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Brief paper

Control Lyapunov-Barrier function-based model predictive control of nonlinear systems☆

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ABSTRACT

In this paper, we propose a Control Lyapunov-Barrier Function-based model predictive control (CLBF-MPC) method for solving the problem of stabilization of nonlinear systems with input constraint satisfaction and guaranteed safety for all times. Specifically, considering the input constraints, a constrained Control Lyapunov-Barrier Function is initially employed to design an explicit control law and characterize a set of initial conditions, starting from which the solution of the nonlinear system is guaranteed to converge to the steady-state without entering a specified unsafe region in the state space. Then, the CLBF-MPC is proposed and is shown to be recursively feasible, and stabilizing and to ensure the avoidance of a set of states in state–space associated with unsafe operating conditions under sample-and-hold control action implementation. Finally, we demonstrate the efficacy of the proposed CLBF-MPC method through application to a chemical process example.

1. Introduction

Safety is an issue of significant importance in the chemical process industries due to the great impact of unsafe operations on human life and the environment. In Sanders (2015), it was reported that since the beginning of the twenty-first century, three major chemical incidents have occurred in the United States, causing huge capital loss, injuries and deaths. The catastrophic industrial chemical incidents (Completed Investigations of Chemical Incidents, 2016) and the resulting severe damage remind us of the need to continue to find methods to improve process operational safety. One important method for doing this at the process design stage is developing inherently safer processes, while another that is important at the level of day-to-day operational safety is improving the design of process control systems (Crowl & Louvar, 2011).

MPC is a widely-used advanced control methodology in industrial chemical plants due to its ability to handle multiple-input multiple-output processes while achieving optimal process performance (Rawlings & Mayne, 2009). Among different MPC formulations, a Lyapunov-based model predictive control (Mhaskar, El-Farra, & Christofides, 2006; Muñoz de la Peña & Christofides, 2008) is developed to provide stabilizability and feasibility based on an explicitly-defined estimate of the region of attraction of the closed-loop system (termed the stability region) via a well-designed control law (e.g., a Lyapunov-based control law). This has motivated significant research work on applications of this control design to nonlinear processes. At this stage, however, the problem of incorporating safety constraints in Lyapunov-based model predictive control (LMPC) has not been studied.

Recently, an intriguing control method accounting for both stability and safety termed Control Lyapunov-Barrier Function (CLBF)-based control has been developed (Jankovic, 2017; Niu & Zhao, 2013; Tee, Ge, & Tay, 2009). A CLBF can be designed through the combination of a Control Lyapunov Function (CLF) and a Control Barrier Function (CBF) (e.g., weighted average in Romdlony & Jayawardhana, 2016 or the quadratic programming combination in Ames, Grizzle, & Tabuada, 2014). Since CBFs can be used to characterize unsafe state–space regions as open and bounded

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sets in the operating region such that these regions may not be defined by direct constraints on state variables, the existing LMPC scheme may not be able to handle safety considerations expressed in terms of CBFs. Incorporating Control Lyapunov-Barrier Functions into MPC to allow for generality in defining unsafe regions and maintaining the closed-loop state only in safe operating regions is an open issue.

Motivated by the above safety considerations, in this work, we propose a CLBF-MPC formulation that integrates a Control Lyapunov-Barrier function with MPC to account for input constraints, safety considerations, and stability of the closed-loop system. The proposed control scheme provides recursive feasibility, guaranteed process safety and stability of the closed-loop system from an explicitly characterized set of initial conditions in the presence of input constraints. The design of the CLBF-MPC uses the universal nonlinear control law (Lin & Sontag, 1991) as a candidate controller to search for the null controllable region (i.e., the set of initial conditions starting from which stabilization at the origin can be achieved) or a subset of it, and to analyze the stability of the closed-loop system. The CLBF-MPC is shown to be recursively feasible, and able to drive the closed-loop state to the origin while avoiding the unsafe region at all times if the state originates from a well-characterized set of initial conditions.

The rest of the paper is organized as follows: in Section 2, the class of systems considered, the stabilizability assumptions, and the concept of the constrained Control Lyapunov-Barrier Function are given. In Section 3, we introduce the sample-and-hold implementation applied in MPC, and develop a CLBF-based model predictive controller that guarantees recursive feasibility, safety and closed-loop stability under sample-and-hold implementation within an explicitly characterized set of initial conditions. In Section 4, a nonlinear chemical process example is used to demonstrate the applicability of the proposed control method.

2. Preliminaries

2.1. Notation

Throughout the paper, the notation $|\cdot|$ is used to denote the Euclidean norm of a vector, the notation $|\cdot|_Q$ denotes a weighted Euclidean norm of a vector (i.e., $|x|_Q = \sqrt{x^TQx}$ where Q is a positive definite matrix). x^T denotes the transpose of x. $\mathbf{R}_+$ denotes the set $[0, \infty)$. The notation $L_fV(x)$ denotes the standard Lie derivative $L_fV(x) := \frac{\partial V(x)}{\partial x}f(x)$. A scalar continuous function $V : \mathbf{R}^n \to \mathbf{R}$ is proper if the set $\{x \in \mathbf{R}^n \mid V(x) \leq k\}$ is compact for all $k \in \mathbf{R}$, or equivalently, V is radially unbounded (Malisoff & Mazenc, 2009). For given positive real numbers β and ϵ, $\mathcal{B}_\beta(\epsilon) := \{x \in \mathbf{R}^n \mid |x - \epsilon| < \beta\}$ is an open ball around ϵ with radius of β. Set subtraction is denoted by "\", i.e., $A\backslash B := \{x \in \mathbf{R}^n \mid x \in A, x \notin B\}$. A function $f(\cdot)$ is of class $\mathcal{C}^1$ if it is continuously differentiable. Given a set $\mathcal{D}$, the boundary and the closure of $\mathcal{D}$ are denoted by $\partial\mathcal{D}$ and $\overline{\mathcal{D}}$, respectively.

2.2. Class of systems

The class of continuous-time nonlinear systems considered is described by the following state–space form:

$$\dot{x} = f(x) + g(x)u + h(x)w, \;\; x(t_0) = x_0 \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ is the manipulated input vector, and $w \in \mathbf{W}$ is the disturbance vector, where $\mathbf{W} := \{w \in \mathbf{R}^l \mid |w| \leq \theta, \; \theta \geq 0\}$. The control action constraint is defined by $u \in U := \{u_{\min} \leq u \leq u_{\max}\} \subset \mathbf{R}^m$, where u_{min} and u_{max} represent the minimum and the maximum value vectors of inputs allowed, respectively. $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ are sufficiently smooth vector and matrix functions of dimensions $n \times 1$, $n \times m$, and $n \times l$, respectively. Without loss of generality, the initial time t_0 is taken to be zero ($t_0 = 0$), and it is assumed that $f(0) = 0$, and thus, the origin is a steady-state of the system of Eq. (1) with $w(t) \equiv 0$, (i.e., $(x_s^*, u_s^*) = (0, 0)$, where x_s^* and u_s^* denote the steady-state of Eq. (1)).

Moreover, we assume that there is an open set $\mathcal{D}$ in state–space within which it is unsafe for the system to be operated (e.g., the temperature or pressure is extremely high). $\mathcal{X}_0 := \{x \in \mathbf{R}^n\backslash\mathcal{D}\}$ where $\{0\} \in \mathcal{X}_0$ represents a set from which the set of initial conditions to be considered will be developed, and $\emptyset$ signifies the null set. In the manuscript, we assume that the measurement of $x(t)$ is available for feedback at each sampling time.

2.3. Stabilizability assumptions expressed via Lyapunov-based control

We assume that the nominal system of Eq. (1) with $w(t) \equiv 0$ admits a positive definite and proper Control Lyapunov Function (CLF) V that satisfies the following condition:

$$L_fV(x) < 0, \forall x \in \{z \in \mathbf{R}^n\backslash\{0\} \mid L_gV(z) = 0\} \tag{2}$$

We also assume that V satisfies the small control property (Sontag, 1989), i.e., for every $\varepsilon > 0$, $\exists\; \delta > 0$, s.t. $\forall\; x \in \mathcal{B}_\delta(0)$, there exists u that satisfies $|u| < \varepsilon$ and $L_fV(x) + L_gV(x)u < 0$.

The CLF assumption implies the existence of a stabilizing feedback control law $\Phi(x)$ that renders the origin asymptotically stable in the sense that Eq. (2) holds for $u = \Phi(x)$, where $\Phi(x) \in U$. An example of a feedback control law that is continuous for all x in a neighborhood of the origin and renders the origin asymptotically stable is the following control law (Lin & Sontag, 1991):

$$k_i(x) = \begin{cases} -\dfrac{p + \sqrt{p^2 + \gamma|q|^4}}{|q|^2}q_i & \text{if} \quad q \neq 0 \\ 0 & \text{if} \quad q = 0 \end{cases} \tag{3a}$$

$$\Phi_{S,i}(x) = \begin{cases} u_{min} & \text{if} \quad k_i(x) < u_{min} \\ k_i(x) & \text{if} \quad u_{min} \leq k_i(x) \leq u_{max} \\ u_{max} & \text{if} \quad k_i(x) > u_{max} \end{cases} \tag{3b}$$

where p denotes $L_fV(x)$, q_i denotes $L_{g_i}V(x)$, $q = [q_1 \cdots q_m]^T$, $f = [f_1 \cdots f_n]^T$, $g_i = [g_{i1} \cdots g_{in}]^T$, $(i = 1, 2, \ldots, m)$ and $\gamma > 0$. $k_i(x)$ of Eq. (3a) represents the i_{th} component of the control law $\Phi_S(x)$ before considering saturation of the control action at the input bounds. $\Phi_{S,i}(x)$ of Eq. (3) represents the i_{th} component of the saturated control law $\Phi_S(x)$ that accounts for the input constraint $u \in U$. Based on the Lyapunov-based control law $\Phi(x)$, a region ϕ_u where the time-derivative of the Control Lyapunov Function is negative under the constrained inputs can be found, $\phi_u = \{x \in \mathbf{R}^n \mid \dot{V} < 0, u = \Phi(x) \in U\}$. Also, we define a level set of $V(x)$ inside ϕ_u as $\Omega_b = \{x \in \phi_u \mid V(x) \leq b, \; b > 0\}$. Since Ω_b is a forward invariant subset of ϕ_u, given any initial states $x_0 \in \Omega_b$, it is guaranteed that for all $t \geq t_0$, $x(t)$ of the system of Eq. (1) with $w(t) \equiv 0$ under the control law of Eq. (3) remains in Ω_b.

2.4. Stabilization and safety via control Lyapunov-Barrier function

In this work, we develop an MPC design that takes advantage of a Control Lyapunov-Barrier Function (defined in this section) in its design to maintain the state in a safe operating region at all times in the following sense:

Definition 1. Consider the nominal system of Eq. (1) with $w(t) \equiv 0$ and input constraints $u \in U$. If there exists a control law $u = \Phi(x) \in U$ such that the state trajectories of the system for any initial state $x(t_0) = x_0 \in \mathcal{X}_0$ satisfy $x(t) \notin \mathcal{D}, \forall\ t \geq 0$, we say that the control law $\Phi(x)$ maintains the process state within a safe operating region at all times.

A number of recent works (Prajna & Jadbabaie, 2004; Xu, Tabuada, Grizzle, & Ames, 2015) have developed control laws for which they have been able to guarantee that the control law maintains safe operation of the process at all times from a set of initial conditions $\mathcal{X}_0$ when a Control Barrier Function (CBF) can be found for the system. The definition of a CBF is as follows: Wieland and Allgöwer (2007)

Definition 2. Given a set of unsafe points $\mathcal{D}$ in state–space, a $\mathcal{C}^1$ function $B(x) : \mathbf{R}^n \to \mathbf{R}$ is a CBF if it satisfies the following properties:

$$B(x) > 0, \quad \forall\, x \in \mathcal{D} \tag{4a}$$

$$L_f B(x) \leq 0,\ \forall\, x \in \{z \in \mathbf{R}^n \backslash \mathcal{D} \mid L_g B(z) = 0\} \tag{4b}$$

$$\mathcal{X}_B := \{x \in \mathbf{R}^n \mid B(x) \leq 0\} \neq \emptyset \tag{4c}$$

A modification of a Control Barrier Function to form a Control Lyapunov-Barrier Function (CLBF) $W_c(x)$ has been employed in Romdlony and Jayawardhana (2016) to show that when a CLBF exists for the system of Eq. (1) with $w(t) \equiv 0$, there exists a controller of the form of Eq. (3) with W_c replacing V, that keeps the closed-loop state inside a level set of $W_c(x)$, and ensures the state does not enter $\mathcal{D}$ at all times for $x_0 \in \mathcal{X}_0$. Since in Romdlony and Jayawardhana (2016), the stabilization and safety-related results were guaranteed only for $u \in \mathbf{R}^m$ (i.e., no input constraints), for the case with input constraints, a new CLBF must be developed to derive similar results. Therefore, based on the definition of a CLBF in Romdlony and Jayawardhana (2016), we propose a modified CLBF (termed a constrained CLBF) in this manuscript that accounts for the presence of input constraints in the system of Eq. (1). Specifically, the definition of a constrained CLBF is as follows:

Definition 3. Given a set of unsafe points in state–space $\mathcal{D}$, a proper, lower-bounded and $\mathcal{C}^1$ function $W_c(x) : \mathbf{R}^n \to \mathbf{R}$ is a constrained CLBF if $W_c(x)$ has a minimum at the origin and also satisfies the following properties:

$$W_c(x) > \rho_c, \quad \forall\, x \in \mathcal{D} \subset \phi_{uc} \tag{5a}$$

$$\left| \frac{\partial W_c(x)}{\partial x} \right| \leq r(|x|) \tag{5b}$$

$$L_f W_c(x) < 0,$$
$$\forall\, x \in \{z \in \phi_{uc} \backslash (\mathcal{D} \cup \{0\} \cup \mathcal{X}_e) \mid L_g W_c(z) = 0\} \tag{5c}$$

$$\mathcal{U}_{\rho_c} := \{x \in \phi_{uc} \mid W_c(x) \leq \rho_c\} \neq \emptyset \tag{5d}$$

$$\overline{\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})} \cap \overline{\mathcal{D}} = \emptyset \tag{5e}$$

where $\rho_c \in \mathbf{R}$, $r(\cdot)$ is a class $\mathcal{K}$ function, and $\mathcal{X}_e := \{x \in \phi_{uc} \backslash (\mathcal{D} \cup \{0\}) \mid \partial W_c(x)/\partial x = 0\}$ is a set of states where $L_f W_c(x) = 0$ (for $x \neq 0$) due to $\partial W_c(x)/\partial x = 0$.

Under a stabilizing control law $\Phi(x)$ (e.g., the Lyapunov-based control law of Eq. (3) with $W_c(x)$ replacing $V(x)$), ϕ_{uc} is defined to be the union of the origin, $\mathcal{X}_e$ and the set where the time-derivative of $W_c(x)$ is negative with constrained input: $\phi_{uc} = \{x \in \mathbf{R}^n \mid \dot{W}_c(x(t), \Phi(x)) = L_f W_c + L_g W_c u < -\alpha |W_c(x) - W_c(0)|, u = \Phi(x) \in U\} \cup \{0\} \cup \mathcal{X}_e$, and α is a positive real number used to characterize the set ϕ_{uc}. Also, we define the set of initial conditions by $\mathcal{X}_{W_c} := \{x \in \phi_{uc} \backslash \mathcal{D}\}$ where $(\{0\} \cup \mathcal{X}_e) \in \mathcal{X}_{W_c}$. From now on, we will denote $\dot{W}_c(x(t), u(t))$, if not otherwise stated, simply by $\dot{W}_c$.

Theorem 1 provides sufficient conditions under which the existence of a constrained CLBF of Eq. (5) for the nominal system of Eq. (1) with $w(t) \equiv 0$ under the control law $\Phi(x)$ guarantees that the solution of the system of Eq. (1) always stays in a safe operating region.

Theorem 1. *Consider that a constrained CLBF $W_c(x) : \mathbf{R}^n \to \mathbf{R}$, that has a minimum at the origin, exists for the nominal system of Eq. (1) (i.e., $w(t) \equiv 0$) with the input constraints $u \in U$, defined with respect to a set of unsafe points $\mathcal{D}$ in state–space. The feedback control law $\Phi(x)$ guarantees that the closed-loop state stays in $\mathcal{X}_{W_c}$ and does not enter $\mathcal{D}$ for all times for $x(0) = x_0 \in \mathcal{X}_{W_c}$.*

Proof. First, we prove that if $x_0 \in \mathcal{X}_{W_c}$, then the closed-loop state will never enter $\mathcal{D}$, for all $t \geq 0$. Consider the first case that $x_0 \in \mathcal{U}_{\rho_c}$. By the definition of ϕ_{uc}, it is guaranteed that $\dot{W}_c$ is negative everywhere in the set $\mathcal{X}_{W_c} \backslash (\{0\} \cup \mathcal{X}_e)$. (e.g., if $L_g W_c(x) = 0$, it follows that $\dot{W}_c(x) = L_f W_c(x) < 0$; if $L_g W_c(x) \neq 0$, it follows that $\dot{W}_c = -\sqrt{L_f W_c^2 + \gamma |L_g W_c|^4} < 0$ using the Lyapunov-based control law of Eq. (3) with $W_c(x)$ replacing $V(x)$). Additionally, if $x \in \mathcal{X}_e$, $\dot{W}_c(x) = 0$ holds. Therefore, it follows that $W_c(x(t)) \leq W_c(x(0))$ for all $x(t) \in \mathcal{U}_{\rho_c}$ by $\dot{W}_c \leq 0$, i.e., $x(t)$ stays in the set $\mathcal{U}_{\rho_c}$ for all $t \geq 0$ if $x_0 \in \mathcal{U}_{\rho_c}$.

Also, $\mathcal{U}_{\rho_c}$ is a compact invariant set due to the properness of W_c and the property $\dot{W}_c \leq 0$. Due to the fact that $\mathcal{U}_{\rho_c} \cap \mathcal{D} = \emptyset$, it follows that for any $x_0 \in \mathcal{U}_{\rho_c}$, the closed-loop state does not enter the set of unsafe states at any time (i.e., it is maintained within the set of safe states at all times). Additionally, since any subset of $\mathcal{U}_{\rho_c}$, $\mathcal{U}_{\rho} := \{x \in \phi_{uc} \mid W_c(x) \leq \rho \leq \rho_c\} \subset \mathcal{U}_{\rho_c}$, is also a compact invariant set, we can show that if $x_0 \in \mathcal{U}_{\rho}$, $x(t) \in \mathcal{U}_{\rho}$, $\forall t \geq 0$. It remains to be shown that for all other initial states $x_0 \in \phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$, $x(t) \notin \mathcal{D}, \forall\ t \geq 0$. Given an initial state x_0 that belongs to the set $\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$, $W_c(x_0) > \rho_c$ holds because it is not within the set $\mathcal{U}_{\rho_c}$ defined in Eq. (5d). However, since Eq. (5c) holds within $\phi_{uc} \backslash (\mathcal{D} \cup \{0\})$, the conclusion that $\dot{W}_c(x)$ is negative along the trajectory of $x(t)$ holds using the same steps as performed above when x_0 was within $\mathcal{U}_{\rho_c}$. Furthermore, since the set $\overline{\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})}$ does not intersect with $\overline{\mathcal{D}}$, any trajectory starting in $\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$ will reach the boundary of $\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$ before reaching the boundary of $\mathcal{D}$. Because Eq. (5e) holds (i.e., $\overline{\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})} \cap \overline{\mathcal{D}} = \emptyset$), it must hold that $\overline{\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})} \cap \mathcal{U}_{\rho_c}$, is a nonempty set. Because $W_c(x) > \rho_c$ within $\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$ but $W_c(x) \leq \rho_c$ within $\mathcal{U}_{\rho_c}$ from Eq. (5d), $W_c(x) = \rho_c$, $\forall x \in \partial \phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$ due to the continuity of W_c, which means that the trajectory will continue to enter and remain in $\mathcal{U}_{\rho_c}$ after it reaches the boundary of $\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$. This completes the proof that for all $x_0 \in \mathcal{X}_{W_c}$, $x(t) \notin \mathcal{D}, \forall\ t \geq 0$.

Remark 1. In Theorem 1, simultaneous stability (boundedness of the closed-loop state) and safety are proved for the nominal system of Eq. (1) with any $x_0 \in \mathcal{X}_{W_c}$ under $u = \Phi(x)$. Note that the set of initial condition $\mathcal{X}_{W_c}$ contains two parts. One is $\mathcal{U}_{\rho_c}$ of Eq. (5d), where it satisfies $W_c(x) \leq \rho_c$; the other one is $\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})$, which is required to satisfy Eq. (5e). Therefore, if we restrict the initial conditions to $\mathcal{U}_{\rho_c}$ or any subset of it, the conditions of a constraint CLBF in Definition (5) can be reduced to Eqs. (5a)–(5d). Otherwise, if the set $\overline{\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})}$ is considered as a part of initial conditions, all the conditions in Definition (5) are required to hold for W_c. The additional condition of Eq. (5e) for the case of $x_0 \in \overline{\phi_{uc} \backslash (\mathcal{D} \cup \mathcal{U}_{\rho_c})}$ also implies that $W_c(x) = \rho_c$ for all $x \in \partial \mathcal{D}$, which can be readily shown by contradiction.

Remark 2. Since $\dot{W}_c(x_e) = 0$ holds for all $x_e \in \mathcal{X}_e$, the origin of the closed-loop nominal system of Eq. (1) with $w(t) \equiv 0$ cannot be rendered asymptotically stable with a continuous controller (e.g., Eq. (3) with $W_c(x)$ replacing $V(x)$) (Braun & Kellett, 2018).

Therefore, in order to drive the state to the origin instead of converging to $x_e \neq 0$, a CLBF-based MPC will be introduced in Section 3 to ensure that the closed-loop state converges to a small neighborhood around the origin.

2.5. Design of constrained CLBF

Based on the constructive methods for an unconstrained CLBF from Romdlony and Jayawardhana (2016), the methods for constructing a constrained CLBF are developed in this subsection. Specifically, a constrained CLBF can be constructed by combining a CLF and a CBF that have been separately designed, and we present a practical method for designing a CLBF that satisfies the properties in Eq. (5). Proposition 1 provides the guidelines for choosing the CLF and CBF, and the corresponding weights, through which the global minimum of $W_c(x)$ is achieved at the origin.

Proposition 1. *Given an open set $\mathcal{D}$ of unsafe states for the system of Eq. (1) with $w(t) \equiv 0$, assume that there exist a $\mathcal{C}^1$ CLF $V : \mathbf{R}^n \to \mathbf{R}_+$, and a $\mathcal{C}^1$ CBF $B : \mathbf{R}^n \to \mathbf{R}$, such that the following conditions hold:*

$$c_1 |x|^2 \leq V(x) \leq c_2 |x|^2 , \ \forall x \in \mathbf{R}^n, c_2 > c_1 > 0 \tag{6}$$

$$\mathcal{D} \subset \mathcal{H} \subset \phi_{uc}, \ 0 \notin \mathcal{H} \tag{7}$$

$$\begin{aligned} &B(x) = -\eta < 0, \ \forall x \in \mathbf{R}^n \backslash \mathcal{H}; \ B(x) \geq -\eta, \ \forall x \in \mathcal{H}; \\ &B(x) > 0, \ \forall x \in \mathcal{D} \end{aligned} \tag{8}$$

where $\mathcal{H}$ is a compact and connected set within ϕ_{uc}. Define $W_c(x)$ to have the form $W_c(x) := V(x) + \mu B(x) + \nu$, where:

$$\left| \frac{\partial W_c(x)}{\partial x} \right| \leq r(|x|) \tag{9}$$

$$\begin{aligned} &L_f W_c(x) < 0, \\ &\forall \ x \in \{z \in \phi_{uc} \backslash (\mathcal{D} \cup \{0\} \cup \mathcal{X}_e) \mid L_g W_c(z) = 0\} \end{aligned} \tag{10}$$

$$\mu > \frac{c_2 c_3 - c_1 c_4}{\eta}, \tag{11a}$$

$$\nu = \rho_c - c_1 c_4, \tag{11b}$$

$$c_3 := \max_{x \in \partial \mathcal{H}} |x|^2, \tag{11c}$$

$$c_4 := \min_{x \in \partial \mathcal{D}} |x|^2 \tag{11d}$$

then for initial states $x_0 \in \phi_{uc} \backslash \mathcal{D}_{\mathcal{H}}$, where $\mathcal{D}_{\mathcal{H}} := \{x \in \mathcal{H} \mid W_c(x) > \rho_c\}$, the control law $\Phi(x)$ (with $W_c(x)$ replacing $V(x)$) guarantees that the closed-loop state is bounded in $\phi_{uc} \backslash \mathcal{D}_{\mathcal{H}}$ and does not enter the unsafe region $\mathcal{D}_{\mathcal{H}}$ for all times.

Proof. We define a new compact and connected set $\mathcal{H}$, which satisfies Eq. (7), and an expanded unsafe region $\mathcal{D}_{\mathcal{H}}$, such that all the states with $W_c(x) > \rho_c$ inside the region $\mathcal{H}$ are included in $\mathcal{D}_{\mathcal{H}}$. We prove that the proposed constrained CLBF, $W_c(x)$, meets all the requirements of Eq. (5) with $\mathcal{D}_{\mathcal{H}}$ replacing $\mathcal{D}$ and has a global minimum at the origin. Firstly, it is trivial to show that Eq. (5a) holds by the definition of $\mathcal{D}_{\mathcal{H}}$. Additionally, we can use Eqs. (6), (8), and (11) to show that for all $x \in \mathcal{D}$, $W_c(x) > \rho_c$ also holds as follows:

$$\begin{aligned} W_c(x) &= V(x) + \mu B(x) + \nu \\ &> c_1 |x|^2 + \rho_c - c_1 c_4 \\ &> \rho_c \end{aligned} \tag{12}$$

Eqs. (5b) and (5c) are also trivially satisfied by the proposed CLBF via the required property of Eqs. (9) and (10). To prove that Eq. (5d) holds, we obtain the following inequalities for all $x \in \partial \mathcal{H}$,

$$\begin{aligned} W_c(x) &= V(x) + \mu B(x) + \nu \\ &\leq c_2 |x|^2 - \mu \eta + \rho_c - c_1 c_4 \\ &< \rho_c \end{aligned} \tag{13}$$

Hence, Eq. (5d) holds due to the fact that $\mathcal{U}_{\rho_c} \neq \emptyset$ obtained from Eq. (13), which also implies that $\partial \mathcal{H} \cap \partial \mathcal{D}_{\mathcal{H}} = \emptyset$. Following this, we have $\mathcal{D}_{\mathcal{H}} \subset \mathcal{H} \subset (\mathcal{D}_{\mathcal{H}} \cup \mathcal{U}_{\rho_c})$, which implies the boundary of $\phi_{uc} \backslash (\mathcal{D}_{\mathcal{H}} \cup \mathcal{U}_{\rho_c})$ does not intersect with the boundary of $\mathcal{D}_{\mathcal{H}}$, (i.e., Eq. (5e) holds, $\overline{\phi_{uc} \backslash (\mathcal{D}_{\mathcal{H}} \cup \mathcal{U}_{\rho_c})} \cap \overline{\mathcal{D}_{\mathcal{H}}} = \emptyset$). Additionally, $W_c(x)$ has a global minimum at the origin since the minimums of $V(x)$ and $B(x)$ are both at the origin. Therefore, we can conclude that for any initial states $x_0 \in \phi_{uc} \backslash \mathcal{D}_{\mathcal{H}}$, the control law $\Phi(x)$ guarantees that the closed-loop state is bounded in $\phi_{uc} \backslash \mathcal{D}_{\mathcal{H}}$ and does not enter $\mathcal{D}_{\mathcal{H}}$ for all times.

Remark 3. In Braun and Kellett (2018), it is shown that there exists a set of points $x_e \in \mathcal{X}_e$ where $\partial W_c(x_e)/\partial x_e = 0$. This implies that the closed-loop state may converge to x_e instead of the origin under a continuous controller. Therefore, in order to avoid the convergence to x_e, a different control action has to be applied at x_e such that the state is able to leave x_e and move towards a smaller level set of $W_c(x)$. To that end, $W_c(x)$ needs to be well-designed such that x_e is a saddle point on $\mathbf{R}^n$. Subsequently, in the following section, a CLBF-based MPC will be designed with a constraint to find a path that leaves saddle points and moves towards a smaller level set of $W_c(x)$.

3. CLBF-based model predictive control

3.1. Sample-and-hold implementation of CLBF-based controller

In the proof of Theorem 1, it was noted that when a constrained CLBF exists for the nominal system of Eq. (1) with $w(t) \equiv 0$, the controller $\Phi(x)$ when continuously implemented, can maintain the state in a safe region of operation. Because the CLBF will be used to design constraints for MPC, for which control actions are implemented in sample-and-hold, the sample-and-hold properties of the controller $\Phi(x)$ (with a sampling period Δ) must be investigated in the presence of disturbances. The next two propositions and their proofs develop these results.

Proposition 2. *Consider the nominal system of Eq. (1) with a constrained CLBF W_c that meets the requirements of Definition 3 and has a minimum at the origin, and the set of initial conditions $\mathcal{U}_{\rho_c} \subset \mathcal{X}_{W_c}$. Let $u(t) = \Phi(x(t_k))$ for all $t_k \leq t < t_{k+1}$, $x(t_k) \in \mathcal{U}_{\rho_c} \backslash \mathcal{B}_\delta(x_e)$ where $\delta > 0$, $x_e \in \mathcal{X}_e$ and t_k represents the time instance $t = k\Delta$, $k = 0, 1, 2, \ldots$, and $u(t) = \bar{u}(x) \in U$ such that if $x(t_k) \in \mathcal{B}_\delta(x_e)$, then $W_c(x(t_{k+1})) < W_c(x(t_k))$ for any $\Delta > 0$. Then, given any positive real number d, there exists a real number Δ^*, such that, if $\Delta \in (0, \Delta^*]$ and $x_0 \in \mathcal{U}_{\rho_c}$, then $x(t) \in \mathcal{U}_{\rho_c}$, and $\lim_{t\to\infty} |x(t)| \leq d$.*

Proof. We need to show that under sample-and-hold implementation, any states originating in $\mathcal{U}_{\rho_c}$ converge to a level set around the origin $\mathcal{U}_{\rho_{min}} := \{x \in \phi_{uc} \mid W_c(x) \leq \rho_{min}\}$ as $t \to \infty$ where $\rho_{min} < \rho_c$. Following this, it is trivial to show that $x(t) \in \mathcal{U}_{\rho_{min}}$ as $t \to \infty$ implies $\lim_{t\to\infty} |x(t)| \leq d$ by the continuity of $W_c(x)$. To prove that the state will converge to $\mathcal{U}_{\rho_{min}}$, we first show that $\forall \, x(t_k) \in \mathcal{U}_{\rho_c} \backslash (\mathcal{U}_{\rho_s} \cup \mathcal{B}_\delta(x_e))$, where $\rho_s < \rho_{min} < \rho_c$, $\dot{W}_c(x(t), u(t)) < -\epsilon$ holds in the set $\mathcal{Z} := \{x \in \phi_{uc} \backslash \mathcal{B}_\delta(x_e) \mid \rho_s \leq W_c(x) \leq \rho_c\}$ with

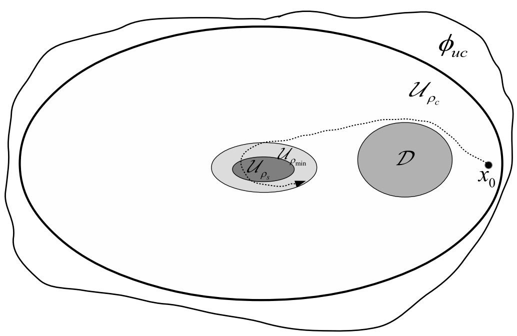

Fig. 1. A schematic representing the sets $\mathcal{U}_{\rho_c}$, $\mathcal{U}_{\rho_{min}}$ and $\mathcal{U}_{\rho_s}$, where an example of the closed-loop trajectory that originates from $x_0 \in \mathcal{U}_{\rho_c}$ (dotted line) is shown to avoid the unsafe region $\mathcal{D}$, and ultimately enter and remain in $\mathcal{U}_{\rho_{min}}$ under the sample-and-hold implementation of $u = \Phi(x) \in U$.

$u(t) = u(t_k) = \Phi(x(t_k)), \forall\ t \in [t_k, t_k + \Delta^*)$ as below:

$$\begin{aligned}\dot{W}_c(x(t), u(t)) =& \dot{W}_c(x(t_k), u(t_k)) + (\dot{W}_c(x(t), u(t)) \\ &-\ \dot{W}_c(x(t_k), u(t_k))) \\ =& L_f W_c(x(t_k)) + L_g W_c(x(t_k))u(t_k) \\ &+\ (L_f W_c(x(t)) - L_f W_c(x(t_k))) \\ &+\ (L_g W_c(x(t)) - L_g W_c(x(t_k)))u(t)\end{aligned} \tag{14}$$

Due to the smoothness of $f(\cdot)$ and $g(\cdot)$, and the fact that $W_c(x)$ is a $\mathcal{C}^1$ function that satisfies Eq. (5b), there exist positive real numbers k_1 and k_2, such that $|(L_f W_c(x(t)) - L_f W_c(x(t_k))| \leq k_1 |x(t) - x(t_k)|$, $|(L_g W_c(x(t)) - L_g W_c(x(t_k)))u(t)| \leq k_2|x(t) - x(t_k)|$. Since $f(x)$ and $g(x)$ are continuous functions, and $\mathcal{Z}$ is bounded, there exists a positive real number k_4 and a sampling period Δ', such that $|x(t) - x(t_k)| \leq k_4\Delta'$ for all $t \in [t_k, t_k + \Delta')$. Also, by the definition of ϕ_{uc}, it follows that $\dot{W}_c(x(t_k)) < -\alpha|W_c(x) - W_c(0)| < -\alpha\rho_m$ holds for all $x \in \mathcal{Z}$, where $\rho_m := \min_{x\in\mathcal{Z}} |W_c(x) - W_c(0)|$. Let $\Delta' < \frac{\alpha\rho_m-\epsilon}{k_4(k_1+k_2)}$ and $0 \leq \epsilon < \alpha\rho_m$, where α is used to characterize ϕ_{uc}, and substitute the above inequalities obtained from Lipschitz conditions into Eq. (14), then it follows that

$$\begin{aligned}\dot{W}_c(x(t), u(t)) &\leq \dot{W}_c(x(t_k), u(t_k)) + k_4(k_1 + k_2)\Delta' \\ &< -\alpha\rho_m + k_4(k_1 + k_2)\Delta' \\ &< -\epsilon\end{aligned} \tag{15}$$

Eq. (15) implies that $W_c(x(t)) < W_c(x(t_k)) \leq \rho_c, \forall\ t > t_k$ and within finite steps, the closed-loop state trajectory $x(t)$ will enter $\mathcal{U}_{\rho_s}$. Hence, $x(t)$ is shown to be bounded in $\mathcal{U}_{\rho_c}$, for all $t \in [t_k, t_k + \Delta')$.

Additionally, consider $x(t_k) \in \mathcal{B}_\delta(x_e)$ where x_e are designed to be saddle points. Since we assume that there exists a set of control actions $\bar{u}(x)$ that decreases $W_c(x)$, $x(t_{k+1})$ is able to move to a smaller level set of $W_c(x)$ and within finite sampling steps leaves $\mathcal{B}_\delta(x_e)$. Moreover, $x(t)$ never returns to $\mathcal{B}_\delta(x_e)$ once it leaves since Eq. (15) (i.e., $W_c(x(t)) < W_c(x(t_k)), \forall\ t > t_k$) holds thereafter.

It remains to show that given $x(t_k) \in \mathcal{U}_{\rho_s}$, the trajectory of $x(t)$ will stay in $\mathcal{U}_{\rho_{min}}, \forall\ t \in [t_k, t_k + \Delta'')$. Consider Δ'' such that

$$\rho_{min} = \max_{\Delta t\in[0,\Delta'')} \{W_c(x(t_k + \Delta t)) \mid x(t_k) \in \mathcal{U}_{\rho_s},\ u \in U\}. \tag{16}$$

Again, there exists a sufficiently small Δ'' such that Eq. (16) holds. Therefore, let $\Delta^* = \min\{\Delta', \Delta''\}$, and now we are able to show that for any state $x(t_k) \in \mathcal{U}_{\rho_c}$, $x(t)$ will remain in $\mathcal{U}_{\rho_c}$ during one sampling period $\Delta \in (0, \Delta^*]$. An example of the closed-loop trajectory under the sample-and-hold implementation of $u = \Phi(x)$ and the relationship among the sets $\mathcal{U}_{\rho_c}$, $\mathcal{U}_{\rho_{min}}$ and $\mathcal{U}_{\rho_s}$ are shown in Fig. 1.

Remark 4. The above proof is based on the assumption that the system of Eq. (1) is undisturbed, i.e., $w(t) \equiv 0$. However, when taking the bounded disturbance $|w(t)| \leq \theta$ into account and the CLBF-based controller applied in a sample-and-hold fashion, we can show that Proposition 2 still holds for the system of Eq. (1) subject to the bounded disturbance. Specifically, we first derive the similar result for $L_h W_c(x)$ via the local Lipschitz property of $h(\cdot)$: $\exists\ k_3 > 0$, s.t. $|L_h W_c(x(t)) - L_h W_c(x(t_k))| \leq k_3|x(t) - x(t_k)|$. Following that, we obtain similar results for $\dot{W}_c(x(t), u(t))$ and ρ'_{min} that account for $w(t)$ as follows:

$$\begin{aligned}\dot{W}_c(x(t), u(t)) &\leq \dot{W}_c(x(t_k), u(t_k)) + k_4(k_1 + k_2 + k_3\theta)\Delta' \\ &< -\alpha\rho_m + k_4(k_1 + k_2 + k_3\theta)\Delta' \\ &< -\epsilon\end{aligned} \tag{17}$$

$$\begin{aligned}\rho'_{min} = \max_{\Delta t\in[0,\Delta'')} &\{W_c(x(t_k + \Delta t), u, w) \mid x(t_k) \in \mathcal{U}_{\rho_s}, \\ &u \in U,\ |w| \leq \theta\}.\end{aligned} \tag{18}$$

where $\Delta' < \frac{\alpha\rho_m-\epsilon}{k_4(k_1+k_2+k_3\theta)}$ and $0 \leq \epsilon < \alpha\rho_m$, respectively. Therefore, by choosing appropriate Δ' and ϵ for the sufficiently small bounded disturbance (i.e., θ is sufficiently small), $\dot{W}_c$ still remains negative during each sampling period in the presence of disturbance. Additionally, if $x(t_k) \in \mathcal{B}_\delta(x_e)$, we again assume that there exists a set of feasible control actions $\bar{u}(x)$ that satisfies $W_c(x(t_{k+1})) < W_c(x(t_k)), \forall\ |w| \leq \theta$. On the other hand, based on the definition of ρ'_{min} of Eq. (18), it is trivial to show that for any $x(t_k) \in \mathcal{U}_{\rho_s}$, the trajectory of $x(t)$ is guaranteed to stay in $\mathcal{U}_{\rho'_{min}}, \forall\ t \in [t_k, t_k + \Delta'')$. The above proof implies that our CLBF-MPC is robust to the sufficiently small bounded disturbance.

Remark 5. We assume that there exists a set of feasible solutions $\bar{u}(x) \in U$ in $\mathcal{B}_\delta(x_e)$ such that the closed-loop state leaves $\mathcal{B}_\delta(x_e)$ in the direction of decreasing $W_c(x)$. For example, $\bar{u}(x)$ can be determined as $\bar{u}(x(t_k)) = \arg\min_{u\in U}\{W_c(x(t_{k+1})) \mid W_c(x(t_{k+1})) < W_c(x(t_k))\}$. However, in the absence of input constraints, the fact that x_e is a saddle point ensures that there is always a control action (maybe large) that would make $W_c(x)$ decrease. Once the state leaves $\mathcal{B}_\delta(x_e)$ in the direction of decreasing $W_c(x)$, it continues to move towards the origin under $u = \Phi(x)$.

3.2. Formulation of CLBF-MPC

The CLBF-MPC design is represented by the following optimization problem:

$$\mathcal{J} = \min_{u\in S(\Delta)} \int_{t_k}^{t_k+P_N\Delta} L(\tilde{x}(t), u(t))dt \tag{19a}$$

$$\text{s.t}\quad \dot{\tilde{x}}(t) = f(\tilde{x}(t)) + g(\tilde{x}(t))u(t) \tag{19b}$$

$$u(t) \in U,\ \forall\ t \in [t_k, t_k + P_N\Delta) \tag{19c}$$

$$\tilde{x}(t_k) = x(t_k) \tag{19d}$$

$$\begin{aligned}&\dot{W}_c(x(t_k), u(t_k)) \leq \dot{W}_c(x(t_k), \Phi(x(t_k))), \\ &\text{ if } W_c(x(t_k)) > \rho'_{min} \text{ and } x(t_k) \notin \mathcal{B}_\delta(x_e)\end{aligned} \tag{19e}$$

$$\begin{aligned}&W_c(\tilde{x}(t)) \leq \rho'_{min},\ \forall\ t \in [t_k, t_k + P_N\Delta), \\ &\text{ if } W_c(x(t_k)) \leq \rho'_{min}\end{aligned} \tag{19f}$$

$$\begin{aligned}&W_c(\tilde{x}(t)) < W_c(x(t_k)),\ \forall\ t \in (t_k, t_k + P_N\Delta), \\ &\text{ if } x(t_k) \in \mathcal{B}_\delta(x_e)\end{aligned} \tag{19g}$$

where $\tilde{x}(t)$ is the predicted state trajectory, $S(\Delta)$ is the set of piecewise constant functions with period Δ, and P_N is the number of sampling periods in the prediction horizon. $\dot{W}_c(x, u)$ is used to represent $\frac{\partial W_c(x)}{\partial x}(f(x) + g(x)u)$. The cost function $L(\tilde{x}(t), u(t))$ satisfies $L(0, 0) = 0$ and $L(\tilde{x}(t), u(t)) > 0, \forall(\tilde{x}(t), u(t)) \neq (0, 0)$ such that the minimum value of the cost function will be attained at the equilibrium point of the system of Eq. (1). Let $u^*(t)$ be the optimal solution of the optimization problem of Eq. (19) over the prediction horizon $P_N\Delta$. We assume that the states of the closed-loop system are measured at each sampling time. Specifically, the above optimization problem is solved based on the measured state $x(t_k)$ at $t = t_k$. After $u^*(t)$, where $t \in [t_k, t_k + P_N\Delta)$, is obtained from the CLBF-MPC optimization problem, only the first control action of $u^*(t)$ is sent to the control actuators to be applied over the next sampling period. Then, at the next instance of time $t_{k+1} := t_k + \Delta$, the optimization problem is solved again, and the horizon will be rolled one sampling period.

In the optimization problem of Eq. (19), the objective function of Eq. (19a) that is minimized is the integral of $L(\tilde{x}(t), u(t))$ over the prediction horizon, where the function $L(x, u)$ is not restricted to a traditional quadratic function. The constraint of Eq. (19b) is the nominal system of Eq. (1) (i.e., $w(t) \equiv 0$) and is used to predict the evolution of the closed-loop state. Eq. (19c) defines the input constraints for all the inputs over the entire prediction horizon. Eq. (19d) defines the initial condition of the nominal process system of Eq. (19b). The constraint of Eq. (19e) forces $W_c(\tilde{x})$ along the predicted state trajectories to decrease at least at the rate under $u = \Phi(x)$ when $W_c(x(t_k)) > \rho'_{min}$ and $x(t_k) \notin \mathcal{B}_\delta(x_e)$, while the constraint of Eq. (19f) activates if $W_c(x(t_k)) \leq \rho'_{min}$ (i.e., $x(t_k)$ enters a small ball around the origin $\mathcal{B}_d(0) := \{x \in \mathbf{R}^n \mid |x| \leq d\}$) so that the states of the closed-loop system will remain inside $\mathcal{B}_d(0)$ afterwards. Additionally, if $x(t_k) \in \mathcal{B}_\delta(x_e)$, the constraint of Eq. (19g) is activated to decrease $W_c(x)$. Once the state leaves $\mathcal{B}_\delta(x_e)$, it is guaranteed that the state does not return to $\mathcal{B}_\delta(x_e)$ because the state will be driven to smaller level sets of $W_c(x)$ under the constraint of Eq. (19e) thereafter.

Theorem 2 shows that the control actions computed by the CLBF-MPC of Eq. (19) guarantee that the state of the closed-loop system of Eq. (1) is always bounded in $\mathcal{U}_{\rho_c}$, and is ultimately bounded in a small region around the origin. In addition, the optimization problems are recursively feasible.

Theorem 2. *Consider the system of Eq. (1) with a constrained CLBF W_c which has a minimum at the origin, and the set of initial conditions $\mathcal{U}_{\rho_c}$. Given any initial state $x_0 \in \mathcal{U}_{\rho_c}$, it is guaranteed that the optimization problem is feasible for all times under the CLBF-MPC scheme of Eq. (19) with sampling period $\Delta \in (0, \Delta^*]$, which is defined in Proposition 1. Additionally, for $x_0 \in \mathcal{U}_{\rho_c}$, it is guaranteed that $x(t) \in \mathcal{U}_{\rho_c},\ \forall\ t \geq 0$, and $\limsup_{t\to\infty} |x(t)| \leq d$.*

Proof. The proof of this proposition consists of two parts. In the first part, we show that for all $x_0 \in \mathcal{U}_{\rho_c}$, the optimization problem of Eq. (19) is recursively feasible throughout the entire prediction horizon. Then, we show that under the CLBF-MPC, the trajectory of $x(t)$ is always bounded in $\mathcal{U}_{\rho_c}$, and is ultimately bounded in a small region around the origin $\mathcal{U}_{\rho'_{min}}$.

Part 1 : Assuming that $x(t_k) \in \mathcal{U}_{\rho_c} \backslash \mathcal{U}_{\rho'_{min}}$, $t_k \geq 0$ where ρ'_{min} is defined in Eq. (18), the sample-and-hold control law $u(t) = \Phi(x(t_k + i\Delta))$, $i = 0, 1, \ldots, P_N - 1$, and $u(t) = \bar{u}(x)$ are feasible solutions to the optimization problem of Eq. (19). Specifically, when $x(t_k) \in \mathcal{U}_{\rho_c} \backslash (\mathcal{U}_{\rho'_{min}} \cup \mathcal{B}_\delta(x_e))$, $u(t) = \Phi(x(t_k + i\Delta))$ satisfies both the input constraint of Eq. (19c) and the constraint of Eq. (19e) when the controller is applied in the sample-and-hold fashion. However, if $x(t_k) \in \mathcal{B}_\delta(x_e)$, $u(t) = \bar{u}(x) \in U$ is a set of feasible solutions that satisfies the constraints of Eq. (19c) and of Eq. (19g). The constraint of Eq. (19f) is not activated in this case.

When $x(t_k) \in \mathcal{U}_{\rho'_{min}}$, $u(t) = \Phi(x(t_k + i\Delta))$, $i = 0, 1, \ldots, P_N - 1$ is again a feasible solution that satisfies the constraints of Eqs. (19c), (19f). Specifically, if $x(t_k) \in \mathcal{U}_{\rho_s} \subset \mathcal{U}_{\rho'_{min}}$, it is guaranteed that the constraint of Eq. (19f) is satisfied according to the definition of ρ'_{min} of Eq. (18). However, if $x(t_k) \in \mathcal{U}_{\rho'_{min}} \backslash \mathcal{U}_{\rho_s}$, based on the proof in Proposition 2, it follows that the sample-and-hold controller $u(t) = \Phi(x(t_k + i\Delta))$ guarantees $\dot{W}_c(x) < -\epsilon$ over a sampling period, which implies that $W_c(x(t_{k+1})) \leq W_c(x(t_k)) \leq \rho'_{min}$. Therefore, at every sampling time, if $x(t_k) \in \mathcal{U}_{\rho_c}$, a feasible solution to the optimization problem of Eq. (19) exists.

Part 2 : We now prove that if $x_0 \in \mathcal{U}_{\rho_c}$, $x(t) \in \mathcal{U}_{\rho_c},\ \forall\ t \geq 0$. Since the initial condition x_0 is in the set $\mathcal{U}_{\rho_c}$, it follows that under the constraints of Eqs. (19c)–(19g), $x(t) \in \mathcal{U}_{\rho_c},\ \forall\ t \geq 0$ by letting $t_k = 0$ for the result of $W_c(x(t)) < W_c(x(t_k)) \leq \rho_c, \forall\ t > t_k$ from Proposition 2. Therefore, the assumption that $x(t_k) \in \mathcal{U}_{\rho_c}$ at $t = t_k$, $t_k \geq 0$ in Part 1 is also proved.

Finally, let $x_0 \in \mathcal{U}_{\rho_c} \backslash \mathcal{U}_{\rho'_{min}}$, we will show that $x(t)$ ultimately enters $\mathcal{U}_{\rho'_{min}}$ and remains there for all subsequent times. It follows that $W_c(x(t + \Delta)) < W_c(x(t))$ holds when $x(t) \in \mathcal{U}_{\rho_c} \backslash (\mathcal{U}_{\rho'_{min}} \cup \mathcal{B}_\delta(x_e))$ and $x(t) \in \mathcal{B}_\delta(x_e)$ from the proof in Proposition 2. This implies that within finite time t_s, the trajectory will enter $\mathcal{U}_{\rho'_{min}}$.

Also, it has been shown in Part 1 that if $x(t) \in \mathcal{U}_{\rho'_{min}}$, the constraint of Eq. (19f) is satisfied according to the definition of $\mathcal{U}_{\rho'_{min}}$, and hence there always exists a set of control actions such that $W_c(x(t)) \leq \rho'_{min},\ \forall\ t \geq t_s$. Note that $W_c(\cdot)$ is a continuous function of the state, thus given the real number ρ'_{min}, one can find a positive real number d, such that $W_c(x(t)) \leq \rho'_{min}$ implies $\limsup_{t\to\infty} |x(t)| \leq d$.

Remark 6. We note that Theorem 2 applies to both the nominal closed-loop system of Eq. (1) with $w(t) \equiv 0$ and the closed-loop system of Eq. (1) subject to bounded disturbances (i.e., $|w| \leq \theta$) under the sample-and-hold implementation of the CLBF-MPC of Eq. (19). The case of nominal closed-loop system is straightforward since the nominal system of Eq. (1) is used as the prediction process model in the formulation of CLBF-MPC of Eq. (1), which implies that the real closed-loop states are consistent with the predicted states under the CLBF-MPC and therefore closed-loop stability and safety are guaranteed following the above proof. However, considering the system subject to bounded disturbances, it is shown in Proposition 2 that for sufficiently small disturbances θ and sufficiently small sampling period Δ, $\dot{W}_c$ of the uncertain closed-loop system of Eq. (1) continues to satisfy $\dot{W}_c < -\epsilon$ for all $x \in \mathcal{U}_{\rho_c} \backslash (\mathcal{U}_{\rho'_{min}} \cup \mathcal{B}_\delta(x_e))$. Additionally, if $x \in \mathcal{U}_{\rho'_{min}}$ or $x \in \mathcal{B}_\delta(x_e)$, the constraints of Eq. (19f) and of Eq. (19g) still hold since ρ'_{min} of Eq. (18) and $\bar{u}(x)$ are determined accounting for the impact of the bounded disturbances. Therefore, all the constraints of Eq. (19) are still satisfied and Theorem 2 holds for the uncertain closed-loop system of Eq. (1) with $|w| \leq \theta$.

Remark 7. It should be noted that the problem of convergence to x_e instead of the origin can be solved by taking advantage of the CLBF-MPC of Eq. (19). The constraint of Eq. (19g) requires $W_c(x)$ to decrease if $x(t_k) \in \mathcal{B}_\delta(x_e)$, which drives the state out of $\mathcal{B}_\delta(x_e)$ in the direction of decreasing $W_c(x)$. Additionally, since in general, the objective function of the CLBF-MPC of Eq. (19a) penalizes the distances between states and the origin and also control actions, the objective function value becomes large if the state converges to any points other than the origin (e.g., x_e). Therefore, CLBF-MPC will try to avoid converging to x_e by optimizing control actions in a sample-and-hold fashion (i.e., discontinuous control actions) and taking future cost values into account.

Remark 8. The control law of MPC is discontinuous in x due to the two modes of operation of CLBF-MPC. However, continuity of MPC control law is not required to achieve closed-loop stability in the sense that the closed-loop state can be driven to a neighborhood around the origin $\mathcal{U}_{\rho'_{min}}$ for any initial condition $x(0) \in \mathcal{U}_{\rho_c}$ instead of asymptotically stability under continuous

Table 1
Parameter values of the CSTR.

$T_0 = 310$ K	$F = 100 \times 10^{-3}$ m^3/min
$V_L = 0.1$ m^3	$E = 8.314 \times 10^4$ kJ/kmol
$k_0 = 72 \times 10^9$ min^{-1}	$\Delta H = -4.78 \times 10^4$ kJ/kmol
$C_p = 0.239$ kJ/(kg K)	$R = 8.314$ kJ/(kmol K)
$\rho_L = 1000$ kg/m^3	$C_{A0_s} = 1.0$ kmol/m^3
$Q_s = 0.0$ kJ/min	$C_{A_s} = 0.57$ kmol/m^3
$T_s = 395.3$ K	

Lyapunov-based control law of Eq. (3). We note that the closed-loop system $\dot{x} = f(x) + g(x)u(t)$ has a right hand side which is sufficiently smooth in x and piecewise continuous in t, therefore it has a solution which is unique and via the fact that it stays in a level set of V this solution exists for all times.

4. Application to a chemical process example

In this section, we utilize a chemical process example to illustrate the application of the proposed CLBF-MPC method. Consider a well-mixed, non-isothermal continuous stirred tank reactor (CSTR) where an irreversible first-order exothermic reaction takes place. The reaction converts the reactant A to the product B via the chemical reaction $A \rightarrow B$. A heating jacket that supplies or removes heat from the reactor is used. The CSTR dynamic model derived from material and energy balances is given below:

$$\frac{dC_A}{dt} = \frac{F}{V_L}(C_{A0} - C_A) - k_0 e^{-E/RT} C_A \tag{20a}$$

$$\frac{dT}{dt} = \frac{F}{V_L}(T_0 - T) - \frac{\Delta H k_0}{\rho_L C_p} e^{-E/RT} C_A + \frac{Q}{\rho_L C_p V_L} \tag{20b}$$

where C_A is the concentration of reactant A in the reactor, T is the temperature of the reactor, Q denotes the heat supply/removal rate, and V_L is the volume of the reacting liquid in the reactor. The feed to the reactor contains the reactant A at a concentration C_{A0}, temperature T_0, and volumetric flow rate F. The liquid has a constant density of ρ_L and a heat capacity of C_p. k_0, E and ΔH are the reaction pre-exponential factor, activation energy and the enthalpy of the reaction, respectively. Process parameter values are listed in Table 1. The control objective is to operate the CSTR at the equilibrium point $(C_{As}, T_s) = (0.57 \text{ kmol/m}^3, 395.3 \text{ K})$ and maintain the state in a safe region of state–space by manipulating the heat input rate $\Delta Q = Q - Q_s$, and the inlet concentration of species A, $\Delta C_{A0} = C_{A0} - C_{A0_s}$. The input constraints for ΔQ and ΔC_{A0} are $|\Delta Q| \leq 0.0167$ kJ/min and $|\Delta C_{A0}| \leq 1$ kmol/m^3, respectively.

To place Eq. (20) in the form of nonlinear systems of Eq. (1), deviation variables are used in this example, such that the equilibrium point of the system is at the origin of the state–space. $x^T = [C_A - C_{As} \; T - T_s]$ represents the state vector in deviation variable form, $u^T = [\Delta C_{A0} \; \Delta Q]$ represents the manipulated input vector in deviation variable form, and $w^T = [w_1 \; w_2]$ is the bounded disturbance vector of Gaussian distribution with zero mean and variance $\sigma_1 = 1.0$ kmol/m^3, $\sigma_2 = 3.5$ K. The upper bound for disturbances $|w_1| \leq 1.0$ kmol/m^3 and $|w_2| \leq 3.17$ K are approximated via simulation runs under various sizes of disturbances.

We construct a Control Lyapunov Function using the standard quadratic form $V(x) = x^T P x$ with $P = \begin{bmatrix} 9.35 & 0.41 \\ 0.41 & 0.02 \end{bmatrix}$. In order to show the effectiveness of the proposed CLBF-MPC control scheme to maintain closed-loop stability and safety, we define the unsafe region as a region located in the middle of the set ϕ_{uc}, such that it is possible for the trajectory of the closed-loop system to encounter the unsafe region when converging to the origin. The unsafe region is defined as an ellipse: $\mathcal{D} := \{x \in \mathbf{R}^2 \mid F(x) = \frac{(x_1+0.22)^2}{1} + \frac{(x_2-4.6)^2}{1\times 10^4} < 2 \times 10^{-4}\}$. $\mathcal{H}$ is defined as $\mathcal{H} := \{x \in \mathbf{R}^2 \mid F(x) < 4 \times 10^{-4}\}$ such that it satisfies $\mathcal{D} \subset \mathcal{H} \subset \phi_{uc}$ in Proposition 1. The Control Barrier Function $B(x)$ is defined as follows.

$$B(x) = \begin{cases} e^{\frac{aF^2(x)}{F(x)-4\times 10^{-4}}} - e^{-2a\times 10^{-4}}, & \text{if } x \in \mathcal{H} \\ -e^{-2a\times 10^{-4}}, & \text{if } x \notin \mathcal{H} \end{cases} \tag{21}$$

where a is a parameter that can be adjusted in characterizing the set ϕ_{uc}. From Eq. (21), it is guaranteed that $B(x)$ is positive in the unsafe region $\mathcal{D}$. Then, the Control Lyapunov-Barrier Function $W_c(x) = V(x) + \mu B(x) + \nu$ is constructed following the rules in Proposition 1, where the parameters are determined as follows, $a = 0.001$, $\rho_c = 0$, $c_1 = 0.001$, $c_2 = 10$, $c_3 = \max_{x\in\partial\mathcal{H}} |x|^2 = 34.8$, $c_4 = \min_{x\in\partial\mathcal{D}} |x|^2 = 16.85$, and $\nu = \rho_c - c_1 c_4 = -1.685 \times 10^{-2}$. Hence, μ is chosen to be 5000 to satisfy Eq. (13). Based on the above $W_c(x)$, x_e is calculated to be a saddle point $(-0.235, 4.83)$ in state–space.

The objective function of the CLBF-MPC in this example is formulated to seek to drive the system to its equilibrium point while minimizing the heat supply and removal rate, and the feed reactant concentration as well, and is given as follows,

$$L(\tilde{x}, u) = |\tilde{x}(t)|^2_{Q_L} + |u(t)|^2_{R_L} \tag{22}$$

where the weighting matrices for the states and inputs are chosen to be $Q_L = \begin{bmatrix} 1000 & 0 \\ 0 & 10 \end{bmatrix}$ and $R_L = \begin{bmatrix} 1 & 0 \\ 0 & 100 \end{bmatrix}$, respectively, such that the term related to the states and the term related to the inputs are on the same order of magnitude in Eq. (22) to penalize both state and input deviations from the steady-state significantly. In the simulations below, the process model of Eq. (20) was integrated numerically using the explicit Euler method with an integration time step of $h_c = 10^{-5}$ min. The MPC sampling period and the prediction horizon were chosen to be $\Delta = 2 \times 10^{-3}$ min and $P_N = 10$, under which the desired closed-loop performance is achieved with high computational efficiency (i.e., the control action calculation is done within the sampling period). The constrained nonlinear optimization problem was solved using the IPOPT software package (Wächter & Biegler, 2006) with a 4-core CPU desktop.

We first demonstrate the implementation of the CLBF-MPC for the case that the trajectory of the closed-loop system does not reach the border of the unsafe region when converging to the origin. To this end, we first chose the subset $\mathcal{U}_\rho \subset \mathcal{U}_{\rho_c}$ as the safe operating region and set an initial condition that is far away from the set $\mathcal{D}$. Starting from the initial condition $(x_1, x_2) = (0.2, -5)$, it is demonstrated that the stabilization of the closed-loop system can be achieved (the green trajectory in Fig. 2), and the states always remain in $\mathcal{U}_\rho$. Additionally, another three initial conditions $(-0.19, 5.5)$, $(-0.35, 7)$ and $(-0.235, 6.5)$ are chosen to start the system from where the state encounters the unsafe region $\mathcal{D}$ on its way to the origin under the CLBF-MPC as shown in Fig. 2. All three demonstrate that the states avoid the unsafe region $\mathcal{D}$ and ultimately converge to the origin.

We now study the case where the trajectory that the state takes from the initial condition $(-0.235, 6.5)$ under the CLBF-MPC reaches the border of $\mathcal{D}$, and also compare it with the same initial condition under a non-Lyapunov-based MPC with a state constraint to avoid $\mathcal{D}$ and a terminal constraint to guarantee closed-loop stability. It is demonstrated in Fig. 3 that under the CLBF-MPC (black solid line), the state will first reach the boundary of $\mathcal{H}$, then avoid entering the unsafe region $\mathcal{D}$ by passing around it, and finally, the state trajectory is moving towards the origin. However, under MPC with state constraints, it is demonstrated that the optimization problem becomes infeasible when

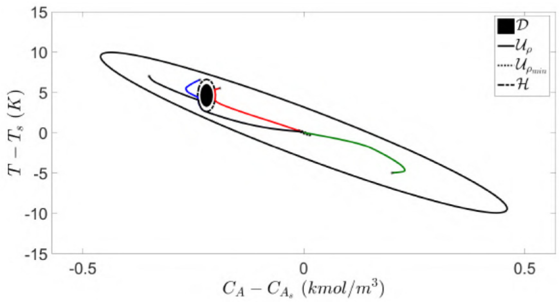

Fig. 2. Closed-loop state trajectories under the CLBF-MPC for four different initial conditions (0.2, −5) (green), (−0.19, 5.5) (red), (−0.35, 7) (black) and (−0.235, 6.5) (blue). The set of unsafe states $\mathcal{D}$ is shaded in solid black area and the set $\mathcal{U}_\rho$ is the region between the level set of the Lyapunov function (the largest ellipse) and the set $\mathcal{H}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

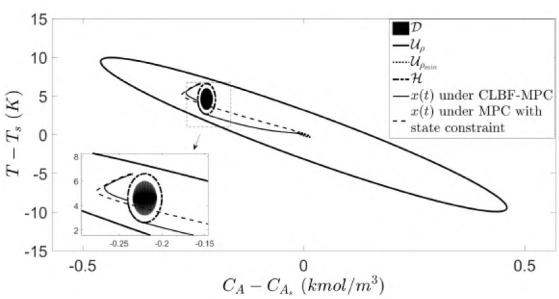

Fig. 3. Closed-loop state profiles under the CLBF-MPC of Eq. (19) (solid line) and under the MPC with state constraints (dashed line), where the unsafe region $\mathcal{D}$ is an obstacle for the closed-loop state trajectory starting from the initial condition (−0.235, 6.5).

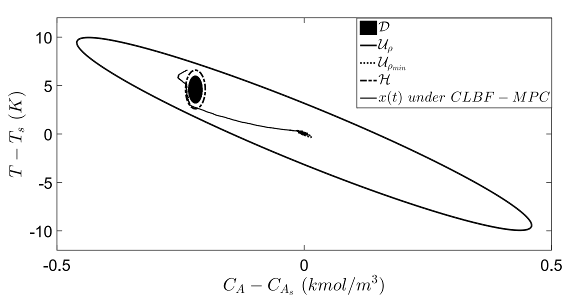

Fig. 4. Closed-loop state profile for the initial condition (−0.235, 6.5) under the CLBF-MPC of Eq. (19) (solid line) subject to bounded disturbance.

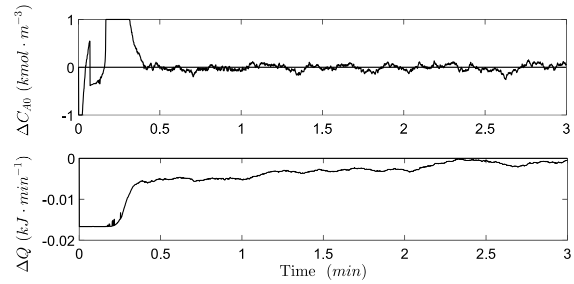

Fig. 5. Manipulated input profiles ($u_1 = \Delta C_{A0}$ and $u_2 = \Delta Q$) for the initial condition (−0.235, 6.5) under the CLBF-MPC of Eq. (19) subject to bounded disturbance.

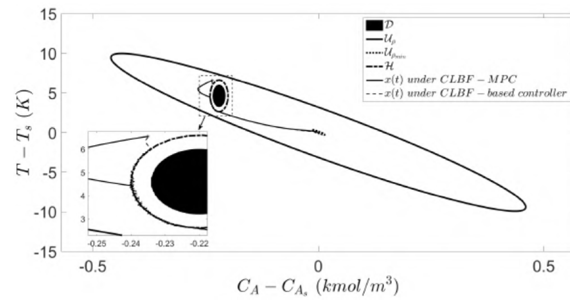

Fig. 6. Closed-loop state profiles for the initial condition (−0.235, 6.5) under the CLBF-MPC of Eq. (19) (solid line) and under the CLBF-based controller of Eq. (3) (dashed line).

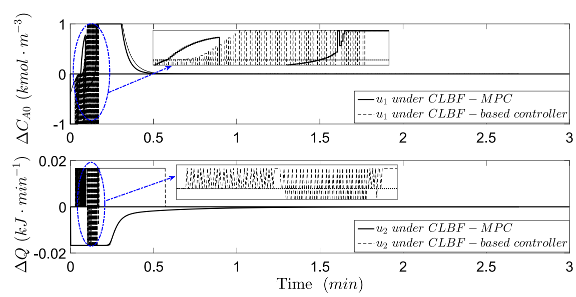

Fig. 7. Manipulated input profiles ($u_1 = \Delta C_{A0}$ and $u_2 = \Delta Q$) for the initial condition (−0.235, 6.5) under the CLBF-MPC of Eq. (19) (solid line) and under the CLBF-based controller of Eq. (3) (dashed line).

the trajectory gets close to the boundary of the unsafe region. In this case, we deactivate the state constraint and apply the feasible solution of the optimization problem of MPC with terminal constraint only such that the trajectory crosses the unsafe region but can still move towards the origin. Therefore, CLBF-MPC outperforms the standard MPC with state constraints since it reconciles the tasks of safety and closed-loop stability with guaranteed recursive feasibility.

Additionally, when the bounded disturbance is added into the process model, it is demonstrated in Fig. 4 that the CLBF-MPC can still guarantee safety and closed-loop stability. The corresponding input profiles are also shown in Fig. 5, in which it is seen that the control actions oscillate around the steady state due to the disturbance.

To demonstrate the advantages of the proposed CLBF-MPC control scheme compared to the case of using explicit CLBF-based control law of Eq. (3) all the time, the simulation results of the closed-loop state and inputs profiles for the same initial condition (−0.235, 6.5) are shown in Figs. 6 and 7, respectively. In Fig. 7, it is observed that under the CLBF-based control law of Eq. (3), the inlet concentration of the reactant and the heat input rate start oscillating heavily from $t = 0.003$ min to $t = 0.2$ min, and correspondingly, the oscillation arises in the state trajectory near the boundary of $\mathcal{H}$. The reason for the oscillation is that the intrinsic dynamics of the closed-loop system force the states to go towards and cross the unsafe region, yet the constraints of CLBF-MPC prevent this undesirable behavior due to the dramatic increase in the values of W_c inside the unsafe region (barrier function dominates). By balancing these two opposite effects, the control action becomes oscillating when the state moves around the boundary of $\mathcal{H}$. Additionally, under the proposed CLBF-MPC control scheme, the dynamic performance was improved since MPC has the ability to anticipate future state behavior and can take control actions accordingly.

Also, from the simulation results, it is shown that under the CLBF-MPC, the total consumptions of reactant ΔC_{A0} and of energy ΔQ within the operating time $t_s = 3$ min are 0.268 kmol/m^3

and 0.006 kJ, respectively, which represent improvements of 13% and 25%, respectively, compared to 0.308 $kmol/m^3$ and 0.008 kJ under the explicit CLBF-based controller. Therefore, in this case, the CLBF-MPC of Eq. (19) outperforms the explicit CLBF-based controller of Eq. (3) due to the smoother control actions and reduced control energy consumptions.

Remark 9. It is noted that the chosen region $\mathcal{D}$ is an illustrative case of a bounded unsafe set embedded fully within the closed-loop system stability region. Such an unsafe set poses both theoretical as well as implementation challenges for CLBF-MPC (and other CLBF-based controller designs) as the controller has to drive the state around the unsafe region and to the steady-state. Furthermore, safeness of a point/region of the state space is a function of multiple process states (e.g., combination of temperature and concentration of reactants that determine magnitude of reaction rates) and not only a function of the process temperature, and as a result, the states to the left of the bounded unsafe set that is of higher temperature but lower reactant concentration may be safer than the states in the unsafe set. Finally, in the case of an unsafe region which is characterized by an upper bound on the reactor temperature (i.e., all states with temperature above a certain value are considered as unsafe, resulting in an unbounded unsafe set), the proposed CLBF-MPC can be readily implemented to drive the state to the steady-state and avoid the unsafe region.

5. Conclusion

In this work, we considered the problem of simultaneous stabilization of nonlinear systems subject to input constraints and meeting the safety task of avoiding an unsafe region in state–space. We proposed a CLBF-based model predictive control design that can guarantee stability and safety simultaneously from a well-characterized set of initial conditions. The constrained CLBF was first used to characterize the set of initial conditions starting from which both closed-loop stability and safety can be achieved under the CLBF control law. Then, the CLBF-based control law was used as a candidate control law to design constraints for the CLBF-MPC and establish simultaneous closed-loop stability and safety with guaranteed recursive feasibility under CLBF-MPC. The application of the CLBF-MPC design was demonstrated through a chemical process example.

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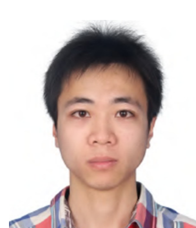

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