designed controller must have to handle the copresence of process uncertainty and actuator constraints.

Remark 4. Theorem 1 provides an explicit characterization of the region in state space where the desired stability and performance properties outlined in the theorem are guaranteed. This characterization takes the form of the inequality in Eq. (16) which, given an arbitrary startup condition for the process, can be used to check a priori, whether the closed-loop properties can be guaranteed under the controller of Eq. (14), in the presence of process uncertainty and actuator constraints. This aspect of the proposed design has important practical implications for efficient process operation since it provides the plant operators with a systematic and clear guide to identify feasible process operating conditions. This is particularly significant in the case of unstable plants (e.g. exothermic chemical reactor) where lack of such a priori knowledge can lead to disastrous consequences.

Remark 5. The output feedback controller design of Eq. (14) is near-optimal in the sense that the cost incurred by implementing this controller to the system of Eq. (1) tends to the optimal (minimal) cost achieved by implementing the bounded robust optimal state feedback controller (i.e. u of Eq. (14) with $\hat{x}=x$), for all times, when the gain of the observer is sufficiently large. The near-optimality of the controller of Eq. (14) is therefore a consequence of both the optimality of the static component of the controller (state feedback problem), and the high-gain nature of the observer which can be exploited to make the performance of the output feedback controller arbitrarily close to that of the robust optimal state feedback controller. Instrumental in this regard is the use of the saturation function (see Remark 1) which allows the use of arbitrarily large values of the observer gain to achieve the desired degree of near-optimality without the detrimental effects of observer peaking.

Remark 6. In El-Farra and Christofides (2000a), it was shown that the inequality of Eq. (16) describes the region of guaranteed closed-loop stability obtained under the bounded robust optimal static component of Eq. (14) (state feedback), which, in general, is an unbounded region. This region remains practically preserved under the output feedback design of Theorem 1 in the sense that, given an initial state and uncertainty that belong to *any* compact subset of the state feedback region, there always exists an observer gain such that the closed-loop properties of Theorem 1 are satisfied. As expected, the nature of this result is consistent with the semi-global result obtained in El-Farra & Christofides (2000b) for the unconstrained case.

4. Application to a chemical reactor

Consider a well-mixed continuous stirred tank reactor where three parallel irreversible elementary exothermic reactions of the form $A\xrightarrow{k1}P_1$, $A\xrightarrow{k2}P_2$ and $A\xrightarrow{k3}P_3$ take place, where A is the reactant species, P_1 is the desired product and P_2, P_3 are undesired byproducts. The feed to the reactor consists of pure A at flow rate F, molar concentration C_{A0} and temperature T_{A0}. A cooling jacket is used to remove heat from the reactor. Under standard modeling assumptions, the process model takes the following form

$$\frac{dC_A}{dt}=\frac{F}{V}(C_{A0}-C_A)-\sum_{i=1}^{3}R_i(C_A,\ T)$$

$$\frac{dC_{P1}}{dt}=-\frac{F}{V}C_{P_1}+R_1(C_A,\ T)$$

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{F}{V}(T_{A0} - T) + \sum_{i=1}^{3} \frac{(-\Delta H_i)}{\rho c_p} R_i(C_A, T) + \frac{UA}{\rho c_p V}(T_c - T)$$

where $R_i(C_A, T) = k_{io} e^{-Ei/RT} C_A$ (see (El-Farra & Christofides, 2000c) for parameter values). It was verified that these conditions correspond to an unstable equilibrium point of the system of the above equation.

The control problem for the process is that of regulating the concentration of the desired product C_{P_1} by manipulating the temperature of the fluid in the cooling jacket T_c. The enthalpies of the three reactions $\triangle H_1, \triangle H_2, \triangle H_3$, and the feed temperature T_{A0} are assumed to be the main uncertain variables present. Cooling water at 300 K is available to cool the reactor but must be returned at a temperature no higher than 360 K, thus placing a constraint on the magnitude of allowable cooling. Defining $\theta_i = \triangle H_i - \triangle H_{i0}$, $i = 1, 2, 3$ and $\theta_4 = T_{A0} - T_{A0s}$, where the subscript s denotes the steady state values and ΔH_{i0} are the nominal values for the enthalpies, the process model of the above equation can be recast in the form of Eq. (1) and easily verified to satisfy the assumptions of Theorem 1.

The controller of Eq. (14) (whose practical implementation requires measurements of C_{P_1} only) was used in the simulations and the following time-varying uncertain variables were considered in all of the simulation runs: $\theta_i(t) = 0.4(-\triangle H_{i0})[1 + \sin(2t)]$, $i = 1, 2, 3$, $\theta_4(t) = 0.03 T_{A0s}[1 + \sin(2t)]$. The upper bounds on the uncertain variables were taken to be $\theta_{bi} = 0.8|(-\triangle H_{i0})|, i = 1, 2, 3$, $\theta_{b4} = 0.06 T_{A0s}$ and the magnitude of actuator constraints present was set at $u_{\max} = 60$ K. Moreover, the following values were used for the tuning parameters of the controller and observer: $\phi = 0.01$, $\chi = 1.1$, $L = 3000$, $a_1 = 100$, $a_2 = 2000$, to guarantee that the output of the closed-loop system satisfies relation of the form $\lim\sup_{t\to\infty}|y - v| \leq 0.005$.

The performance, robustness, and near-optimality properties of the dynamic bounded robust output feedback controller of Eq. (14) were tested through simulations in the presence of the specified active constraints. In particular, we tested the ability of the controller to drive the output of the process close to the desired (unstable) steady state starting from an initial condition in the region of guaranteed closed-loop stability (i.e. satisfies Eq. (16)) despite the presence of uncertainty and actuator constraints. Fig. 1 shows the controlled output and manipulated input profiles. One can immediately see that process has been successfully stabilized, the effect of the uncertainty significantly reduced (compare with the unstable output of the open-loop system), and that the output of the process remains very close to the desired steady state. Included in the figure also are the controlled output and manipulated input profiles for the process under the optimal state feedback controller. It is clear from the figure that the profiles obtained under the output feedback controller follow closely those obtained under the optimal state feedback controller and therefore, the robust output feedback controller of Eq. (14) is near-optimal.

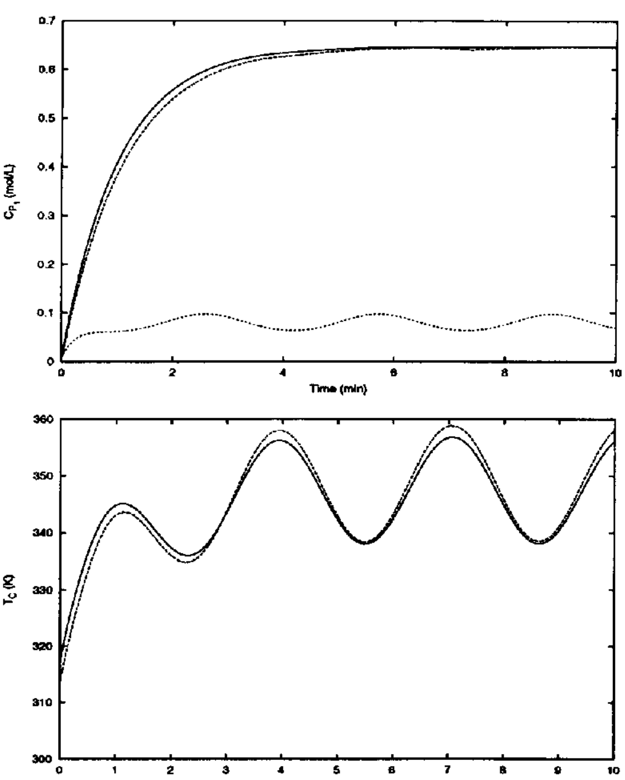

Fig. 1. Closed-loop output and input profiles under the controller of Eq. (14) (solid), the state feedback controller (dashed) and open-loop profile (dotted).

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