DISTRIBUTED NONLINEAR CONTROL OF DIFFUSION-REACTION PROCESSES

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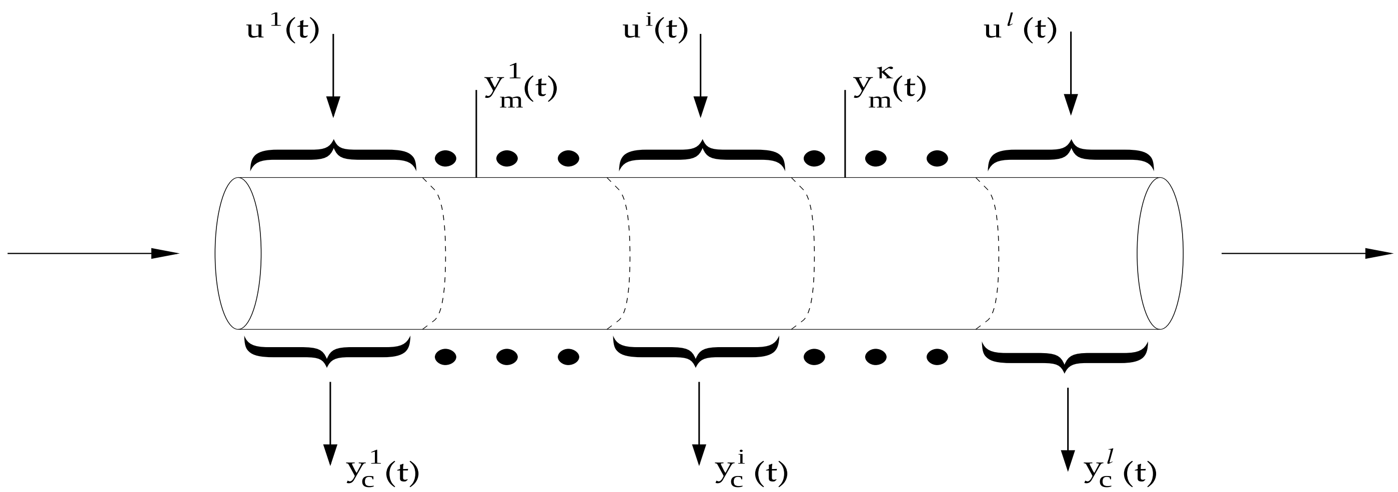
INTRODUCTION

- Spatially-distributed systems.
  - ◇ Representative examples:
    - ▷ Transport-reaction processes.
    - ▷ Fluid dynamic systems.
  - ◇ Control problem: Stabilization and/or regulation of spatially distributed variables using a small number of spatially-distributed control actuators and measurement sensors.



  - ◇ Highly dissipative partial differential equation (PDE) systems.

BACKGROUND ON CONTROL OF DISSIPATIVE PDEs

- Standard approach:

  (e.g., Balas, IJC, 1979; Ray, 1981, Curtain, 1982).

  ◇ Derivation of ODE models using eigenfunction expansions.

  ◇ Controller design using methods for ODEs.

  ◇ High-dimensionality of the controller?

- Synthesis of nonlinear low-order controllers:

  (Christofides, Birkhäuser, 2001).

  ◇ Derivation of low-order ODE models using Galerkin's method and approximate inertial manifolds.

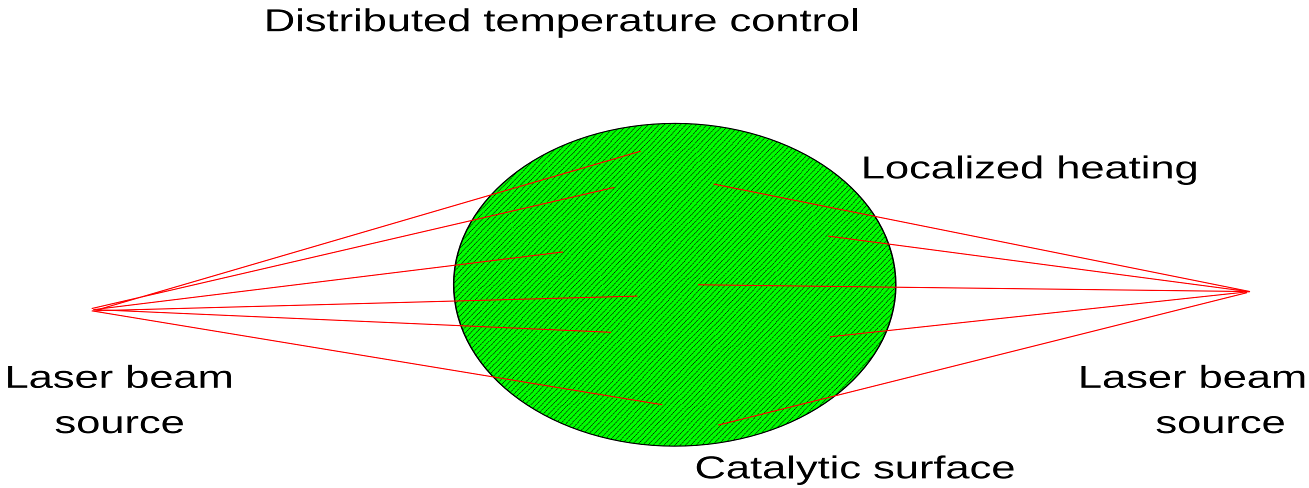
  ◇ Nonlinear, robust and constrained control.

- Other approaches:

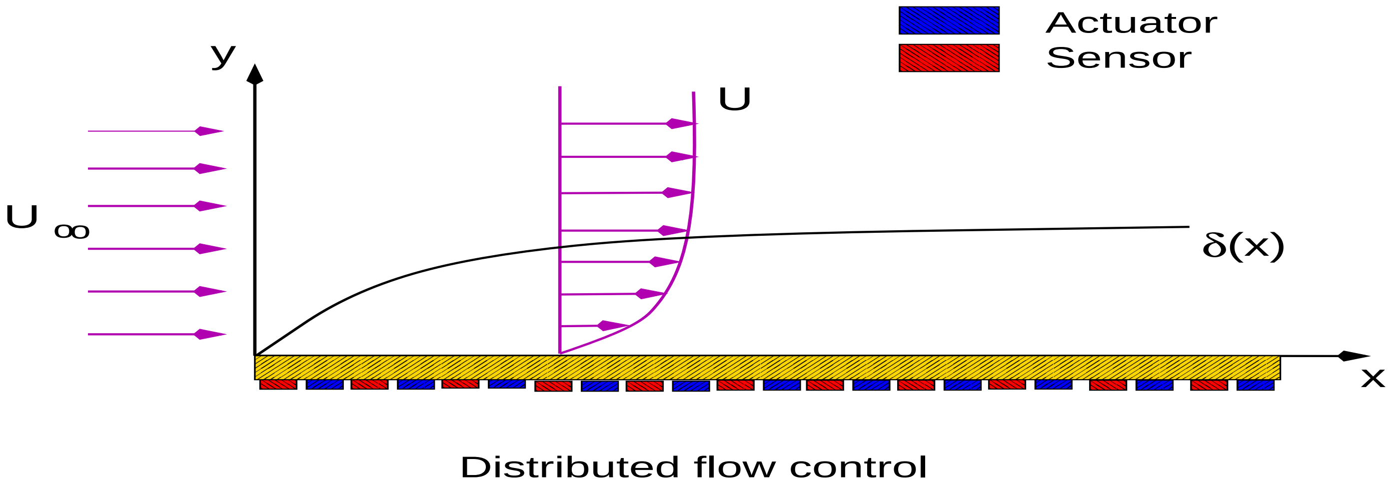
  ◇ Controller design based on infinite-dimensional system (e.g., Burns and King, J. Vibr. & Contr., 1998).

  ◇ Stabilization via Lyapunov-based boundary feedback (e.g., Liu and Krstic, NA, 2001).

DISTRIBUTED ACTUATION AND SENSING



(Papathanasiou *et. al.*, CPL, 2002)



(Ho and Tai, ARFM, 1998)

- Exploit large number of degrees of freedom in actuation/sensing to create complex spatio-temporal behavior in the closed-loop system.

PRESENT WORK

(Dubljevic, Christofides and Kevrekidis, IJRNC, 2003)

- Distributed nonlinear control of diffusion-reaction processes described by quasi-linear parabolic PDEs.

  ◇ Control problem formulation:

    ▷ Large numbers of actuators/sensors are available.

    ▷ Desired behavior described by a "target parabolic PDE".

  ◇ Methodology for controller design:

    ▷ Order reduction using Galerkin's method.

    ▷ Nonlinear state/output feedback controller design using geometric control methods.

  ◇ Application to diffusion-reaction processes.

QUASI-LINEAR PARABOLIC PDE SYSTEMS

- **System description**

$$\frac{\partial \bar{x}}{\partial t} = B\frac{\partial^2 \bar{x}}{\partial z^2} + w\sum_{i=1}^{m} b_i(z)u_i + f(\bar{x})$$

$$y_m^{\kappa} = \int_0^{\pi} s^{\kappa}(z)\omega\bar{x}(z,t)dz \qquad \kappa = 1,\ldots,p$$

- **Boundary conditions:**

$$\bar{x}(0,t) = 0, \quad \bar{x}(\pi,t) = 0$$

- ◇ $\bar{x}(z,t)$: state variable
- ◇ $u^i(t)$: manipulated input
- ◇ $y_m^{\kappa}(t)$: measured output
- ◇ m : number of actuators
- ◇ $z \in [0,\pi]$: spatial coordinate
- ◇ $b^i(z)$: actuator function
- ◇ $s^{\kappa}(z)$: sensor shape function
- ◇ $f(\bar{x})$: nonlinear function

DESIRED SPATIO-TEMPORAL BEHAVIOR

- **"Target parabolic PDE system":**

$$\frac{\partial \bar{x}}{\partial t} = B\frac{\partial^2 \bar{x}}{\partial z^2} + \hat{f}(\bar{x}, z, t)$$

- **Boundary conditions:**

$$\bar{x}(0,t) \ = \ 0, \ \ \bar{x}(\pi,t) \ = \ 0$$

◇ $\bar{x}(z,t)$: state variable ◇ $\hat{f}(\bar{x},z,t)$: nonlinear function

- "Target parabolic PDE system" can be generated for a given closed-form expression of desired $\bar{x}(z,t)$.

GALERKIN'S METHOD

- $\mathcal{H}_s = span\{\phi_1, \phi_2, \ldots, \phi_m\}$, $\mathcal{H}_f = span\{\phi_{m+1}, \phi_{m+2}, \ldots, \}$

  P_s, P_f : orthogonal projection operators

  $x_s(t) = P_s x(t)$: state vector corresponding to first m eigenmodes

  $x_f(t) = P_f x(t)$: state vector corresponding to remaining eigenmodes

- Set of infinite ODEs:

$$\begin{aligned} \frac{dx_s}{dt} &= A_s x_s + \mathcal{B}_s u + f_s(x_s, x_f) \\ \frac{\partial x_f}{\partial t} &= A_f x_f + \mathcal{B}_f u + f_f(x_s, x_f) \end{aligned}$$

- Neglecting the x_f-subsystem:

$$\frac{d\tilde{x}_s}{dt} = A_s \tilde{x}_s + \mathcal{B}_s u + f_s(\tilde{x}_s, 0), \quad \tilde{x}_f = 0$$

- Other spatial discretization techniques may be used.

NONLINEAR CONTROL DESIGN

- Approximation of PDE systems using Galerkin's method:
  - ◇ Approximation of original PDE system:

$$\frac{d\tilde{x}_s}{dt} = \mathcal{A}_s\tilde{x}_s + \mathcal{B}_s u + f_s(\tilde{x}_s, 0)$$

  - ◇ Approximation of target PDE system:

$$\frac{d\tilde{x}_s}{dt} = \mathcal{A}_s\tilde{x}_s + \hat{f}_s(\tilde{x}_s, t)$$

  - ◇ Orders of both finite-dimensional systems are the same and are determined based on the desired accuracy.
- Assumption: Number of control actuators is equal to the order of approximations, and $\mathcal{B}_s$ is invertible.
- Nonlinear state feedback controller:

$$u = \mathcal{B}_s^{-1}\left(-f_s(\tilde{x}_s, 0) + \hat{f}_s(\tilde{x}_s, t)\right)$$

- Closed-loop finite-dimensional system:

$$\dot{\tilde{x}}_s = \mathcal{A}_s\tilde{x}_s + \hat{f}_s(\tilde{x}_s, t)$$

STATE ESTIMATION

$$\begin{aligned}\frac{dx_s}{dt} &= \mathcal{A}_s x_s + \mathcal{B}_s u + f_s(x_s, x_f) \\ \frac{\partial x_f}{\partial t} &= \mathcal{A}_f x_f + \mathcal{B}_f u + f_f(x_s, x_f) \\ y_m &= \mathcal{S} x_s + \mathcal{S} x_f\end{aligned}$$

- **Estimate** x_s **from the measurements** y_m .
- **Assumptions:**
  - ◇ $\kappa = m$ (# of measurements = order of the finite-dimensional systems)
  - ◇ $\mathcal{S}^{-1}$ exists (ODE approximation observable via static output feedback)
    - ⋆ Appropriate selection of sensor locations

$$\hat{x}_s = \mathcal{S}^{-1} y_m, \ \hat{x}_s \text{ is an estimate of } x_s$$

NONLINEAR OUTPUT FEEDBACK CONTROL

- **Controller structure**:

  ◇ Computation of the control action:

$$u(\hat{x}_s) = \mathcal{B}_s^{-1}\left(-f_s(\hat{x}_s, 0) + \hat{f}_s(\hat{x}_s, t)\right)$$

  ◇ Estimate of finite-dimensional system state:

$$\hat{x}_s = \mathcal{S}^{-1} y_m$$

- **Enforcement of the "target" spatio-temporal behavior in the closed-loop system** as $m \to \infty$.

LINEAR PDE SYSTEM UNDER LINEAR STATE FEEDBACK CONTROL

- **Original PDE:**

$$
\begin{aligned}
\frac{\partial \bar{x}}{\partial t} &= \frac{\partial^2 \bar{x}}{\partial z^2} + (\beta_T e^{-\gamma}\gamma - \beta_U)\bar{x} + \beta_U b(z)u(t) \\
\bar{x}(0,t) &= 0, \quad \bar{x}(\pi,t) = 0
\end{aligned}
$$

- **Target PDE:**

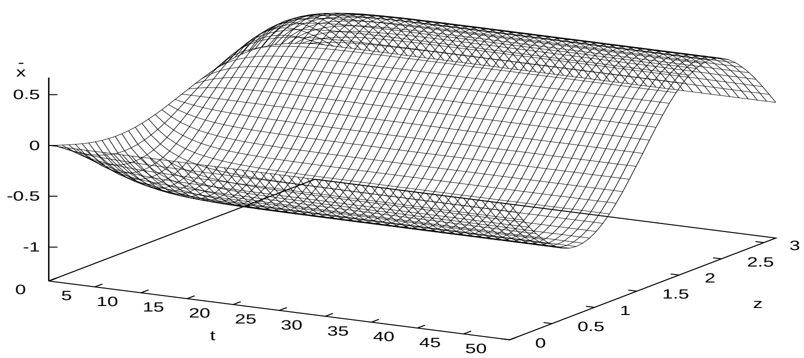
$$
\begin{aligned}
\frac{\partial \bar{x}}{\partial t} &= b\frac{\partial^2 \bar{x}}{\partial z^2} - 4sin(2z) \\
\bar{x}(0,t) &= 0, \quad \bar{x}(\pi,t) = 0
\end{aligned}
$$

- Computation of the sufficient number of actuators and the corresponding state feedback law to make the solution of original system "almost identical" to the solution of the target PDE.

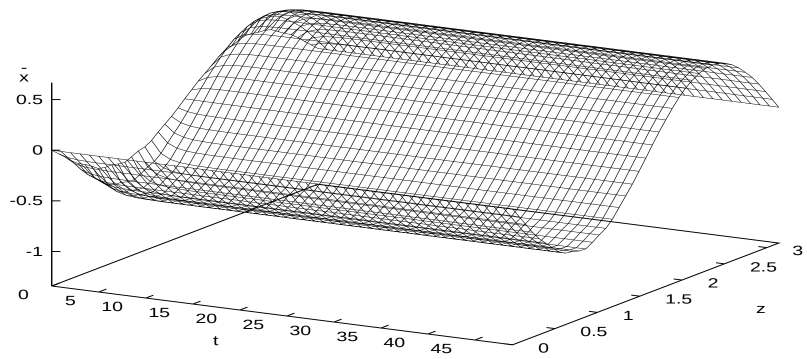
CLOSED-LOOP SIMULATION: LINEAR PDE SYSTEM UNDER LINEAR STATE FEEDBACK CONTROL

- Large number of control actuators is needed.

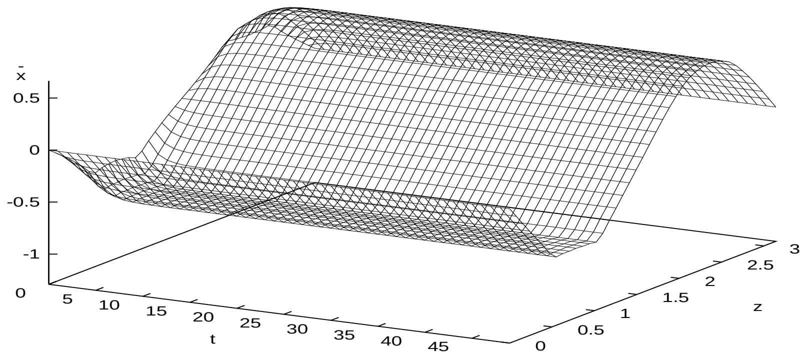
Profile of the target PDE.



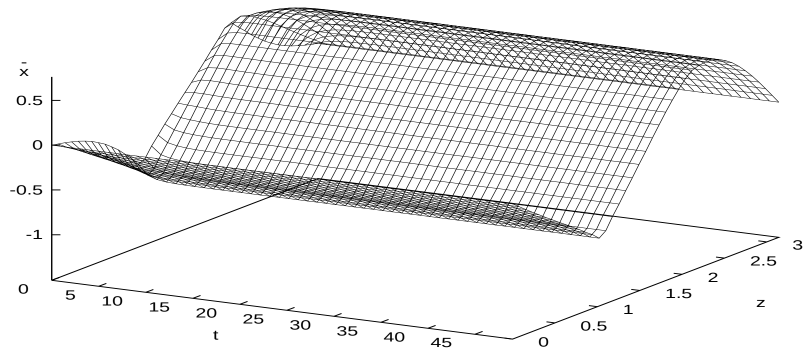
Closed-loop profile (10 actuators).



Closed-loop profile (5 actuators).



Closed-loop profile (2 actuators).



NONLINEAR PDE SYSTEM UNDER LINEAR STATE FEEDBACK CONTROL

- **Original PDE:**

$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial^2 \bar{x}}{\partial z^2} + \beta_T e^{-\frac{\gamma}{1+\bar{x}}} + \beta_U(b(z)u(t) - \bar{x}) - \beta_T e^{-\gamma}$$

- **Target PDE:**

$$\frac{\partial \bar{x}}{\partial t} = b\frac{\partial^2 \bar{x}}{\partial z^2} - 4sin(2z)$$

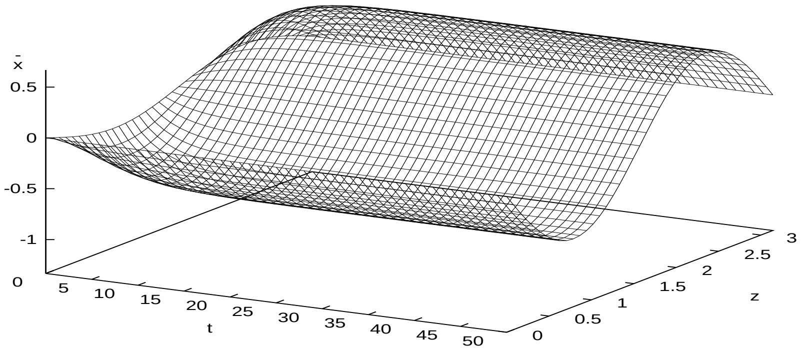
- Is linear feedback control adequate to enforce target behavior with a large number of control actuators?

- Linearize the nonlinear state feedback laws around the steady-state, $\bar{x}(z,t) = 0$, to obtain linear state feedback laws.

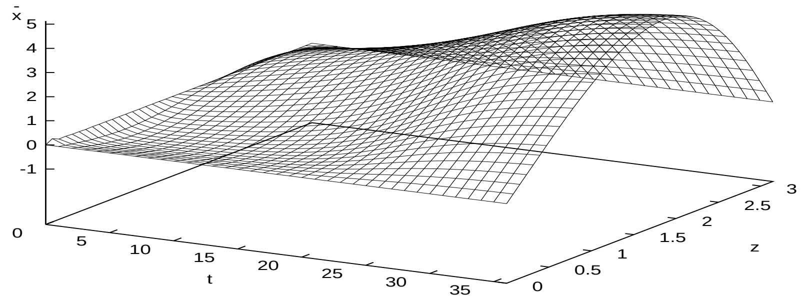
CLOSED-LOOP SIMULATION: NONLINEAR PDE SYSTEM UNDER LINEAR STATE FEEDBACK CONTROL

- Failure to enforce the target behavior using linear feedback control with a large number of actuators:

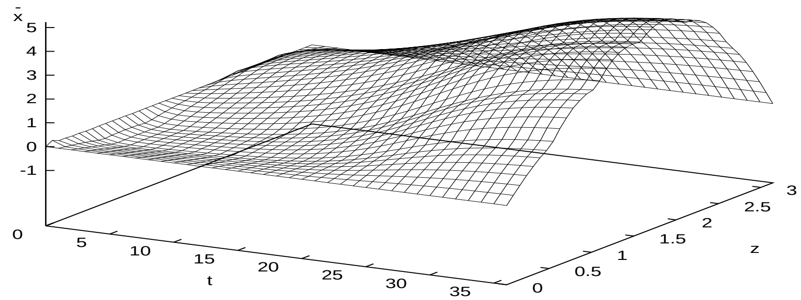
Profile of the target PDE.



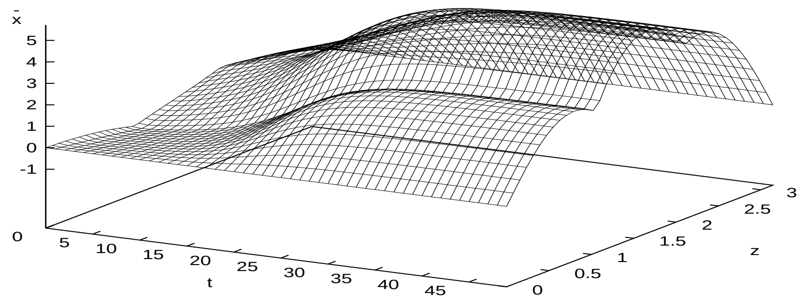
Closed-loop profile (20 actuators).



Closed-loop profile (5 actuators).



Closed-loop profile (2 actuators).



NONLINEAR PDE SYSTEM UNDER NONLINEAR STATE FEEDBACK CONTROL

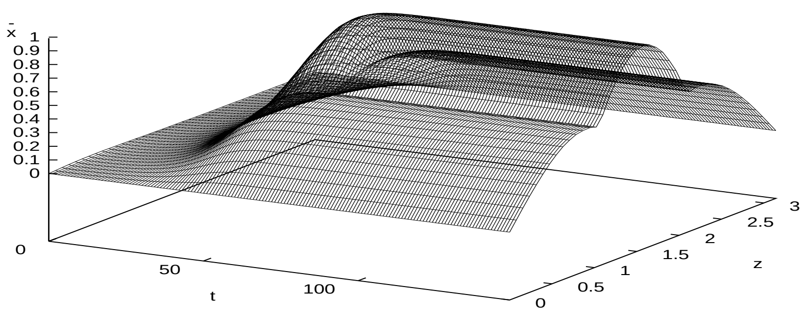
- **Original PDE:**

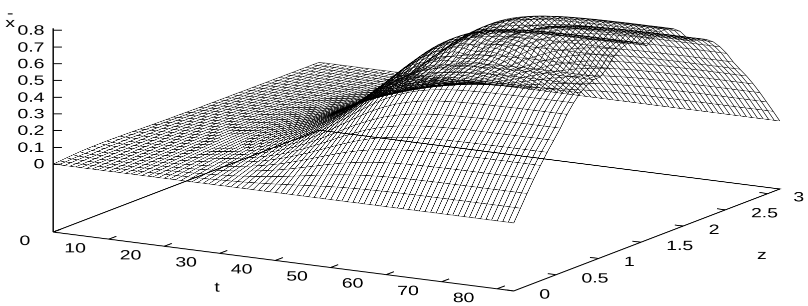
$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial^2 \bar{x}}{\partial z^2} + \beta_T e^{-\frac{\gamma}{1+\bar{x}}} + \beta_U (b(z)u(t) - \bar{x}) - \beta_T e^{-\gamma}$$

- **Target PDE:**

$$\frac{\partial \bar{x}}{\partial t} = b\frac{\partial^2 \bar{x}}{\partial z^2} + \alpha \bar{x} - \beta \bar{x}^3$$

- The use of 5 actuators and nonlinear state feedback controller enforces the desired closed-loop behavior: Closed-loop profiles using 2 actuators (left plot) and 5 actuators (right plot).





NONLINEAR TIME-VARYING TARGET PDE

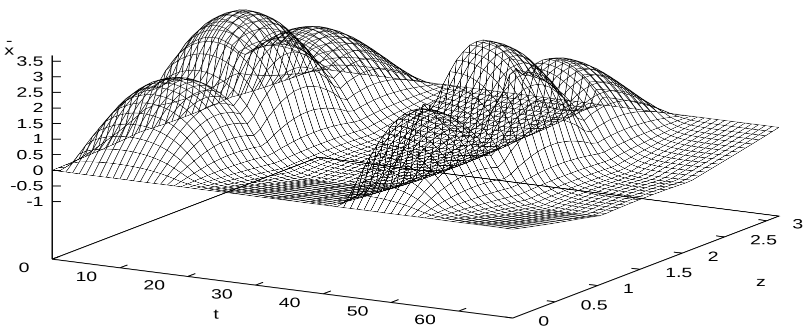
- **Original PDE:**

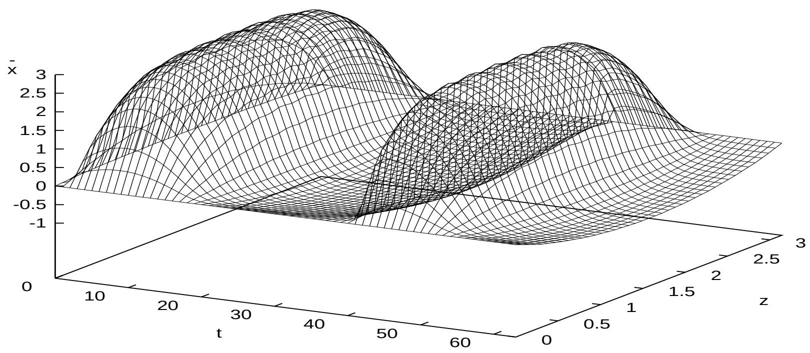
$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial^2 \bar{x}}{\partial z^2} + \beta_T e^{-\dfrac{\gamma}{1+\bar{x}}} + \beta_U (b(z)u(t) - \bar{x}) - \beta_T e^{-\gamma}$$

- **Target PDE:**

$$\frac{\partial \bar{x}}{\partial t} = b\frac{\partial^2 \bar{x}}{\partial z^2} + \alpha \bar{x} - \beta \bar{x}^3 + 2.5 sin(0.5t)\beta_T e^{-\frac{\gamma}{1+\bar{x}}}$$

- The use of 10 actuators and nonlinear state feedback controller enforces the desired closed-loop behavior: Closed-loop profiles using 2 actuators (left plot) and 10 actuators (right plot).





CLOSED-LOOP SIMULATION: NONLINEAR PDE SYSTEM UNDER NONLINEAR OUTPUT FEEDBACK CONTROL

- **Original PDE:**

$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial^2 \bar{x}}{\partial z^2} + \beta_T e^{-\frac{\gamma}{1+\bar{x}}} + \beta_U(b(z)u(t) - \bar{x}) - \beta_T e^{-\gamma}$$

- **Target PDE:**

$$\frac{\partial \bar{x}}{\partial t} = b\frac{\partial^2 \bar{x}}{\partial z^2} - 4sin(2z)$$

- State measurements are not available.
- Number of measurement sensors is equal to the number of control actuators.
- Estimation of the states of the finite-dimensional system from the measurements.

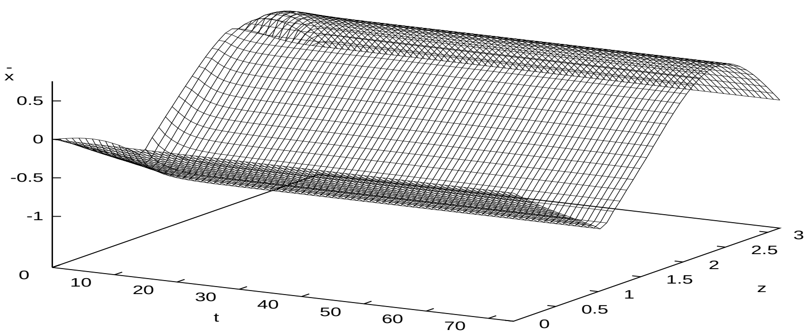
NONLINEAR PDE SYSTEM UNDER NONLINEAR OUTPUT FEEDBACK CONTROL

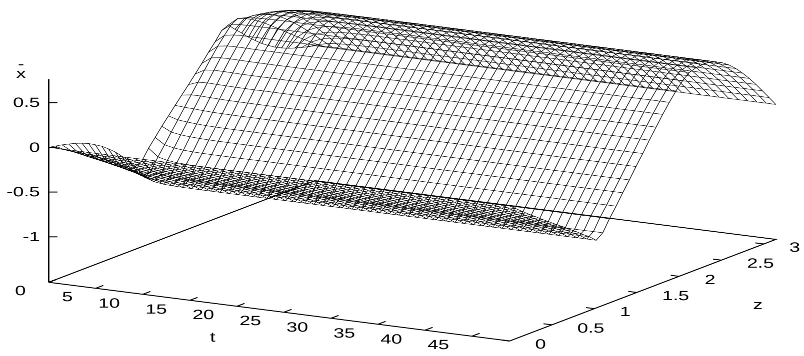
- Comparison of closed-loop profiles under state and output feedback control.

  ◇ Number of control actuators and measurement sensors: 2.

  Left: Closed-loop profile under output feedback control.

  Right: Closed-loop profile under state feedback control.

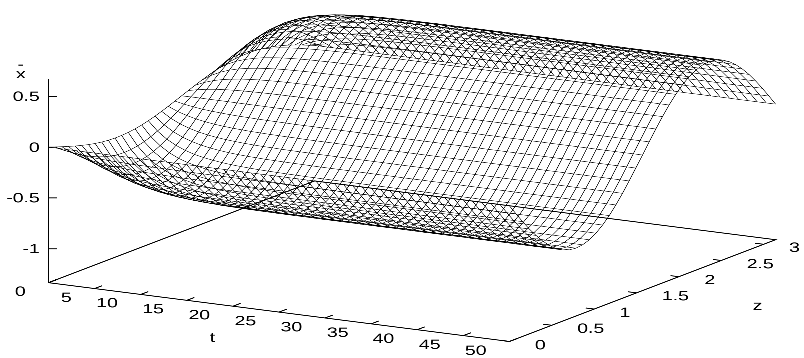


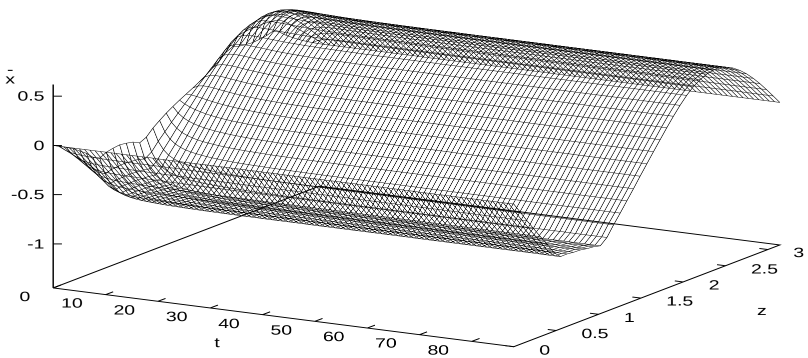


- Discrepancy due to estimation error.

CLOSED-LOOP SIMULATION: NONLINEAR PDE SYSTEM UNDER NONLINEAR OUTPUT FEEDBACK CONTROL

- Comparison of closed-loop profile under nonlinear output feedback control with 5 control actuators and measurement sensors (right plot) to the target profile (left plot).





- Five actuators/sensors suffice to achieve the desired control objective.

CLOSED-LOOP SIMULATION: $2D$ NONLINEAR PDE SYSTEM UNDER NONLINEAR STATE FEEDBACK CONTROL

- $2D$ **Nonlinear PDE system:**

$$\frac{\partial \bar{x}}{\partial t} = \frac{\partial^2 \bar{x}}{\partial z^2} + \frac{\partial^2 \bar{x}}{\partial y^2} + \hat{f}(\bar{x}, t), \quad \bar{x}(z, y, t) = 0 \ \forall \ (z, y) \in \Gamma$$

- **Original PDE:**

$$\hat{f}(\bar{x}, t) = 0$$

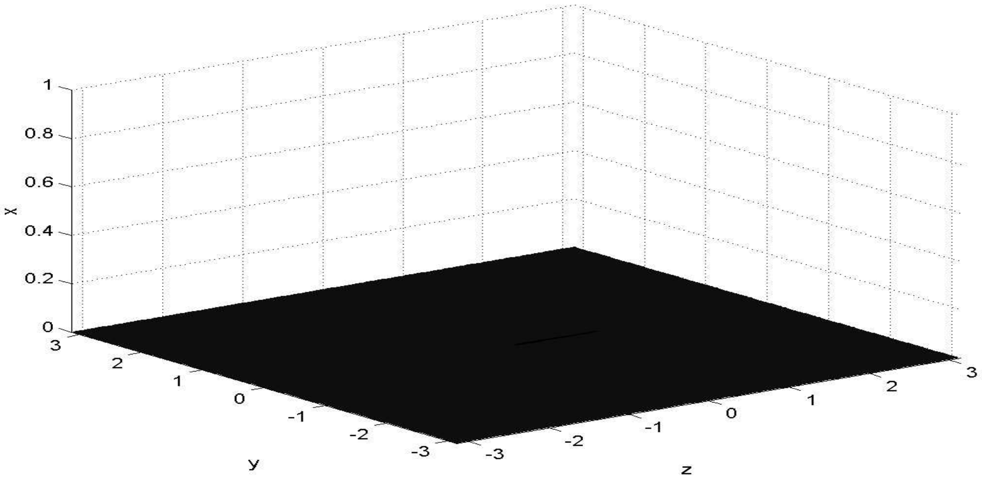
- **Target PDE:**

$$\hat{f}(\bar{x}, t) = \alpha \bar{x} - \beta \bar{x}^3$$

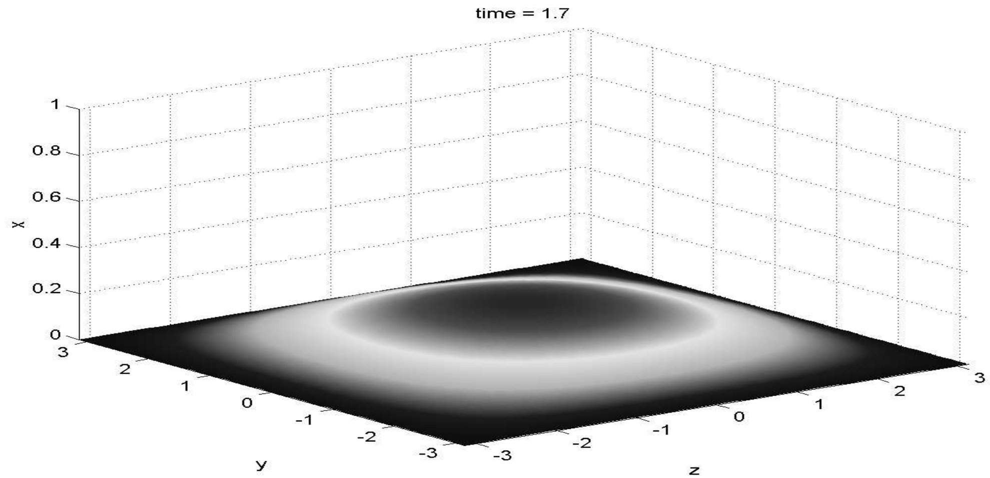
- State measurements are available.
- Number of control actuators 100.

CLOSED-LOOP SIMULATION: $2D$ NONLINEAR PDE SYSTEM UNDER NONLINEAR STATE FEEDBACK CONTROL

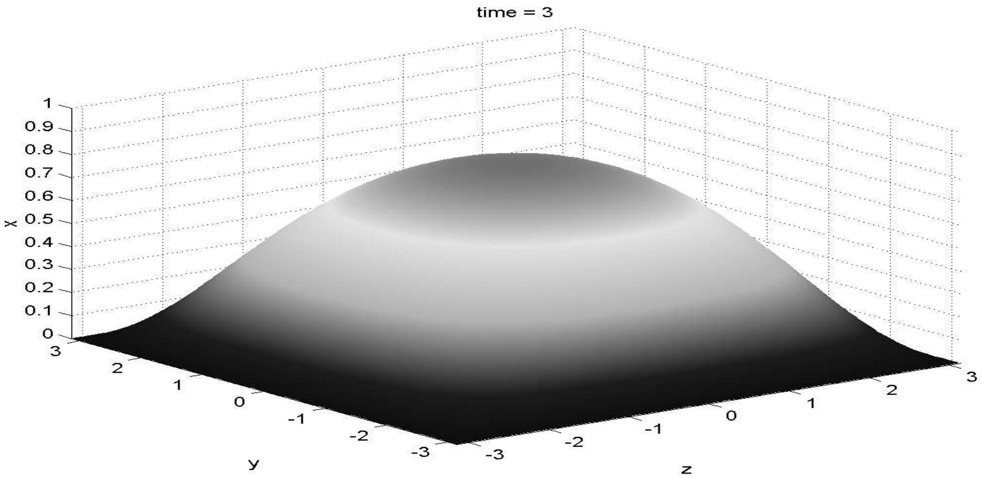
Profile of the original PDE.



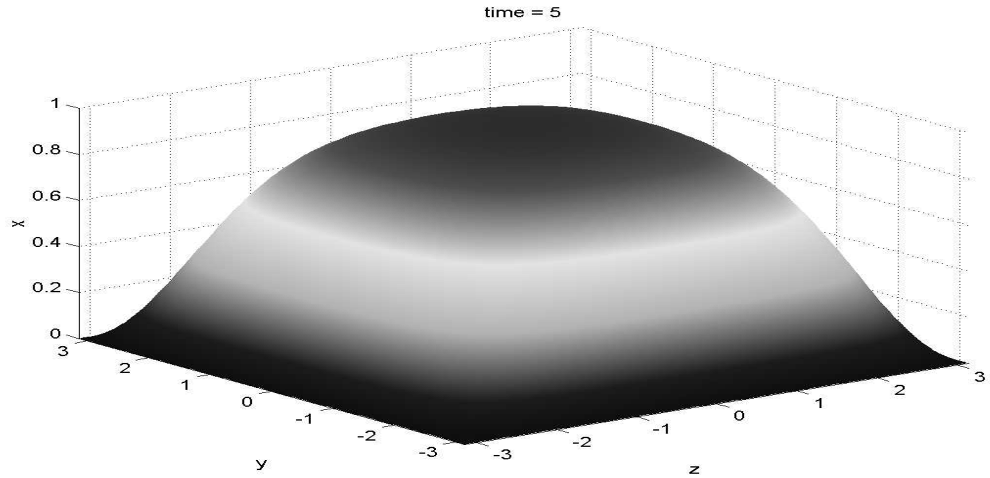
Closed-loop profile (time= 1.7)



Closed-loop profile (time= 3)



Closed-loop profile (time= 5)



CONCLUSIONS

- Distributed nonlinear control of diffusion-reaction processes described by quasi-linear parabolic PDEs.
  - ◇ Control problem formulation:
    - ▷ Large numbers of actuators/sensors are available.
    - ▷ Desired behavior described by a "target parabolic PDE".
  - ◇ Methodology for controller design:
    - ▷ Order reduction using Galerkin's method.
    - ▷ Nonlinear state/output feedback controller design using geometric control methods.
  - ◇ Application to diffusion-reaction processes.

ACKNOWLEDGMENT

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