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DYNAMIC FEEDFORWARD/OUTPUT FEEDBACK CONTROL OF NONLINEAR PROCESSES

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Abstract—This paper addresses the feedforward/output feedback control problem for single-input single-output minimum-phase nonlinear processes. Combination of dynamic feedforward/static state feedback laws and state observers is employed to synthesize nonlinear dynamic feedforward/output feedback controllers that completely eliminate the effect of measurable disturbances and induce a desired input/output behavior. The developed methodology in applied to an exothermic continuous chemical reactor and extensive simulations illustrate the controller performance and robustness.

INTRODUCTION

The majority of control problems in continuous chemical processes concern the regulation of output variables to desired steady states in the presence of disturbance inputs. Moreover, typical control problems in batch and semi-batch processes involve tracking of output profiles in the presence of disturbance inputs. Whenever measurements of the disturbances are available, feedforward compensation combined with output feedback, is widely used in linear process control to improve the controller performance. The solution of the associated linear feedforward/output feedback control problem is by now well understood [see e.g. Morari and Zafiriou (1989)].

In the context of nonlinear control, the recent years have witnessed a flourishing research activity within the methodological and mathematical framework of differential geometry [see e.g. the tutorial papers of Kravaris and Kantor (1990a, b) and the review paper by Kravaris and Arkun (1991)]. Within this framework, significant research effort was devoted in the study of disturbance inputs for analysis and controller synthesis purposes; geometric conditions were derived for the solvability of the problem of decoupling the disturbances from the outputs via static state feedback (Hirchorn, 1981; Isidori *et al.*, 1981; Nijmeijer and van der Schaft, 1983) or static feedforward/state feedback (Moog and Glumineau, 1983); the problem of feedforward compensation of disturbances within the context of exact state-space linearization was also addressed and solved (Calvet and Arkun, 1988a, b); a general dynamic feedforward/static state feedback synthesis problem was posed and solved by Daoutidis and Kravaris (1993) achieving complete elimination of the effect of measured disturbances on the controlled outputs and a well-characterized input/output behavior in the closed-loop system.

In this paper we address the feedforward/*output* feedback control problem for general single-input single-output minimum-phase nonlinear processes. The objective is to develop explicit reduced-order realizations of dynamic controllers that use measurements of the output and the disturbances to enforce desired objectives in the closed-loop system. The approach followed for the solution of this problem is a state-space one, based on combination of feedforward/static state feedback laws and state observers. To overcome the difficulties associated with the existence and construction of nonlinear state observers [see e.g. Tsinias (1989, 1990) and Grizzle and Moraal (1990)], and in analogy with the approach for the pure output feedback problem (Daoutidis and Kravaris, 1992), the natural modes of the process (i.e. the process dynamics or the process zero dynamics) are used for the state observation.

In what follows, we will start with a brief discussion on key differential geometric concepts, and a brief review of the feedforward/static state feedback problem and its solution. Then, the feedforward/output feedback synthesis problem will be formulated precisely for the class of processes under consideration. The synthesis problem will be addressed and solved, initially for open-loop stable processes and subsequently for processes with possible open-loop instability. Controller implementation issues will be addressed and conditions that guarantee the stability of the closed-loop system will be also derived. Finally, the developed control methodology will be applied to an exothermic continuous reactor example and will be evaluated through simulations.

PRELIMINARIES

We will consider nonlinear processes with a continuous-time state-space description of the form:

$$\dot{x} = f(x) + u(t)g(x) + \sum_{\kappa=1}^{p} d_\kappa(t)w_\kappa(x)$$
$$y = h(x) \tag{1}$$

where x denotes the vector of state variables, u denotes the manipulated input, d_κ denotes a measurable disturbance input, and y denotes the output (to be controlled). It is assumed that $x \in X \subset \mathbb{R}^n$, where X is open and connected, while $u(t) \in \mathbb{R}$ and $d(t) = [d_1(t), \ldots, d_p(t)]^T \in \mathbb{R}^p$, $\forall t \in [0, \infty)$, $y \in \mathbb{R}$, and $d_\kappa(t)$ are sufficiently smooth functions of time. $f(x)$, $g(x)$, $w_\kappa(x)$ denote analytic vector fields on X, and $h(x)$ denotes an analytic scalar field on X. For the theoretical development it is also assumed that the input variables in eq. (1) represent deviations from some nominal values.

Throughout the paper we will be using the standard Lie derivative notation, where

$$L_f h(x) = \sum_{l=1}^{n} \frac{\partial h(x)}{\partial x_l} f_l(x)$$

and $f_l(x)$ denotes the lth row element of $f(x)$. One can define higher-order Lie derivatives, $L_f^k h(x) = L_f L_f^{k-1} h(x)$ as well as mixed Lie derivatives, $L_g L_f^{k-1} h(x)$, in an obvious way.

Referring to the nonlinear system of the form of eq. (1), we define the relative order of the output y with respect to the manipulated input u as the smallest integer r such that

$$L_g L_f^{r-1} h(x) \not\equiv 0 \tag{2}$$

for $x \in X$. If no such integer exists, $r = \infty$. Without loss of generality, it will be assumed that X does not contain any singular points, i.e. points $x \in \mathbb{R}^n$ for which $L_g L_f^{r-1} h(x) = 0$. In particular, as long as $L_g L_f^{r-1} h(x_0) \neq 0$, where x_0 is the nominal equilibrium point, by the continuity of $L_g L_f^{r-1} h(x)$ one can always redefine X as an open and connected set that contains x_0 and is such that $L_g L_f^{r-1} h(x) \neq 0$, $\forall x \in X$. We also define the relative order of the output y with respect to the disturbance input vector d as the smallest integer ρ for which

$$[L_{w_1} L_f^{\rho-1} h(x) \cdots L_{w_p} L_f^{\rho-1} h(x)] \not\equiv [0 \cdots 0] \tag{3}$$

for $x \in X$.

In the rest of this section we will briefly review the solution to the feedforward/static state feedback control problem for processes of the form of eq. (1). In this problem, we seek a feedforward/static state feedback law of the form:

$$u = p(x) + q(x)v + Q'(x, d(t), d(t)^{(1)}, d(t)^{(2)}, \cdots) \tag{4}$$

where $p(x)$, $q(x)$ are algebraic functions of the states with $q(x)$ invertible on X, $d(t)^{(k)}$ denotes the kth order derivative of $d(t)$, and Q' is an algebraic function which is nonsingular under nominal conditions (i.e. remains finite when $d(t) = 0$ for a finite time interval), that induces a desired input/output behavior independent of the values of the disturbance inputs. The basic result is taken from Doutidis and Kravaris (1993) and is summarized in Theorem 1 that follows.

Theorem 1: *Consider the nonlinear process described by eq. (1). Let* r, ρ *denote the relative orders of the output* y *with respect to the manipulated input* u *and the disturbance input vector* d, *respectively, and assume that* $\rho <$ r. *Then, the conditions*

$$L_g \phi_l(x, d(t)) \equiv 0, \quad l = 0, 1, \ldots, r - \rho - 1 \tag{5}$$

where

$$\phi_l(x, d(t)) = \sum_{\mu=0}^{l} L_f^{l-\mu} \left(\sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} + \frac{\partial}{\partial t} \right) \times \left(L_f + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} + \frac{\partial}{\partial t} \right)^{\mu} L_f^{\rho-1} h(x) \tag{6}$$

are necessary and sufficient in order for a feedforward/static state feedback law of the form of eq. (4) to induce the input/output behavior:

$$\sum_{k=0}^{r} \gamma_k y^{(k)} = v \tag{7}$$

where γ_k *are adjustable parameters, independently of the disturbance inputs. If these conditions are satisfied, the appropriate control law takes the form:*

$$u = [\gamma_r L_g L_f^{r-1} h(x)]^{-1} \left\{ v - \sum_{k=0}^{r} \gamma_k L_f^k h(x) - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(x, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \right\}. \tag{8}$$

Remark 1: Theorem 1 provides a precise characterization of the class of nonlinear systems of the form of eq. (1) for which the posed problem is solvable, as well as an explicit synthesis formula for its solution. In the case that $\rho < r$, derivatives of certain disturbance inputs up to order $r - \rho$ are required in the control law in order to achieve complete elimination of their effect on the output. Whenever these disturbance signals are not sufficiently smooth, and, thus, exact evaluation of their derivatives is not possible, standard filtering (or smoothing) techniques can be employed for the approximate evaluation of the derivatives. Obviously, when such approximations are employed, disturbance *attenuation* instead of complete disturbance *elimination* will be achieved in the closed-loop system, with transient characteristics depending on the filter design.

FORMULATION OF THE FEEDFORWARD/OUTPUT FEEDBACK SYNTHESIS PROBLEM

We will now formulate a feedforward/output feedback controller synthesis problem for nonlinear processes of the form of eq. (1) that satisfy the conditions of eq. (5) and exhibit minimum-phase behavior (in the usual sense of stability of the zero dynamics) when $d = 0$. The objective is to calculate state-space realizations of feedforward/output feedback controllers that enforce certain properties in the closed-loop system (see Fig. 1). The desirable closed-loop properties include:

- input/output stability and tracking of changes in the output set-point,

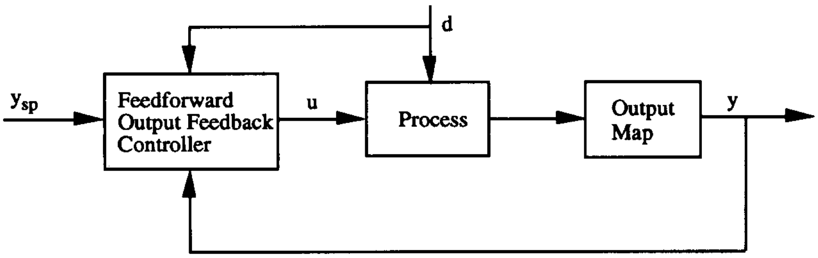

Fig. 1. General feedforward/output feedback control structure.

- complete elimination of the measured disturbances and asymptotic rejection of unmeasured disturbances and modeling errors,
- asymptotic stability of the unforced closed-loop system.

In particular, we will request a disturbance-free linear input/output behavior with no zeros, of the form:

$$\gamma_r y^{(r)} + \cdots + \gamma_1 y^{(1)} + y = y_{sp} \tag{9}$$

where y_{sp} denotes the output set-point. The input/output stability and performance characteristics in closed loop will then be directly related to the choice of the adjustable parameters γ_κ. The above input/output behavior will essentially result by canceling the zero dynamics of the process.

In what follows we will present a solution to the posed synthesis problem, initially for open-loop stable processes and then for processes that may be open-loop unstable. The problem will be addressed within the conceptual framework introduced by Daoutidis *et al.* (1990), i.e. combining the appropriate feedforward/state feedback law with a linear error feedback controller with integral action and an appropriate state observer (see Fig. 2). In analogy with the approach for the pure output feedback case (Daoutidis and Kravaris, 1992), an open-loop state observer will be employed in the case of stable processes, while in the more general case of possibly unstable processes, an observer based on the process zero dynamics will be used instead.

FEEDFORWARD/OUTPUT FEEDBACK CONTROL OF OPEN-LOOP STABLE MINIMUM PHASE PROCESSES

The assumption of open-loop stability of the process dynamics allows us to use the process model itself, forced by the manipulated and disturbance inputs, as an open-loop state observer:

$$\dot{\eta} = f(\eta) + u(t)g(\eta) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(\eta). \tag{10}$$

A feedforward/output feedback controller that solves the posed synthesis problem can then be derived, by combining the state observer of eq. (10) with the feedforward/state feedback law of eq. (8) and a linear error feedback controller with state-space realization:

$$\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\dfrac{\gamma_1}{\gamma_r} & -\dfrac{\gamma_2}{\gamma_r} & \cdots & -\dfrac{\gamma_{r-2}}{\gamma_r} & -\dfrac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \dfrac{1}{\gamma_r} \end{bmatrix} e \tag{11}$$

and output map:

$$v = \xi_1 + e\,. \tag{12}$$

The basic synthesis result, in the form of a reduced-order controller realization, is summarized in theorem 2 that follows (the proof can be found in the Appendix):

Theorem 2: *Consider the nonlinear process described by eq. (1), and assume that the conditions of eq. (5) are satisfied. Then, the dynamic system*

$$\begin{aligned} \dot{\eta} &= f(\eta) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(\eta) + g(\eta)[\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \\ &\quad \times \left\{ e - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) \right. \\ &\quad \left. - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \right\} \\ u &= [\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \\ &\quad \times \left\{ e - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) \right. \\ &\quad \left. - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \right\} \end{aligned} \tag{13}$$

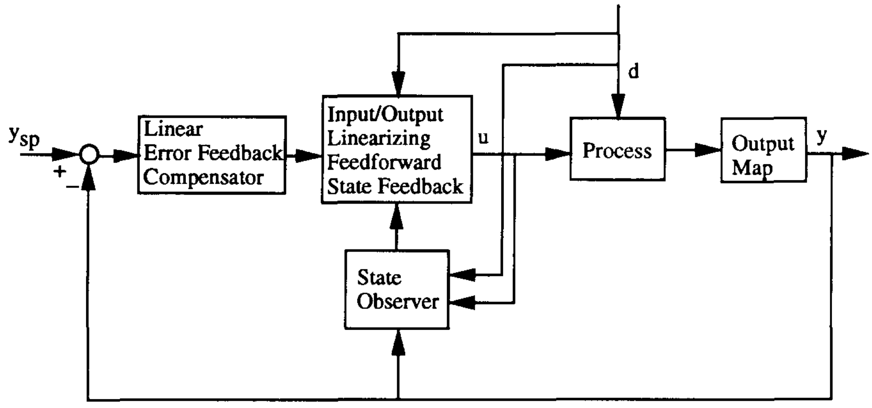

Fig. 2. Proposed feedforward/output feedback control structure.

is a state-space realization of a feedforward/output feedback controller that induces the input/output behavior of eq. (9), *independently of the disturbance inputs*

Remark 2: Whenever $\rho = r$, the following relation holds:

$$\phi_0(x,d) = \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x),$$

and the controller realization of eq. (13) simplifies to

$$\begin{aligned}\dot{\eta} &= f(\eta) + \sum_{\kappa=1}^{r} d_\kappa(t) w_\kappa(\eta) + g(\eta)[\gamma_k L_g L_f^{r-1} h(\eta)]^{-1} \\ &\quad \times \left\{ e - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) \right. \\ &\quad \left. - \gamma_r \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) \right\} \\ u &= [\gamma_k L_g L_f^{r-1} h(\eta)]^{-1} \\ &\quad \times \left\{ e - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) \right. \\ &\quad \left. - \gamma_k \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(x) \right\}.\end{aligned} \tag{14}$$

Feedforward/output feedback control for processes with deadtime

The result of theorem 2 can also be used to develop a solution to the feedforward/output feedback control problem for open-loop stable processes with a state-space description of the form:

$$\begin{aligned}\dot{x} &= f(x) + u(t)g(x) + \sum_{\kappa=1}^{p} d_\kappa(t)(t) w_\kappa(x) \\ y &= h(x(t-\theta))\end{aligned} \tag{15}$$

where θ denotes a deadtime in the output map. The synthesis problem in this case becomes the one of calculating a state-space realization of a feedforward/output feedback controller that induces the input/output behavior:

$$\gamma_r y^{(r)} + \cdots + \gamma_1 y^{(1)} + y = y_{sp}(t-\theta) \tag{16}$$

independently of the disturbance inputs. Motivated by the Smith-type operator structure for nonlinear processes without disturbances (Kravaris *et al.*, 1994), the above synthesis problem can be conveniently addressed within the structure shown in Fig. 3. Within this structure, the appropriate feedforward/output feedback controller is synthesized on the basis of an auxiliary output y^*, which represents the prediction of

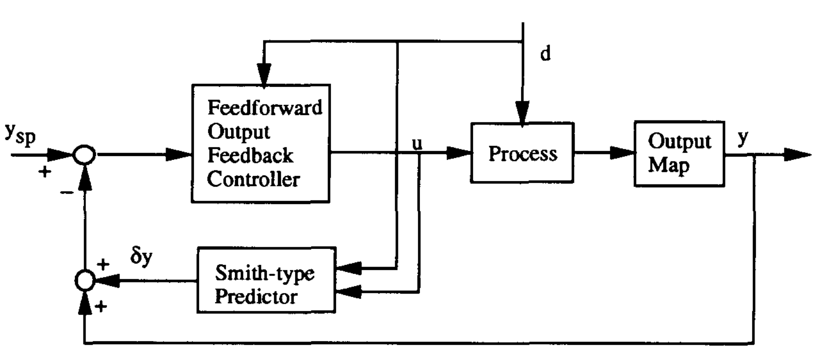

Fig. 3. Feedforward/output feedback control structure for open-loop stable processes with deadtime.

the output if there were no deadtime, and can be obtained by adding a corrective signal δy to the on-line measurement of the actual output y:

$$y^* = y + \delta y.$$

This corrective signal is obtained through a Smith-type predictor, which is driven by both the manipulated and disturbance inputs, and simulates the difference in the responses between the process model without deadtime and the process model with deadtime:

$$\dot{\eta} = f(\eta) + u(t)g(\eta) + \sum_{\kappa=1}^{\kappa} d_\kappa(t)w_\kappa(\eta)$$
$$\delta y = h(\eta) - h(\eta(t-\theta)).$$

It is then straightforward to verify that for processes of the form of eq. (15) and assuming that the conditions of eq. (5) are satisfied, the dynamic system

$$\begin{aligned}\dot{\eta} &= f(\eta) + \sum_{\kappa=1}^{p} d_\kappa(t)w_\kappa(\eta) + g(\eta)[\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \\ &\quad \times \left\{ e + h(\eta(t-\theta)) - h(\eta) - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) \right. \\ &\quad \left. - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \right\} \qquad (17) \\ u &= [\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \\ &\quad \times \left\{ e + h(\eta(t-\theta)) - h(\eta) - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) \right. \\ &\quad \left. - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \right\}\end{aligned}$$

consists a reduced-order realization of a controller that induces the input/output behavior of eq. (16) independent of the disturbance inputs, and thus solves the posed synthesis problem.

Remark 3: The controller realizations of eq. (13) and eq. (17) share a common underlying structure which is depicted in Fig. 4. Referring to this structure, feedback of the model states is combined with feedforward compensation of the disturbance inputs to impose cancellations at the process modes and allow the reduction of the feedforward/output feedback synthesis problem to a feedforward/state feedback synthesis one. As expected, in the absence of disturbance measurements, this structure reduces to the pure model state feedback structure given by Kravaris *et al.* (1994) [see also Coulibaly *et al.* (1992) for a similar structure for linear systems].

FEEDFORWARD/OUTPUT FEEDBACK CONTROL OF GENERAL MINIMUM-PHASE PROCESSES

We will now proceed with the solution of the synthesis problem formulated earlier in the paper, for general minimum-phase processes, with possibly unstable open-loop dynamics. The minimum-phase property of such processes will be exploited through the use of the process zero dynamics for the state reconstruction. The task is complicated significantly by the presence of the disturbance inputs in the process realization. In what follows, we will first describe in detail the proposed scheme of state reconstruction in the presence of disturbances; we will then develop the controller synthesis result, and will finally address controller implementation issues. To facilitate the development, we will be working with the state-space description of the process transformed in appropriate normal form coordinates.

State reconstruction

Consider the nonlinear process of eq. (1) in the general case where $\rho < r$. Consider also the Byrnes–Isidori coordinate transformation:

$$\zeta = \begin{bmatrix} \zeta_1 \\ \vdots \\ \zeta_{n-r} \\ \zeta_{n-r+1} \\ \zeta_{n-r+2} \\ \vdots \\ \zeta_{n-r+\rho} \\ \zeta_{n-r+\rho+1} \\ \vdots \\ \zeta_n \end{bmatrix} = T(x) = \begin{bmatrix} t_1(x) \\ \vdots \\ t_{n-r}(x) \\ h(x) \\ L_f h(x) \\ \vdots \\ L_f^{\rho-1} h(x) \\ L_f^{\rho} h(x) \\ \vdots \\ L_f^{r-1} h(x) \end{bmatrix} \qquad (18)$$

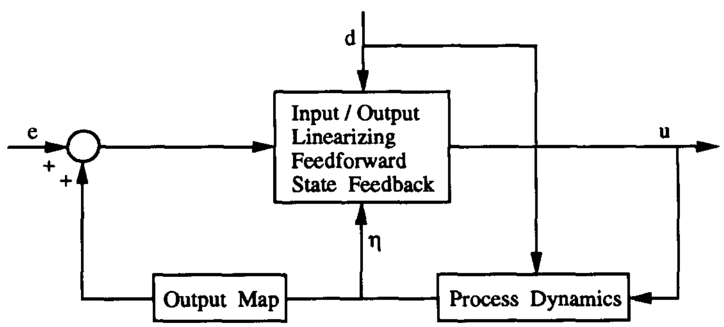

Fig. 4. Feedforward/model state feedback controller structure.

where

- $t_1(x), \ldots, t_{n-r}(x), h(x), L_f h(x), \ldots, L_f^{r-1} h(x)$ are linearly independent scalar fields, and
- $L_g t_i(x) = 0,\ i = 1, \ldots, (n-r)$.

The system of eq. (1), under the coordinate transformation of eq. (18) takes the form (Daoutidis and Kravaris, 1989):

$$\begin{aligned}
\dot{\zeta}_1 &= L_f t_1(\zeta) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-r} &= L_f t_{n-r}(\zeta) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(\zeta) \\
\dot{\zeta}_{n-r+1} &= \zeta_{n-r+2} \\
&\vdots \\
\dot{\zeta}_{n-r+\rho-1} &= \zeta_{n-r+\rho} \\
\dot{\zeta}_{n-r+\rho} &= \zeta_{n-r+\rho+1} + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{\rho-1} h(\zeta) \\
\dot{\zeta}_{n-r+\rho+1} &= \zeta_{n-r+\rho+2} + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{\rho} h(\zeta) \\
&\vdots \\
\dot{\zeta}_{n-1} &= \zeta_n + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{r-2} h(\zeta) \\
\dot{\zeta}_n &= L_f^r h(\zeta) + u(t) L_g L_f^{r-1} h(\zeta) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{r-1} h(\zeta) \\
y &= \zeta_{n-r+1}
\end{aligned} \tag{19}$$

where the ζ-dependence in the expressions of the right-hand side implies their evaluation at $x = T^{-1}(\zeta)$. Furthermore, it is straightforward to verify that whenever the conditions of eq. (5) are satisfied, the expressions for the derivatives of the output y in the ζ-coordinates take the following form:

$$\begin{aligned}
y &= \zeta_{n-r+1} \\
y^{(1)} &= \zeta_{n-r+2} \\
&\vdots \\
y^{(\rho-1)} &= \zeta_{n-r+\rho} \\
y^{(\rho)} &= \zeta_{n-r+\rho+1} + \phi_0(\zeta, d(t)) \\
y^{(\rho+1)} &= \zeta_{n-r+\rho+2} + \phi_1(\zeta, d(t), d(t)^{(1)}) \\
&\vdots \\
y^{(r-1)} &= \zeta_n + \phi_{r-\rho-1}(\zeta, \\
&\quad d(t), d(t)^{(1)}, \ldots, d(t)^{(r-\rho-1)}).
\end{aligned} \tag{20}$$

One can then immediately observe that the state variables $\zeta_{n-r+1}, \zeta_{n-r+2}, \ldots, \zeta_{n-r+\rho}$ can be directly obtained from the measurement of the output y and the evaluation of its derivatives up to order $(\rho - 1)$. Furthermore, provided that that state variables $\zeta_{n-r+\rho+1}, \zeta_{n-r+\rho+2}, \ldots, \zeta_{n-1}, \zeta_n$ can be expressed as functions of $\zeta_1, \ldots, \zeta_{n-r}, \zeta_{n-r+1}, \zeta_{n-r+2}, \ldots, \zeta_{n-r+\rho}, d, \ldots, d^{(r-\rho-1)}$ from the expressions for $y^{(\rho)}$–$y^{(r-1)}$ of eq. (20), the remaining states $\zeta_1, \ldots, \zeta_{n-r}$ can be obtained by simulating the zero dynamics of the process, forced by the output and the disturbance inputs and their derivatives, i.e. the dynamical system:

$$\begin{aligned}
\dot{z}_1 &= L_f t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, \\
&\quad d, \ldots, d^{(r-\rho-1)}) \\
&\vdots \\
\dot{z}_{n-r} &= L_f t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, \\
&\quad d, \ldots, d^{(r-\rho-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, \\
&\quad y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}).
\end{aligned} \tag{21}$$

Remark 4: The state reconstruction scheme described above simplifies significantly whenever $\rho \geqslant r$. In this case, the expressions for the derivatives of the output y in the ζ coordinates take the form:

$$\begin{aligned}
y &= \zeta_{n-r+1} \\
y^{(1)} &= \zeta_{n-r+2} \\
&\vdots \\
y^{(\rho-1)} &= \zeta_{n-r+\rho} \\
y^{(\rho)} &= \zeta_{n-r+\rho+1} \\
y^{(\rho+1)} &= \zeta_{n-r+\rho+2} \\
&\vdots \\
y^{(r-1)} &= \zeta_n.
\end{aligned} \tag{22}$$

Consequently, the state variables $\zeta_{n-r+1}, \zeta_{n-r+2}, \ldots, \zeta_n$ can be directly obtained from the measurement of the output y and the evaluation of its derivatives up to order $(r - 1)$. The remaining states $\zeta_1, \ldots, \zeta_{n-r}$ can be obtained by simulating the forced zero dynamics of the process, which in this case has the form:

$$\begin{aligned}
\dot{z}_1 &= L_f t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}) \\
&\vdots \\
\dot{z}_{n-r} &= L_f t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}).
\end{aligned} \tag{23}$$

Controller synthesis

The state reconstruction scheme described previously will now be combined with the feedforward/state feedback law of eq. (8) and the linear error feedback compensator of eqs (11) and (12) to derive a state-space realization of a feedforward/output feedback controller that solves the posed synthesis problem for general minimum-phase processes. The general synthesis result is summarized in theorem 3 that follows (the proof is completely analogous to the one of theorem 2 and is omitted for brevity).

Theorem 3: *Consider the nonlinear process described by eq. (19) with the forced zero dynamics of eq. (21), and assume that the conditions of eq. (5) are satisfied. Then, the dynamic system:*

$$
\begin{aligned}
\dot{\xi} &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\frac{\gamma_1}{\gamma_r} & -\frac{\gamma_2}{\gamma_r} & \cdots & -\frac{\gamma_{r-2}}{\gamma_r} & -\frac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{\gamma_r} \end{bmatrix} e \\
\dot{z}_1 &= L_f t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) \\
&\;\vdots \\
\dot{z}_{n-r} &= L_f t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) \\
u &= [\gamma_r L_g L_f^{r-1} h(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)})]^{-1} \\
&\quad \times \Bigg\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k y^{(k)} \\
&\quad - \gamma_r L_f^r h(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad - \gamma_r \phi_{r-\rho}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho)}) \Bigg\}
\end{aligned}
\tag{24}
$$

is a state-space realization of a feedforward/output feedback controller that induces the input/output behavior of eq. (9), independently of the disturbance inputs.

Remark 5: In the case that $\rho \geqslant r$ and using the state reconstruction scheme described in remark 4, the controller realization of eq. (24) takes the following form:

$$\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\frac{\gamma_1}{\gamma_r} & -\frac{\gamma_2}{\gamma_r} & \cdots & -\frac{\gamma_{r-2}}{\gamma_r} & -\frac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{\gamma_r} \end{bmatrix} e$$

$$\dot{z}_1 = L_f t_1(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)}) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)})$$

$$\vdots \tag{25}$$

$$\dot{z}_{n-r} = L_f t_{n-r}(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)}) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)})$$

$$u = [\gamma_r L_g L_f^{r-1} h(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)})]^{-1}$$

$$\times \left\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k y^{(k)} - \gamma_r L_f^r h(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)}) - \gamma_r \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} L_f^{r-1}(z_1, \dots, z_{n-r}, y, \dots, y^{(r-1)}) \right\}.$$

Controller implementation

The controller realization of eq. (24) includes derivatives of the measured output up to order $r - 1$, and thus, is not readily implementable whenever $r \geq 2$. This difficulty can be overcome by combining the controller of eq. (24) with an obsever that provides estimates of the output derivatives. For this purpose, we will consider a linear, high-gain observer with the following state-space description (Khalil and Esfandiari, 1992):

$$\begin{aligned} \dot{\tilde{y}}_0 &= \tilde{y}_1 + La_0(y - \tilde{y}_0) \\ \dot{\tilde{y}}_1 &= \tilde{y}_2 + L^2 a_1(y - \tilde{y}_0) \\ &\vdots \\ \dot{\tilde{y}}_{r-2} &= \tilde{y}_{r-1} + L^{r-1} a_{r-2}(y - \tilde{y}_0) \\ \dot{\tilde{y}}_{r-1} &= L^r a_{r-1}(y - \tilde{y}_0) \end{aligned} \tag{26}$$

where $\tilde{y}_i$, $i = 0, \dots, r-1$ denotes the estimate of the ith derivative of the output y, L is the observer gain and $a_0, \dots, a_{r-1}$ are additional adjustable parameters. The linear structure of the observer of eq. (26) allows us to determine its stability characteristics and the rate of convergence of the estimates of the output derivatives to their actual values, by appropriate choice of the parameters $a_0, \dots, a_{r-1}$ and the observer gain L, respectively.

Theorem 4 that follows provides the resulting controller realization and establishes that the synthesis result of theorem 3 is recovered in the limit as $L \to \infty$ (the proof can be found in the appendix).

Theorem 4: *Consider the nonlinear process described by eq. (19) with the forced zero dynamics of eq. (21), and assume that the conditions of eq. (5) are satisfied. Then, the dynamic system:*

$$\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\frac{\gamma_1}{\gamma_r} & -\frac{\gamma_2}{\gamma_r} & \cdots & -\frac{\gamma_{r-2}}{\gamma_r} & -\frac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{\gamma_r} \end{bmatrix} e$$

$$\dot{\tilde{y}} = \begin{bmatrix} -La_0 & 1 & 0 & \cdots & 0 & 0 \\ -L^2a_1 & 0 & 1 & \cdots & 0 & 0 \\ -L^3a_2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -L^{r-1}a_{r-2} & 0 & 0 & \cdots & 0 & 1 \\ -L^ra_{r-1} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tilde{y} + \begin{bmatrix} La_0 \\ L^2a_1 \\ L^3a_2 \\ \vdots \\ L^{r-1}a_{r-2} \\ L^ra_{r-1} \end{bmatrix} y$$

$$\begin{aligned} \dot{z}_1 &= L_f t_1(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\ &\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, y, \tilde{y}_1 \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\ &\;\;\vdots \\ \dot{z}_{n-r} &= L_f t_{n-r}(z_1, \ldots, z_{n-r}, y, \tilde{y}_1 \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\ &\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\ u &= [\gamma_r L_g L_f^{r-1} h(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)})]^{-1} \\ &\quad \times \Bigg\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k \tilde{y}^{(k)} - \gamma_r L_f^r h(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\ &\quad - \gamma_r \phi_{r-\rho}(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho)}) \Bigg\} \end{aligned} \tag{27}$$

is a state-space realization of a feedforward/output feedback controller that induces the input/output behavior of eq. (9) independently of the disturbance inputs, in the limit as $L \to \infty$.

Remark 6: For large values of the observer gain L, the estimates of the output derivatives may exhibit very large values at small times, leading to the so-called peaking phenonenon, and possibly destabilizing the closed-loop system (Khalil and Esfandiari, 1992). To circumvent this problem, the saturation function can be applied to the observer states used in the controller realization.

CLOSED-LOOP STABILITY CONSIDERATIONS

The input/output stability of the closed-loop system resulting from theorems 2 and 3 will be guaranteed if the following condition holds:

(1) The roots of the characteristic polynomial

$$1 + \gamma_1 s + \cdots + \gamma_r s^r = 0$$

lie in the open left-half of the complex plane.

In addition to input/output stability, one must obtain a characterization for the internal stability of the closed-loop system, i.e. the asymptotic stability of the states in the unforced closed-loop system, for perturbations in the initial conditions. Employing standard Lyapunov arguments, it is straightforward to show that the unforced closed-loop system under the controller of theorem 2 will be locally exponentially stable if the following conditions are satisfied:

(2) The open-loop process is locally exponentially stable.

(3) The zero dynamics of the process is locally exponentially stable.

Similarly, it can be shown that the unforced closed-loop system under the controller of theorem 3 will be locally exponentially stable if the second condition from above is satisfied. Finally, under the controller of theorem 4, it can be shown following a standard stability analysis for singularly perturbed systems [see e.g. Isidori (1989) and Khalil (1992)] that the unforced closed-loop system will be locally exponentially stable, for L sufficiently large, if in addition to conditions 1 and 3, the parameters $a_0, \ldots, a_{r-1}$ are chosen such that the roots of the characteristic polynomial

$$s^r + a_0 s^{r-1} + \cdots + a_{r-2} s + a_{r-1} = 0$$

lie in the left-half of the complex plane. Details of the stability analysis are omitted for brevity.

APPLICATION TO A NONISOTHERMAL CSTR

An irreversible elementary reaction A → B is carried out in a perfectly mixed CSTR, shown in Fig. 5. The inlet stream consists of pure A at flowrate F, concentration C_{A0} and temperature T_0. The reaction is exothermic and a cooling jacket with nonnegligible dynamics is used for the heat removal. Cooling water is added to the jacket at a flowrate F_j and an inlet

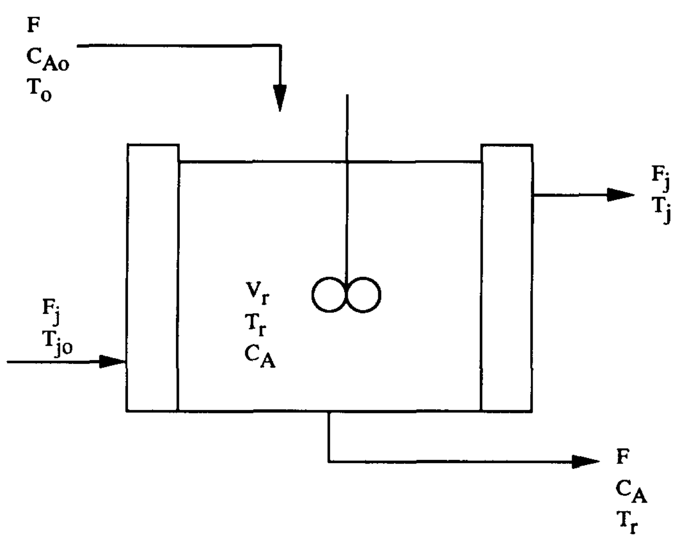

Fig. 5. A continuous stirred tank reactor.

temperature T_{j0}. Under standard assumptions (constant volume in the reactor and the jacket, negligible heat losses, constant densities and heat capacities, uniform coolant temperature in the jacket) the process dynamic model consists of the following equations:

- Reactor mass balance:

$$\frac{dC_A}{dt} = \frac{F}{V_r}(C_{A0} - C_A) - k_0 \exp\left(\frac{-E}{RT_r}\right) C_A \quad (28)$$

- Reactor energy balance:

$$\frac{dT_r}{dt} = \frac{F}{V_r}(T_0 - T_r) + \frac{(-\Delta H_r)}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E}{RT_r}\right) C_A - \frac{UA_r}{\rho_m c_{pm} V_r}(T_r - T_j) \quad (29)$$

- Jacket energy balance:

$$\frac{dT_j}{dt} = \frac{F_j}{V_j}(T_{j0} - T_j) + \frac{UA_r}{\rho_j c_{pj} V_j}(T_r - T_j). \quad (30)$$

The values of the process parameters (Luyben, 1990) are given in Table 1. We consider the control of the reactor temperature at an unstable equilibrium point, by manipulating the jacket flowrate F_j. Measurements of the reactant feed temperature T_0 which acts as a disturbance input are also available. The nominal steady-state for this study corresponds to $C_{As} = 5.33$ kmol m^{-3}, $T_{rs} = 328.93$ K, $T_{js} = 326.29$ K, for $T_{0s} = 300.17$ K and $F_{js} = 5.38$ m^3 h^{-1} (where the subscript s denotes a steady-state value).

Setting

$$x_1 = C_A$$
$$x_2 = T_r$$
$$x_3 = T_j$$
$$u = F_j - F_{js}$$
$$d = T_0 - T_{0s}$$

eqs (28), (29) and (30) can be put in the standard form:

$$\dot{x} = f(x) + g(x)u(t) + w(x)d(t)$$
$$y = h(x) \quad (31)$$

where

$$f(x) = \begin{bmatrix} \frac{F}{V_r}(C_{A0} - x_1) - k_0 \exp(-E/Rx_2)x_1 \\ \frac{F}{V_r}(T_{0s} - x_2) + \frac{(-\Delta H_r)}{\rho_m c_{pm}} k_0 \exp(-E/Rx_2)x_1 - \frac{UA_r}{\rho_m c_{pm} V_r}(x_2 - x_3) \\ \frac{F_{js}}{V_j}(T_{j0} - x_3) + \frac{UA_r}{\rho_j c_{pj} V_j}(x_2 - x_3) \end{bmatrix} \quad (32)$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{T_{j0} - x_3}{V_j} \end{bmatrix}, \quad w(x) = \begin{bmatrix} 0 \\ F/V_r \\ 0 \end{bmatrix}, \quad h(x) = x_2. \quad (33)$$

Table 1. Process and controller parameters

$V_r = 1.00\ m^3$
$V_j = 0.109\ m^3$
$A_r = 23.226\ m^2$
$C_{A0} = 8.0\ kmol\ m^{-3}$
$U = 226\ kcal\ h^{-1}\ m^{-2}\ K^{-1}$
$T_{j0} = 294.4\ K$
$R = 1.987\ kcal\ kmol^{-1}\ K^{-1}$
$\Delta H_r = -1.67 \times 10^4\ kcal\ kgmol^{-1}$
$k_0 = 7.08 \times 10^{10}\ h^{-1}$
$E = 1.67 \times 10^4\ kcal\ kg^{-1}$
$c_{pm} = 0.231\ kcal\ kg^{-1}\ K^{-1}$
$c_{pj} = 0.308\ kcal\ kg^{-1}\ K^{-1}$
$\rho_m = 809\ kg\ m^{-3}$
$\rho_j = 1000\ kg\ m^{-3}$
$F = 1.13\ m^3\ h^{-1}$
$\gamma_1 = 0.22\ h$
$\gamma_2 = 0.01\ h^2$
$\alpha_1 = 0.22$
$\alpha_2 = 0.01$
$L = 2.1$

A straightforward calculation of the relative orders proceeds as follows:

$$L_g h(x) = 0$$

$$L_f h(x) = \frac{F}{V_r}(T_{0s} - x_2) + \frac{(-\Delta H_r)}{\rho_m c_{pm}} k_0 \exp(-E/Rx_2)x_1 - \frac{UA_r}{\rho_m c_{pm} V_r}(x_2 - x_3)$$

$$L_g L_f h(x) = \frac{UA_r}{\rho_m c_{pm} V_r} \frac{T_{j0} - x_3}{V_j} \neq 0$$

$$L_w h(x) = F/V_r \neq 0.$$

Consequently, $r = 2$, $\rho = 1$, and $\rho < r$.

Because of the desired operation around the unstable equilibrium point and the fact that $r = 2$, the controller of theorem 4 was implemented in the simulations. Under the coordinate transformation

$$\zeta = \begin{bmatrix} t(x) \\ h(x) \\ L_f h(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \frac{F}{V_r}(T_{0s} - x_2) + \frac{(-\Delta H_r)}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E}{Rx_2}\right) x_1 - \frac{UA_r}{\rho_m c_{pm} V_r}(x_2 - x_3) \end{bmatrix} \quad (34)$$

the process model can be transformed into the normal form of eq. (19). The zero dynamics of the process takes then the form

$$\dot{\zeta}_1 = \frac{F}{V_r}(C_{A0} - \zeta_1) - k_0 \exp\left(\frac{-E}{R\zeta_2}\right)\zeta_1 \quad (35)$$

and is clearly exponentially stable. According to eq. (27), the necessary controller realization takes then the form

$$\dot{\xi}_1 = \xi_2$$

$$\dot{\xi}_2 = -\frac{\gamma_1}{\gamma_2}\xi_2 + \frac{1}{\gamma_2} e$$

$$\dot{\tilde{y}}_0 = -La_0 \tilde{y}_0 + \tilde{y}_1 + La_0 y$$

$$\dot{\tilde{y}}_1 = -L^2 a_1 \tilde{y}_0 + L^2 a_1 y \quad (36)$$

$$\dot{z}_1 = \frac{F}{V_r}(C_{A0} - z_1) - k_0 \exp\left(\frac{-E}{Ry}\right) z_1$$

$$u = [\gamma_2 L_g L_f^{r-1} h(z_1, y, \tilde{y}_1, d, d^{(1)})]^{-1} \times \{\xi_1 + e - y - \gamma_1 \tilde{y}_1 - \gamma_2 L_f^2 h(z_1, y, \tilde{y}_1, d, d^{(1)}) - \gamma_2 \phi_1(z_1, y, \tilde{y}_1, d, d^{(1)})\}$$

where

$$L_f^2 h = \frac{\partial f_2}{\partial x_1} f_1 + \frac{\partial f_2}{\partial x_2} f_2 + \frac{\partial f_2}{\partial x_3} f_3$$
$$\phi_1 = \frac{F}{V_r}\left(\frac{\partial f_2}{\partial x_2} d(t) + d^{(1)}(t)\right). \quad (37)$$

The quantities $L_g L_f h$, $L_f^2 h$ and ϕ_1 can be easily expressed as functions of $(z_1, y, y^{(1)}, d, d^{(1)})$ through the inverse transformation

$$x_1 = \zeta_1$$

$$x_2 = \zeta_2$$

$$x_3 = \frac{\rho_m c_{pm} V_r}{UA_r}\left(\zeta_3 - \frac{F}{V_r}(T_{0s} - \zeta_2) - \frac{(-\Delta H_r)}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E}{R\zeta_2}\right)\zeta_1 + \frac{UA_r}{\rho_m c_{pm} V_r}\zeta_2\right) \quad (38)$$

and the relations

$$y = \zeta_2$$
$$y^{(1)} = \zeta_3 + \frac{F}{V_r} d(t). \quad (39)$$

The values of the controller parameters are given in Table 1. For the controller implementation, the saturation function was applied to the estimate of the output derivative obtained by the observer, with upper and lower limits equal to 0.3 and -0.3, respectively, chosen by trial and error.

Several simulation runs were performed to evaluate the performance and robustness properties of the controller. In all the simulation runs, a sinusoidal change was imposed in the disturbance input T_0 at time $t = 0$ h, with amplitude 5.0 K and period of oscillations 1.05 h.

In the first set of simulations, we addressed the capability of the proposed controller to maintain the reactor temperature at the nominal steady state. For the sake of comparison, we also implemented the controller realization without incorporating the disturbance measurement. Figure 6 illustrates the resulting output and manipulated input profiles. It can be clearly seen that the feedforward/output feedback controller results in perfect regulation of the output, as predicted by the theory, whereas the pure output feedback controller cannot compensate for the effect of the disturbance, leading to obvious performance degradation.

In the next set of simulation runs, we evaluated the set-point tracking capabilities of the controller. A 2.0 K change in the output set-point was considered at time $t = 0$ h. The corresponding output and manipulated input profiles are shown in Fig. 7. The feedforward/output feedback controller regulates the

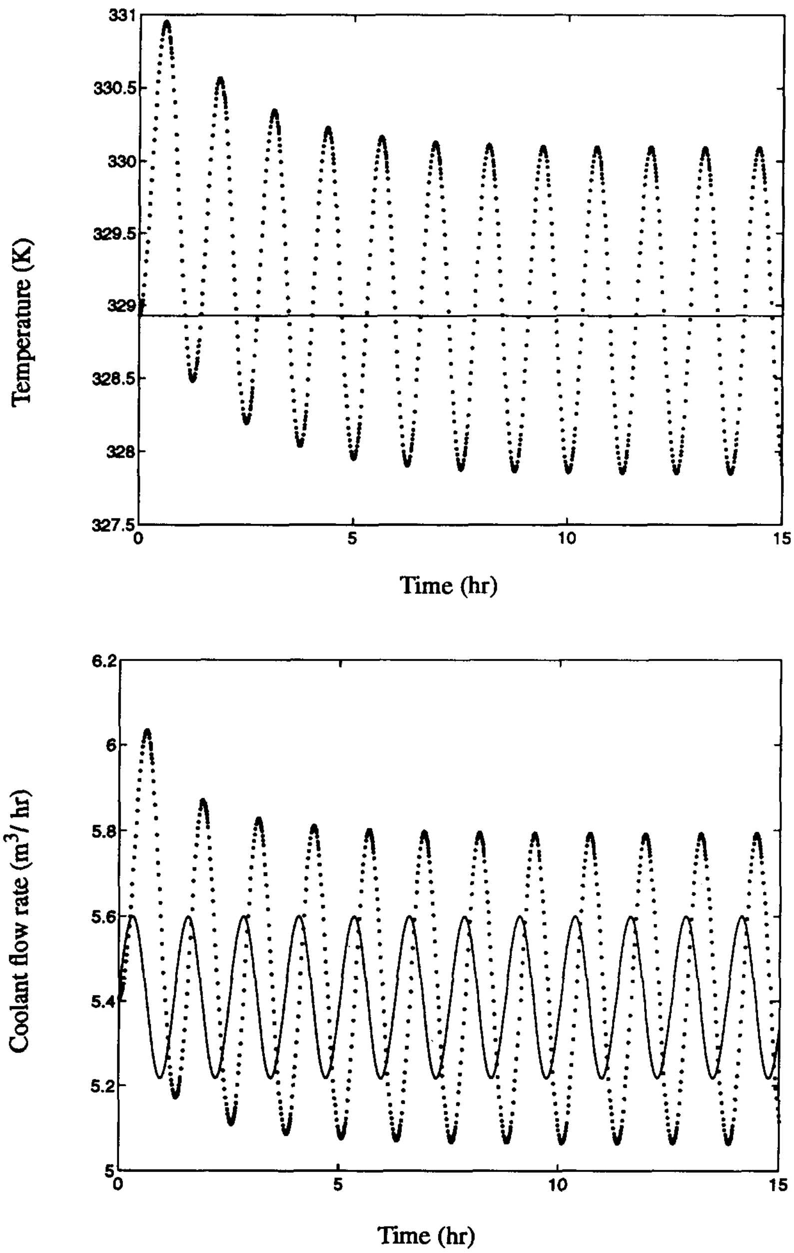

Fig. 6. Output and manipulated input profiles under feedforward/feedback controller (——) and pure feedback controller (....) — regulation.

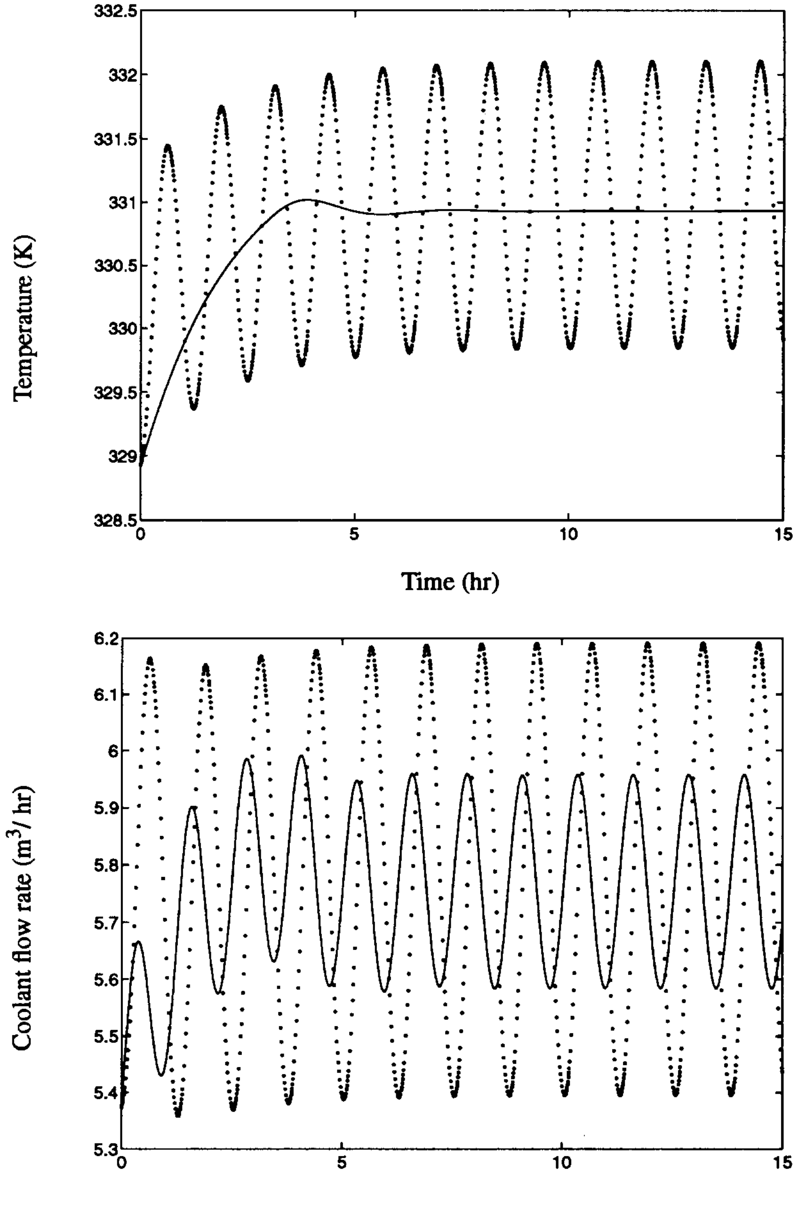

Fig. 7. Output and manipulated input profiles under feedforward/feedback controller (—) and pure feedback controller (...)—set point tracking.

output to the new set-point value, eliminating the effect of the disturbance, whereas the pure output feedback controller cannot attenuate the effect of the disturbance, and thus, fails to drive the output to the new set-point value. Notice that the output response under the feedforward/output feedback controller is very close to the theoretically predicted one, despite the error in the estimate of the output derivative.

In the next set of simulation runs, we tested the robustness of the feedforward/output feedback controller, in the presence of initialization and modeling errors. The same set-point change as previously was considered, and the controller was implemented, initially under a 5% initialization error in the observer states, and subsequently under:

- a 5% change in the inlet concentration, C_{A0}, and
- a 10% change in the inlet temperature of the water in the jacket, T_{j0}.

The resulting output profiles are shown in Figs 8 and 9, respectively; the controller successfully regulates the output to the new set-point value, despite the presence of the initialization or modeling errors.

In the final set of simulation runs, we tested the performance of the controller in the presence of noise in the process measurements, for the same change in the set-point as previously. Initially, the output

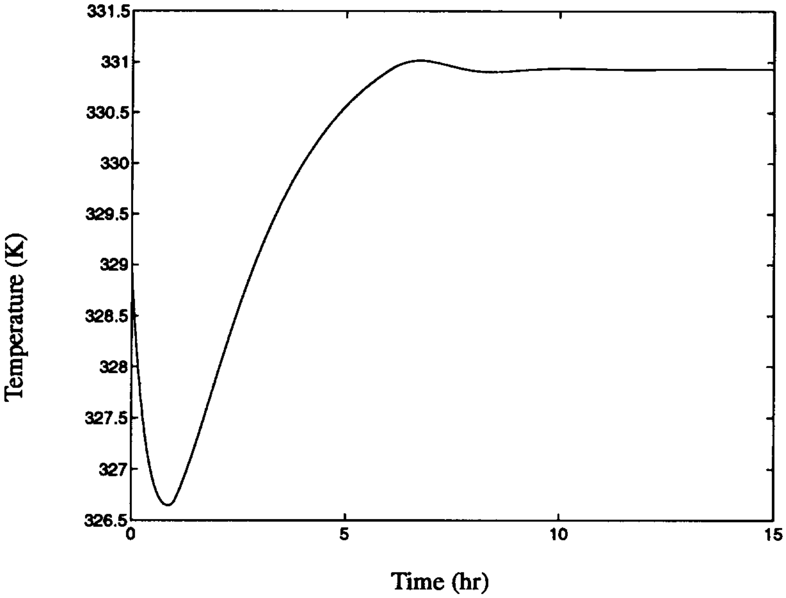

Fig. 8. Output profile—effect of initializaiton errors.

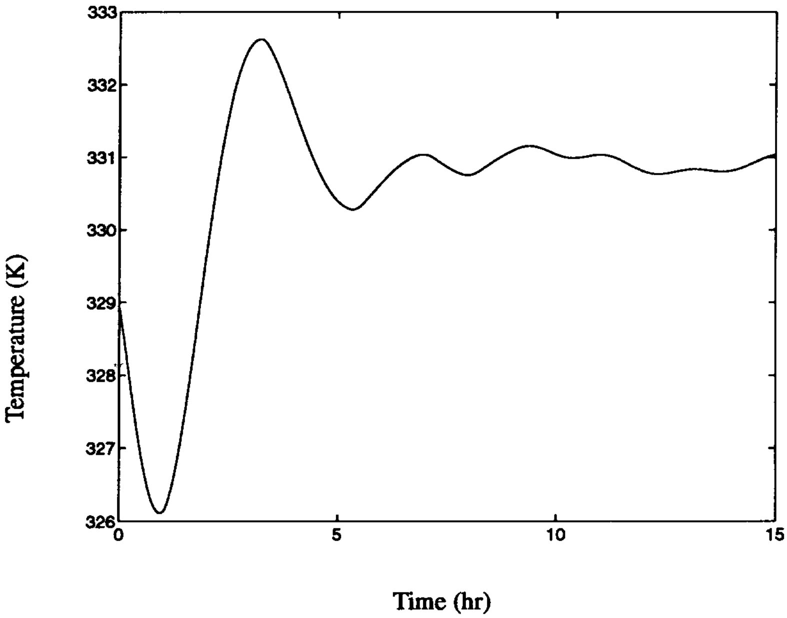

Fig. 9. Output profile—effect of modeling errors.

measurement was assumed to be corrupted by white noise, of amplitude 0.5 K. Figure 10 presents the resulting output profile, illustrating the capability of the controller to regulate the output to its new set-point value. Finally, white noise of the same amplitude was considered in the disturbance and a lead-lag element with transfer function $s/(0.01s + 1)$ was employed to evaluate the derivative of the disturbance used in the controller realization approximately. Figure 11 illustrates the corresponding output profile; clearly the output response is very satisfactory, being very close to the nominal one.

CONCLUSIONS

In this paper we formulated and solved a feedforward/output feedback synthesis problem for single-input single-output minimum-phase nonlinear processes. Explicit reduced-order realizations of dynamic controllers that use measurements of the controlled output and the disturbance inputs to induce a prespecified disturbance-free input/output behavior were developed. The controllers were successfully applied to a continuous stirred tank reactor example and were shown to deal effectively with modeling and initialization errors, and measurement noise.

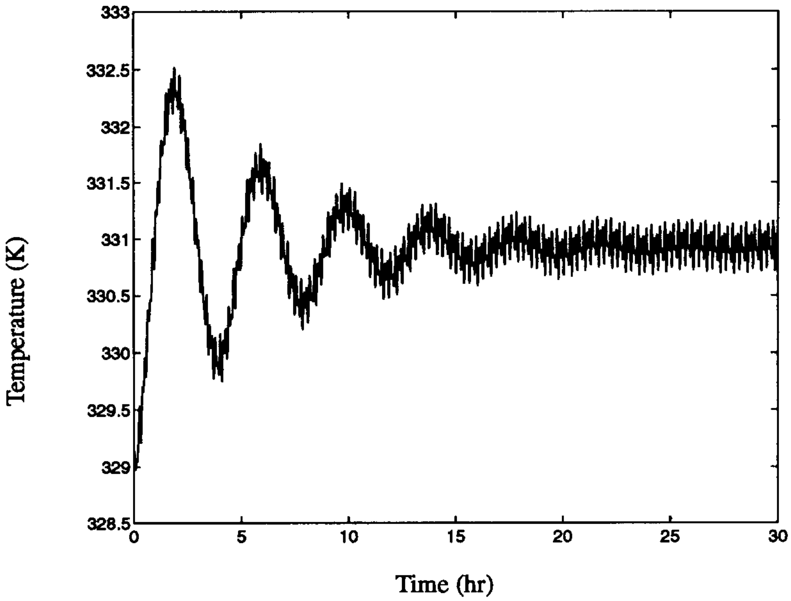

Fig. 10. Output profile—effect of noise in the output measurement.

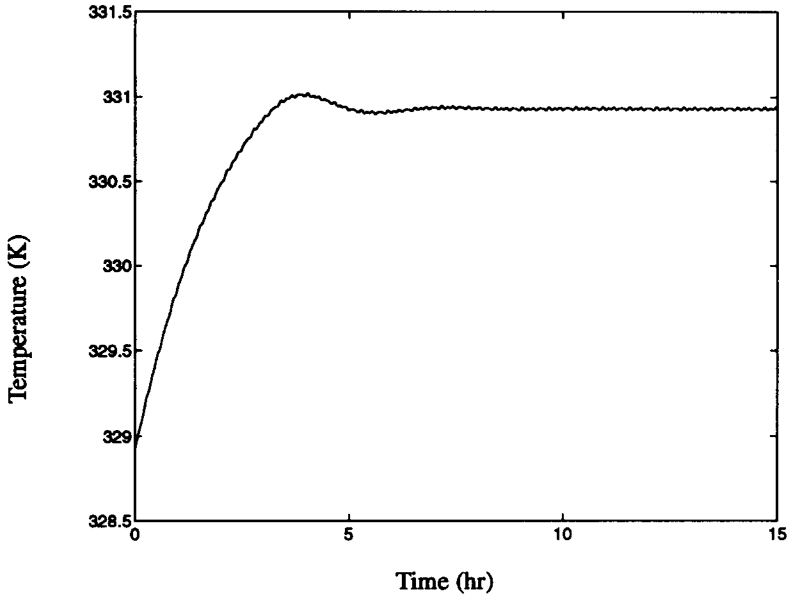

Fig. 11. Output profile—effect of noise in the disturbance measurement.

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NOTATION

a_k	adjustable parameters
A_j	contact surface between surface and jacket
A_r	surface of the reactor
c_{pm}	heat capacity of the reacting mixture
c_{pj}	heat capacity of the water in the jacket
C_{A0}	initial concentration of species A
C_{As}	steady-state concentration of species A
d_κ	disturbance input
$\tilde{e}$	linear observer error coordinate

E activation energy
F flow rate of the inlet stream
F_j flow rate in the jacket
g vector field associated with the manipulated input
h output scalar field
k reaction rate constant
k_0 pre-exponential factor
L linear observer gain
r relative order with respect to the manipulated input
T_j jacket temperature
T_0 inlet stream temperature
T_r reactor temperature
t time
u manipulated input
U heat transfer coefficient
v auxiliary input
V_j volume of the jacket
V_r volume of the reactor
w_κ vector field associated with a disturbance input
x vector of the process state variables
y process output
y_{sp} output set-point
y^* auxiliary output
$y^{(i)}$ ith output derivative
$\tilde{y}_i$ estimate of the ith output derivative
z vector of the observer state variables

Greek letters

γ_k adjustable parameters
ΔH_r heat of the reaction
ε reciprocal of linear observer gain
ζ state vector in normal form coordinates
η vector of controller state variables
θ process output deadtime
ξ vector of controller state variables
ρ relative order with respect to the disturbance vector
ρ_j density of the water in the jacket
ρ_m density of the reacting mixture
ϕ auxiliary functions

Math symbols

$\mathbb{R}$ real line
$\mathbb{R}^n$ n-dimensional Euclidean space
$\in$ belongs to

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APPENDIX

Proof of Theorem 2: Consider the full-order controller realization resulting by combining the control law of eq. (8) with the state observer of eq. (10) and the linear controller of eqs (11) and (12):

$$\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\frac{\gamma_1}{\gamma_r} & -\frac{\gamma_2}{\gamma_r} & \cdots & -\frac{\gamma_{r-2}}{\gamma_r} & -\frac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{\gamma_r} \end{bmatrix} e$$

$$\dot{\eta} = f(\eta) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(\eta) + g(\eta)[\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \times \Big\{ \xi_1 + e - h(\eta) - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) - \sum_{\kappa=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \Big\}$$

$$u = [\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \Big\{ \xi_1 + e - h(\eta) - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \Big\}. \quad \text{(A1)}$$

Under consistent initialization of the controller states, i.e. for

$$\xi_k(0) = \frac{\mathrm{d}^{k-1}[h(\eta)(0)]}{\mathrm{d}t^{k-1}}, \quad k = 1, \ldots, r \quad \text{(A2)}$$

it follows that

$$\xi_1 = h(\eta). \quad \text{(A3)}$$

Substituting the above relation to eq. (A1), we obtain the controller realization of eq. (13). Under the controller of eq. (13), the closed-loop system takes the form

$$\dot{\eta} = f(\eta) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(\eta) + g(\eta)[\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \times \Big\{ e - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \Big\}$$

$$\dot{x} = f(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) + g(x)[\gamma_r L_g L_f^{r-1} h(\eta)]^{-1} \times \Big\{ e - \sum_{k=1}^{r} \gamma_k L_f^k h(\eta) - \sum_{k=\rho}^{r} \gamma_k \phi_{k-\rho}(\eta, d(t), d(t)^{(1)}, \ldots, d(t)^{(k-\rho)}) \Big\} \quad \text{(A4)}$$

$$y = h(x).$$

It is now straightforward to show that, under consistent initialization of the x and η states, the input/output behaviour of the form of eq. (9) is indeed enforced in the closed-loop system. □

Proof of Theorem 4: Under the feedforward/output feedback controller of eq. (27), the closed-loop system takes the form

$$\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\frac{\gamma_1}{\gamma_r} & -\frac{\gamma_2}{\gamma_r} & \cdots & -\frac{\gamma_{r-2}}{\gamma_r} & -\frac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \frac{1}{\gamma_r} \end{bmatrix} e$$

$$\dot{\tilde{y}} = \begin{bmatrix} -La_0 & 1 & 0 & \cdots & 0 & 0 \\ -L^2 a_1 & 0 & 1 & \cdots & 0 & 0 \\ -L^3 a_2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -L^{r-1} a_{r-2} & 0 & 0 & \cdots & 0 & 1 \\ -L^r a_{r-1} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tilde{y} + \begin{bmatrix} La_0 \\ L^2 a_1 \\ L^3 a_2 \\ \vdots \\ L^{r-1} a_{r-2} \\ L^r a_{r-1} \end{bmatrix} y$$

$$\dot{z}_1 = L_f t_1(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, y, \tilde{y}_1 \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)})$$

$$\vdots$$

$$\dot{z}_{n-r} = L_f t_{n-r}(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)})$$

$$
\begin{aligned}
\dot{x} &= f(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) + g(x)[\gamma_r L_g L_f^{r-1} \\
&\quad h(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, d, \ldots, d^{(r-\rho-1)})] \\
&\quad \times \left\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k \tilde{y}_k - \gamma_r L_f^r \right. \\
&\quad h(z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad \times \left\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k \tilde{y}_k - \gamma_r L_f^r \right. \\
&\quad h(z_1, \ldots, z_{n-r}, \tilde{y}_1, \ldots, y, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \} \\
&\quad - \gamma_r \phi_{r-\rho}((z_1, \ldots, z_{n-r}, y, \tilde{y}_1, \ldots, \tilde{y}_{r-1}, d, \ldots, d^{(r-\rho)}) \} \\
y &= h(x).
\end{aligned}
\tag{A5}
$$

Considering the error coordinates: $\tilde{e}_i = L^i(y^{(i)} - \tilde{y}_i)$, $i = 0, \ldots, r-1$, and setting $\varepsilon = 1/L$, the system (A5) takes the form

$$
\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\dfrac{\gamma_1}{\gamma_r} & -\dfrac{\gamma_2}{\gamma_r} & \cdots & -\dfrac{\gamma_{r-2}}{\gamma_r} & -\dfrac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \dfrac{1}{\gamma_r} \end{bmatrix} e
$$

$$
\varepsilon \dot{\tilde{e}} = \begin{bmatrix} -a_0 & 1 & 0 & \cdots & 0 & 0 \\ -a_1 & 0 & 1 & \cdots & 0 & 0 \\ -a_2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_{r-2} & 0 & 0 & \cdots & 0 & 1 \\ -a_{r-1} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tilde{e}
$$

$$
\begin{aligned}
\dot{z}_1 &= L_f t_1(z_1, \ldots, z_{n-r}, \tilde{e}_0, \ldots, \tilde{e}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, \tilde{e}_0, \ldots, \tilde{e}_{r-1}, \\
&\quad d, \ldots, d^{(r-\rho-1)}) \\
&\vdots \\
\dot{z}_{n-r} &= L_f t_{n-r}(z_1, \ldots, z_{n-r}, \tilde{e}_0, \ldots, \tilde{e}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, \\
&\quad \tilde{e}_0, \ldots, \tilde{e}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\
\dot{x} &= f(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) + g(x)[\gamma_r L_g L_f^{r-1} h(z_1, \ldots, z_{n-r}, \\
&\quad \tilde{e}_0, \ldots, \tilde{e}_{r-1}, d, \ldots, d^{(r-\rho-1)})]^{-1} \\
&\quad \times \left\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k \tilde{y}_k - \gamma_r L_f^r h(z_1, \ldots, z_{n-r}, \right. \\
&\quad \tilde{e}_0, \ldots, \tilde{e}_{r-1}, d, \ldots, d^{(r-\rho-1)}) \\
&\quad - \gamma_r \phi_{-\rho}(z_1, \ldots, z_{n-r}, \tilde{e}_0, \ldots, \tilde{e}_{r-1}, d, \ldots, d^{(r-\rho)}) \} \\
y &= h(x).
\end{aligned}
\tag{A6}
$$

For ε sufficiently small (or equivalently, L sufficiently large), the system of eq. (A6) exhibits a two-time-scale behavior and its properties can be analyzed within the framework of singular perturbations (see e.g. Kokotovic *et al.* (1986)]. In particular, the fast dynamics of the system of eq. (A6) can be easily found to be

$$
\frac{d\tilde{e}}{d\tau} = \begin{bmatrix} -a_0 & 1 & 0 & \cdots & 0 & 0 \\ -a_1 & 0 & 1 & \cdots & 0 & 0 \\ -a_2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_{r-2} & 0 & 0 & \cdots & 0 & 1 \\ -a_{r-1} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tilde{e} \tag{A7}
$$

where $\tau = t/\varepsilon$. Equation (A7) possesses an exponentially stable equilibrium, $\tilde{e} = 0$, by the choice of the adjustable parameters a_k. Hence, in the limit as $\varepsilon \to 0$ (i.e. $L \to \infty$), the system of eq. (A6) reduces to

$$
\dot{\xi} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\dfrac{\gamma_1}{\gamma_r} & -\dfrac{\gamma_2}{\gamma_r} & \cdots & -\dfrac{\gamma_{r-2}}{\gamma_r} & -\dfrac{\gamma_{r-1}}{\gamma_r} \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \dfrac{1}{\gamma_r} \end{bmatrix} e
$$

$$\dot{z}_1 = L_f t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_1(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)})$$

$$\vdots$$

$$\dot{z}_{n-r} = L_f t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) + \sum_{\kappa=1}^{p} d_\kappa(t) L_{w_\kappa} t_{n-r}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)})$$

$$\dot{x} = f(x) + \sum_{\kappa=1}^{p} d_\kappa(t) w_\kappa(x) + g(x)[\gamma_k L_g L_f^{r-1} h(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)})]^{-1}$$

$$\times \left\{ \xi_1 + e - y - \sum_{k=1}^{r-1} \gamma_k \tilde{y}_k - \gamma_r L_f^r h(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho-1)}) - \gamma_r \phi_{r-\rho}(z_1, \ldots, z_{n-r}, y, \ldots, y^{(r-1)}, d, \ldots, d^{(r-\rho)}) \right\}$$

$$y = h(x).$$

On the basis of the system of eq. (A8), it is straightforward to show that the input/output behavior of eq. (9) is indeed enforced in the closed-loop system. □