COORDINATING SUPERVISORY CONTROL AND FEEDBACK IN HYBRID PROCESS SYSTEMS

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INTRODUCTION

- **Hybrid nature of process systems**
  - ◇ Interaction of continuous and discrete components
    - ▷ Continuous behavior:
      - ⋆ Mass, energy, momentum conservation
    - ▷ Discrete behavior:
      - ⋆ Physico-chemical (autonomous) discontinuities
        (e.g., phase changes, flow reversals, shocks, transitions)
      - ⋆ Discrete controls and instrumentation
        (e.g., on/off valves, binary sensors, constant-speed motors)
      - ⋆ Changes in process operation modes
      - ⋆ Faults in control system
- **Nonlinear behavior**
  - ◇ Complex reaction mechanisms
  - ◇ Arrhenius reaction rates
- **Model uncertainty**
  - ◇ Unknown process parameters
  - ◇ Time-varying disturbances
- **Input constraints**

BACKGROUND ON HYBRID SYSTEMS

- **Combined discrete-continuous systems:**
  - ◇ Modeling (e.g., Yamalidou et al, C&CE, 1990)
  - ◇ Simulation (e.g., Barton and Pantelides, AIChE J., 1994)
  - ◇ MINLP - Optimization (e.g., Grossman et al., CPC, 2001)
- **Stability of switched and hybrid systems:**
  - ◇ Multiple Lyapunov functions (e.g., Branicky, IEEE TAC,1998)
  - ◇ Dwell-time approach (e.g., Hespanha and Morse, CDC,1999)
- **Control of switched and hybrid systems:**
  - ◇ Mixed Logical Dynamical systems (Morari and co-workers)
  - ◇ Optimal control of switched linear systems (e.g., Xu and Antsaklis, CDC, 2001)
  - ◇ Control of constrained switched nonlinear systems (El-Farra and Christofides, HSCC, 2002)

PRESENT WORK

(El-Farra & Christofides, AIChE J., *submitted*, 2002)

- **Scope:**
  - ◇ Hybrid nonlinear processes with
    - ⋆ Model uncertainty
    - ⋆ Input constraints
- **Objectives:**
  - ◇ Integrated approach for supervisory and feedback control
    - ▷ Design of nonlinear feedback controllers
      - ⋆ Nonlinear behavior
      - ⋆ Input constraints
      - ⋆ Plant-model mismatch
    - ▷ Design of stabilizing switching laws
      - ⋆ Discrete-continuous interactions (changing dynamics)
  - ◇ Application to a switched chemical reactor

HYBRID NONLINEAR PROCESSES WITH UNCERTAINTY & CONSTRAINTS

- **State space description:**

$$\dot{x}(t) = f_i(x(t)) + \sum_{l=1}^{m} g_i^l(x(t))u_i^l(t) + \sum_{k=1}^{q} w_i^k(x(t))\theta_i^k(t)$$

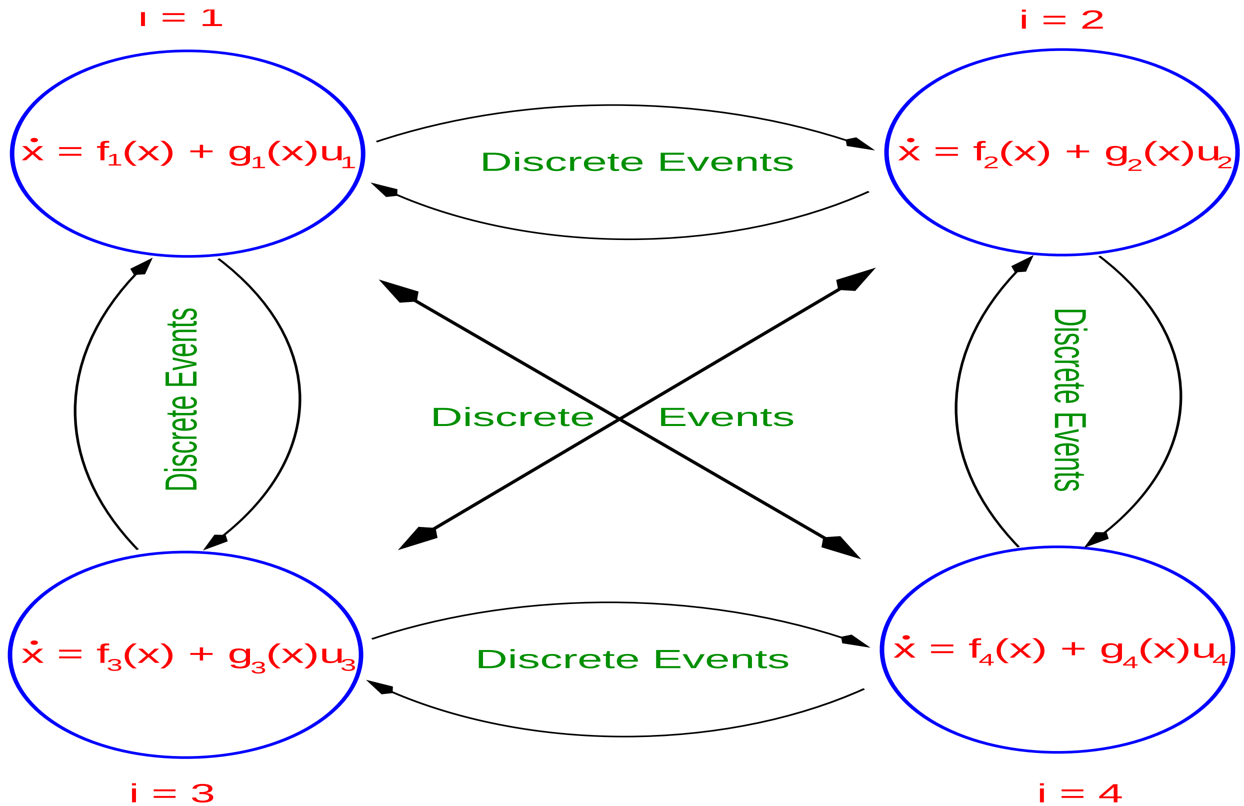
$$i(t) \in \mathcal{I} = \{1, 2, \cdots, N < \infty\}$$

$$u_{i,min}^l \leq u_i^l(t) \leq u_{i,max}^l$$

  - ◇ $x(t) \in \mathbb{R}^n$: vector of continuous state variables
  - ◇ $u_i(t) \in \mathcal{U}_i \subset \mathbb{R}^m$: vector of continuous control inputs for i-th mode
  - ◇ $\theta_i(t) \in \mathcal{W}_i \subset \mathbb{R}^q$: vector of uncertain variables in i-th mode
  - ◇ $i(t) \in \mathcal{I}$: discrete variable controlled by supervisor

- **Combine finite and continuous dynamics:**
  - ◇ Each mode governed by continuous, uncertain dynamics
  - ◇ Transitions between modes governed by discrete events

MULTI-MODAL REPRESENTATION OF HYBRID PROCESSES



- **Autonomous switching:**
  - ◇ i depends **only** on inherent process characteristics
- **Controlled switching:**
  - ◇ i is chosen by a controller / human operator

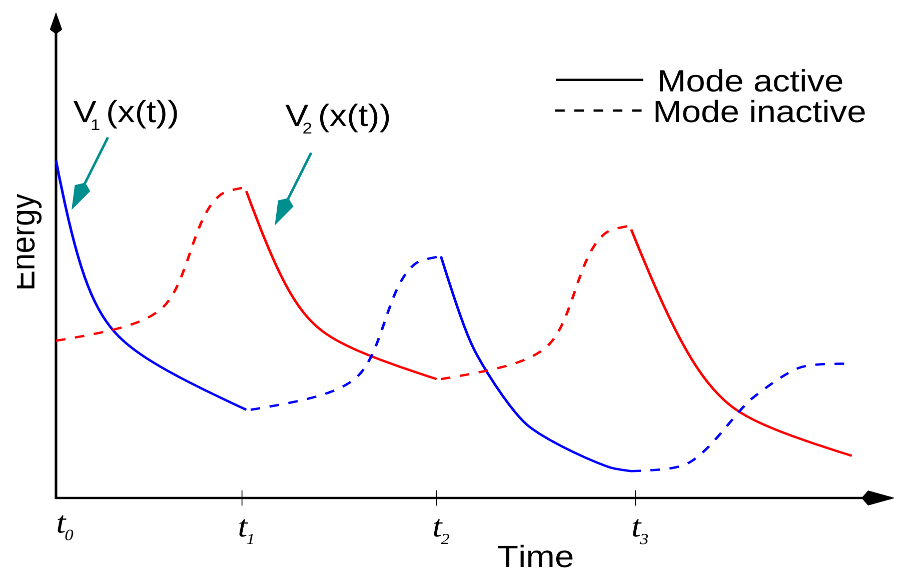
MULTIPLE LYAPUNOV FUNCTIONS

- **A natural and intuitive tool for stability analysis:**
  - ◇ Extends classical Lyapunov stability to switched systems
  - ◇ Sufficient conditions for asymptotic stability:
    - ▷ Stability of constituent subsystems:

$$\frac{dV_i(t)}{dt} < 0,\ t \in [t_{i_k}, t_{i_k+1})$$

    - ▷ Stability of the transitions between the modes:

$$V_i\left(t_{i_k}\right) < V_i\left(t_{i_{k-1}}\right)$$



⋆ **Tool for integrating robust nonlinear feedback control & switching?**

CONTROL PROBLEM FORMULATION

- **Coordinating feedback & switching using MLFs**
  - ◇ Synthesis of family of robust nonlinear controllers
    - ⋆ Model for each mode of the hybrid plant

$$\dot{x} = f_i(x) + G_i(x)u_i + W_i(x)\theta_i$$

    - ⋆ Magnitude of input constraints & size of uncertainty

$$|u_i| \leq u_{i,max}, \quad |\theta_i| \leq \theta_{i,b}$$

    - ⋆ Multiple robust control Lyapunov functions

$$V_i, \quad i = 1, \cdots, N$$

  - ◇ Design of laws that orchestrate mode switching

$$i(t) = \phi(x(t),\ i(t^-),\ t)$$

- **Desired closed-loop properties:**
  - ◇ Asymptotic stability of switched system
  - ◇ Robustness against arbitrarily large uncertainty

BOUNDED ROBUST CONTROLLER DESIGN

(El-Farra & Christofides, Chem. Eng. Sci., 2001)

- **Family of bounded robust feedback laws:**

$$u_i \;=\; -k_i(x, u_{i,max}, \theta_{i,b})(L_{G_i}V_i)^T$$

$$k_i(x,u_{i,max}) = \left(\frac{L_{f_i}^* V_i + \sqrt{(L_{f_i}^* V_i)^2 + (u_{i,max}|(L_{G_i}V_i)^T|)^4}}{|(L_{G_i}V_i)^T|^2 \left[1 + \sqrt{1 + (u_{i,max}|(L_{G_i}V_i)^T|)^2}\right]} \right)$$

$$L_{f_i}^* V_i = L_{f_i}V_i + \chi \sum_{k=1}^{q} |L_{w_i^k} V_i| \theta_{i,b} \left(\frac{|\nabla_x V_i|}{|\nabla_x V_i| + \phi} \right)$$

  - ◇ Nonlinear "gain" reshaping of Sontag's formula (SCL, 1991)
    - ⋆ Arbitrary degree of uncertainty attenuation by tuning χ, ϕ

- **Closed-loop properties for active mode:**
  - ◇ Asymptotic stability
  - ◇ Inverse optimality: $$J_i = \int_0^{\infty} (l_i(x) + u_i^T R_i(x) u_i)dt$$

$$l_i(x) > 0, \qquad R_i(x) > 0, \qquad J_{min} = V_i(x(0))$$

CHARACTERIZATION OF CLOSED-LOOP STABILITY PROPERTIES

$$D_i(u_{i,max}, \theta_{i,b}) \;=\; \{x \in \mathbb{R}^n : L_{f_i}V_i + \chi \sum_{k=1}^{q} |L_{w_i^k}V_i|\theta_{i,b} < u_{i,max}|(L_{G_i}V_i)^T|\}$$

- **Properties of inequality:**
  - ◇ Describes open unbounded region where:
    - ⋆ $|u_i(x)| \leq u_{i,max} \;\; \forall \, x \in D_i$
    - ⋆ $\dot{V}_i(x) < 0 \;\; \forall \;\; 0 \neq x \in D_i$
  - ◇ Captures constraint & uncertainty-dependence of stability region
  - ◇ Explicit guidelines for mode-switching (regions of invariance)
- **Some design implications:**
  - ◇ Given the desired stability region, determine u_{max}
  - ◇ u_{max} determines capacity & size of control actuators
    - ⋆ Valves, pumps, heaters, etc.

STABILIZING SWITCHING LAWS

- **Conditions for asymptotic stability:**

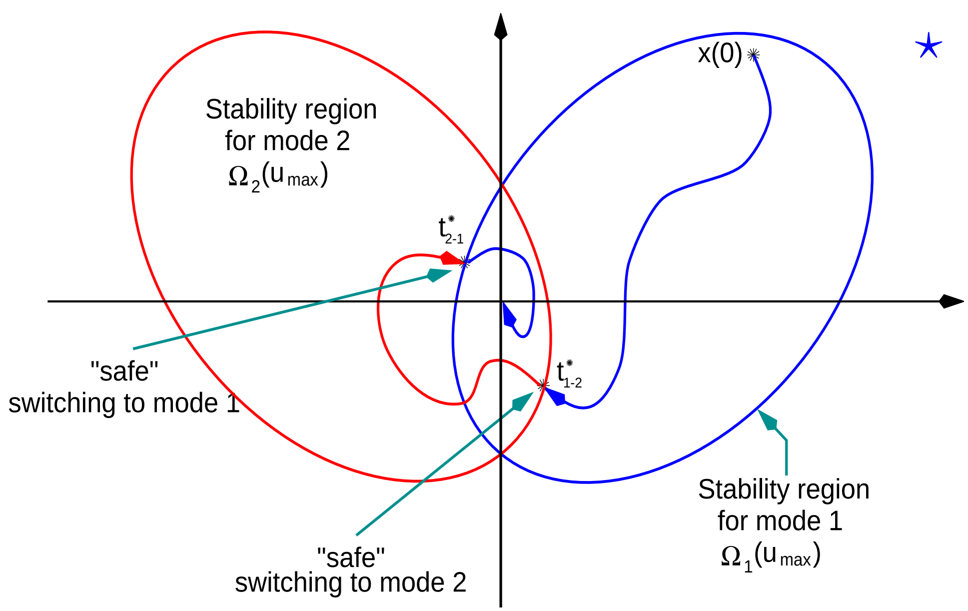
  Switching to mode j at $t = t^*_{j_k}$ is "safe" provided that:

  ◇ State within the stability region of mode j at $t = t^*_{j_k}$

$$x(t^*_{j_k}) \quad \in \quad \Omega_j(u_{j,max}, \theta_{j,b})$$

  ◇ Energy of mode j is less than when it was last activated

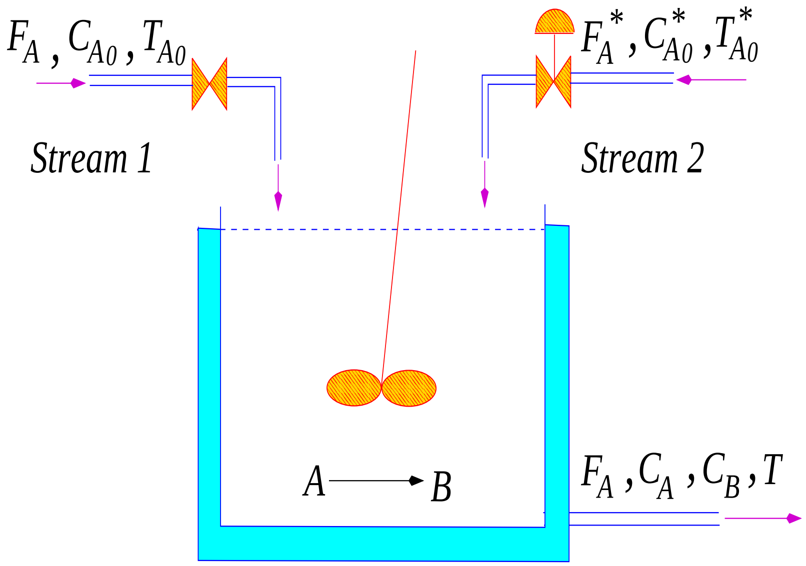
$$V_j(t^*_{j_k}) \quad < \quad V_j(t^*_{j_{k-1}})$$



⋆ Inverse optimality for the switched system:

$$J = \sum_{i=1}^{N} \int_{t_0^i}^{t_f^i} (l_i(x) + u_i^T R_i(x) u_i) dt + \sum_{i=1}^{N} V_i(t_f^i)$$

APPLICATION TO A SWITCHED EXOTHERMIC CSTR



- Process dynamic model:

$$\frac{dC_A}{dt} = \frac{F_A}{V}(C_{A0} - C_A) - k_0 e^{\frac{-E}{RT}} C_A + i(t)\left[\frac{F_A^*}{V}(C_{A0}^* - C_A)\right]$$

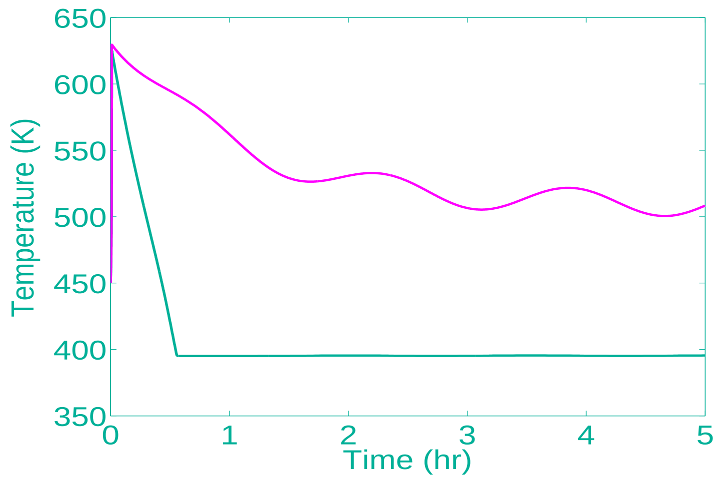
$$\frac{dT}{dt} = \frac{F_A}{V}(T_{A0} - T) + \frac{(-\Delta H_r)}{\rho_m c_{pm}} k_0 e^{\frac{-E}{RT}} C_A + i(t)\left[\frac{F_A^*}{V}(T_{A0}^* - T)\right] + \frac{Q_i(t)}{\rho c_p V}$$

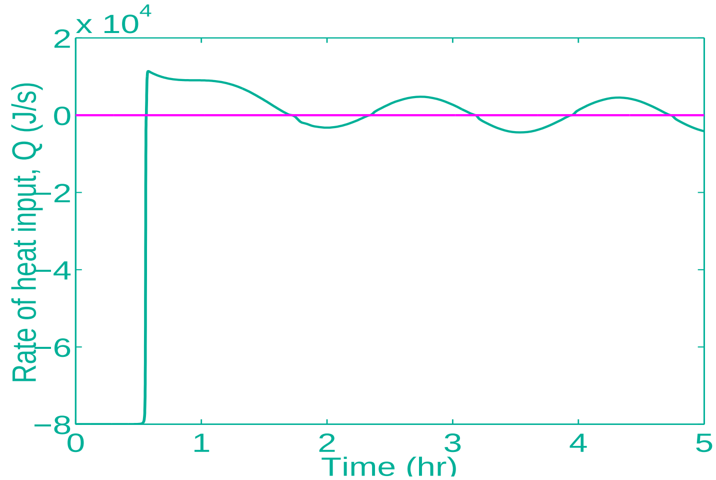
- **Control Problem:**
  - ◇ Controlled Output: $y = T - T_s$
  - ◇ Manipulated Input: $u = Q,\ |u| \leq u_{max}$
  - ◇ Uncertain variables: $\theta_1(t) = T_{A0} - T_{A0s},\ \theta_2(t) = \Delta H_r - \Delta H_{rnom}$
- **Switching problem:**
  - ◇ Discrete control variable: $i(t) \in \{0, 1\}$
  - ◇ Determine the earliest safe switching time between
    - ⋆ mode $i = 0$ (only stream 1 active)
    - ⋆ mode $i = 1$ (both streams 1 & 2 active)

CLOSED-LOOP SIMULATION RESULTS

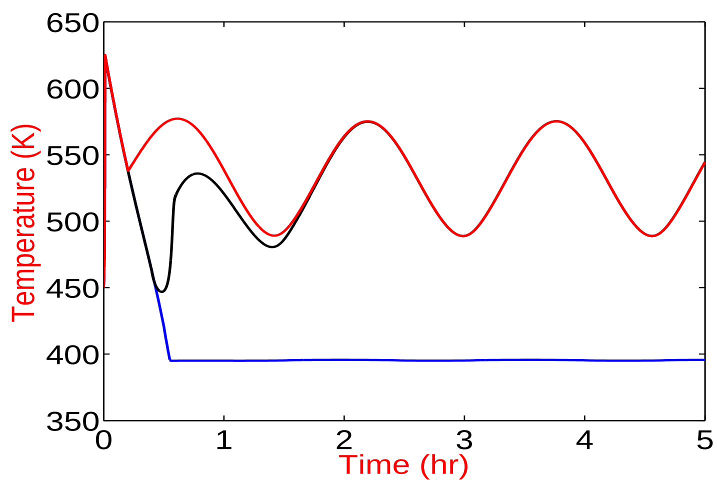
$(|Q| \leq 80 \text{ KJ/min}, \theta_{b1} = 0.1T_{A0}, \theta_{b2} = 0.5\Delta H_{nom})$

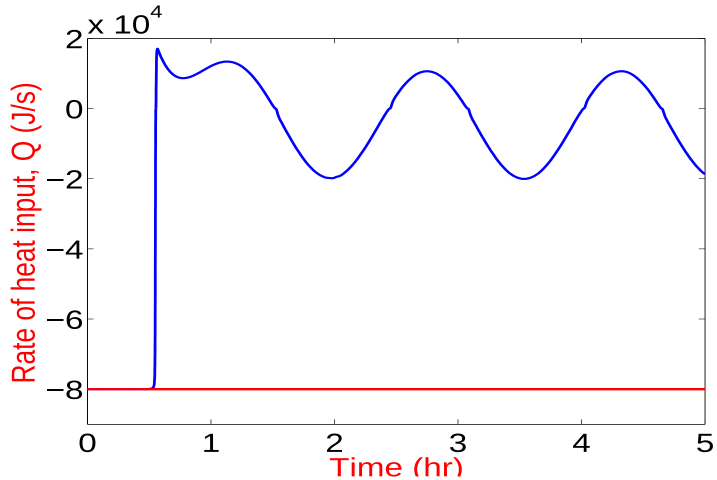
- Valve on stream 2 turned off: open-loop vs. closed-loop response





- Valve on stream 2 turned on at $t = 12$ min (uncertainty unaccounted for), $t = 24$ min (arbitrary), and $t = 30$ min (using robust switching laws)





COORDINATING FEEDBACK & SWITCHING FOR FAULT-TOLERANT PROCESS CONTROL

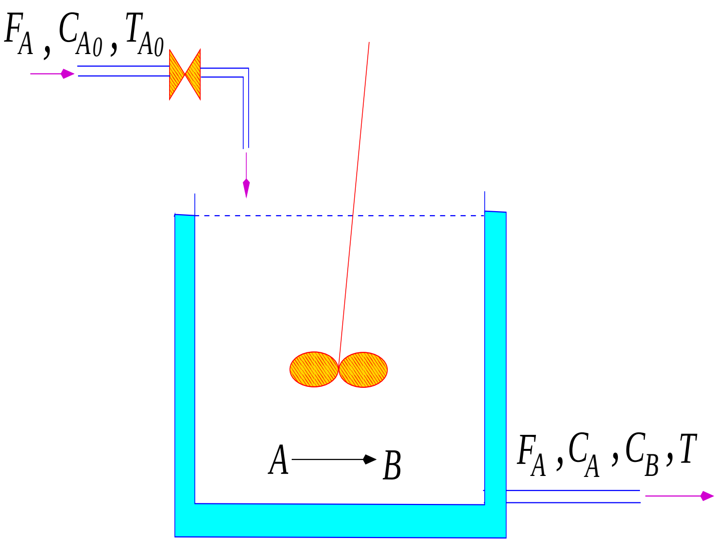
- **Process control system failure:**
  - ◇ Typical sources:
    - ⋆ Failure in control algorithm
    - ⋆ Faults in control actuators and/or measurement sensors
  - ◇ Induce discrete transitions in continuous dynamics
- **Motivation for fault-tolerant control:**
  - ◇ Preserve process integrity & dependability
  - ◇ Minimize negative economic & environmental impact:
    - ⋆ Raw materials waste, production losses, personnel safety, $\cdots$, etc.
- **Approaches for fault-tolerant control:**

Switching between multiple control configurations

◇ Multiple spatially-distributed actuators/sensors
(El-Farra & Christofides, C&CE, *submitted*, 2002)

◇ **Different manipulated inputs**

FAULT-TOLERANT CONTROL OF AN EXOTHERMIC CSTR



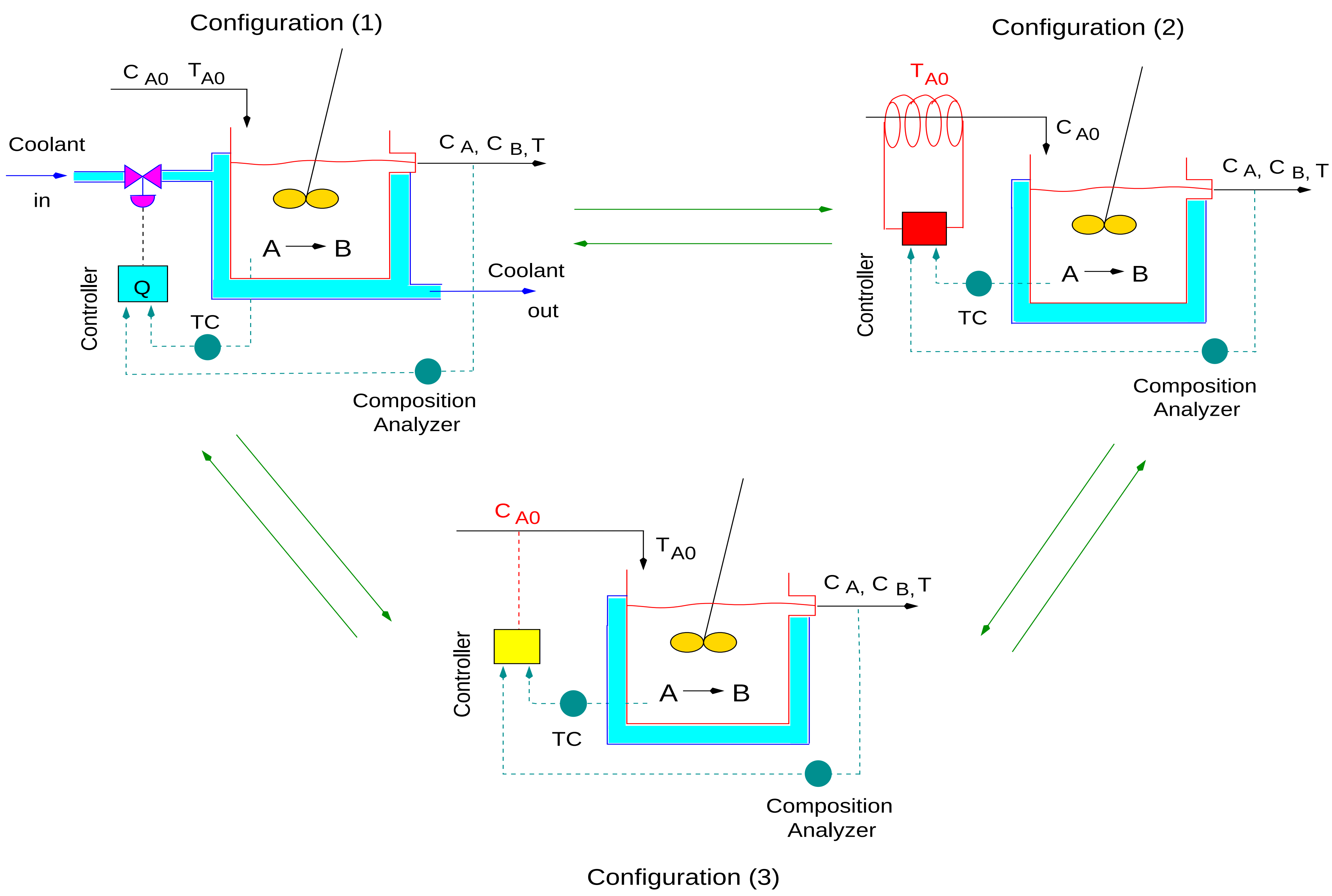
- Process dynamic model:

$$
\begin{aligned}
\frac{dC_B}{dt} &= -\frac{F}{V}C_B + k_{B0}e^{\frac{-E_B}{RT}}C_A \\
\frac{dC_A}{dt} &= \frac{F_A}{V}(C_{A0} - C_A) - \sum_{i=1}^{3} k_{i0}e^{\frac{-E_i}{RT}}C_A \\
\frac{dT}{dt} &= \frac{F_A}{V}(T_{A0} - T) + \sum_{i=1}^{3} \frac{(-\Delta H_{ir})}{\rho c_p} k_{i0}e^{\frac{-E_i}{RT}}C_A \\
&+ \frac{Q(t)}{\rho c_p V}
\end{aligned}
$$

- **Control objective:** stabilize reactor at unstable steady state in the presence of control system failures

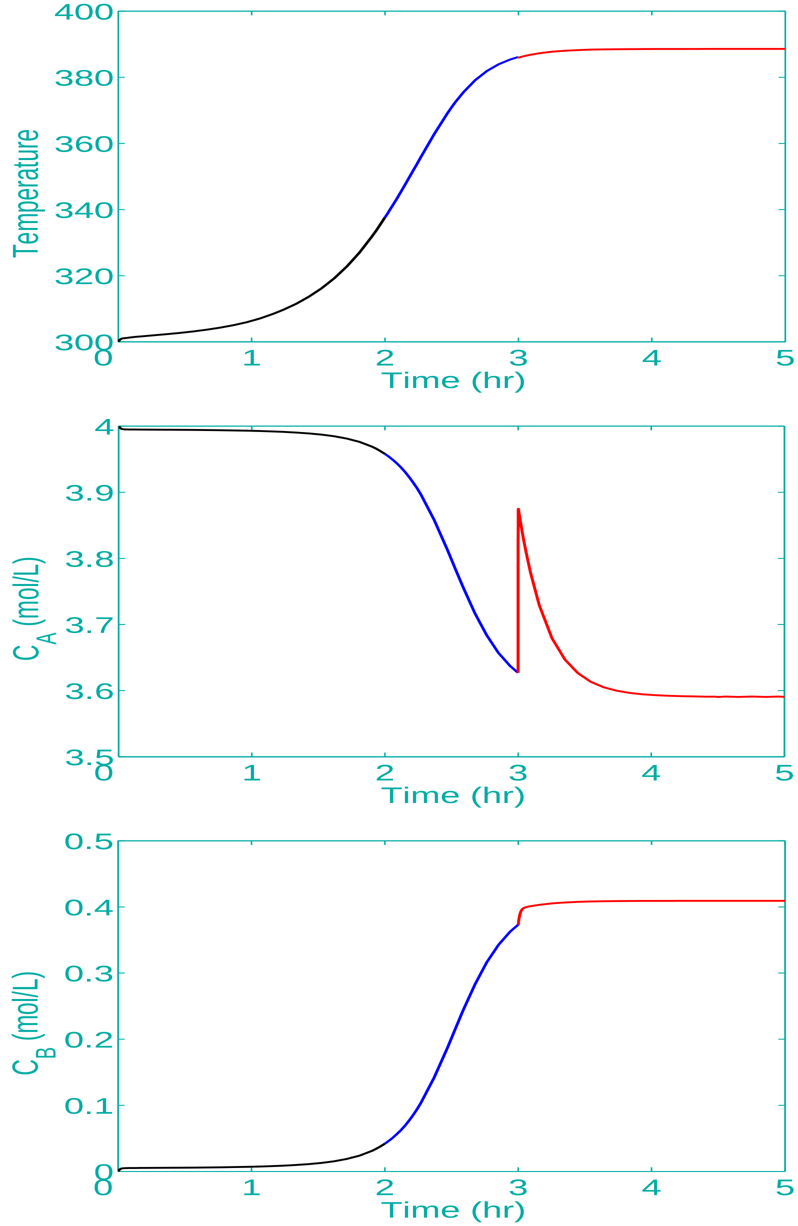
- **Candidate manipulated inputs:**
  - ◇ Rate of heat input: $u_1 = Q$, $|u_1| \leq u_{max}^{(1)}$
  - ◇ Inlet temperature: $u_2 = T_{A0} - T_{A0s}$, $|u_2| \leq u_{max}^{(2)}$
  - ◇ Inlet concentration: $u_3 = C_{A0} - C_{A0s}$, $|u_3| \leq u_{max}^{(3)}$

FAULT-TOLERANT PROCESS CONTROL

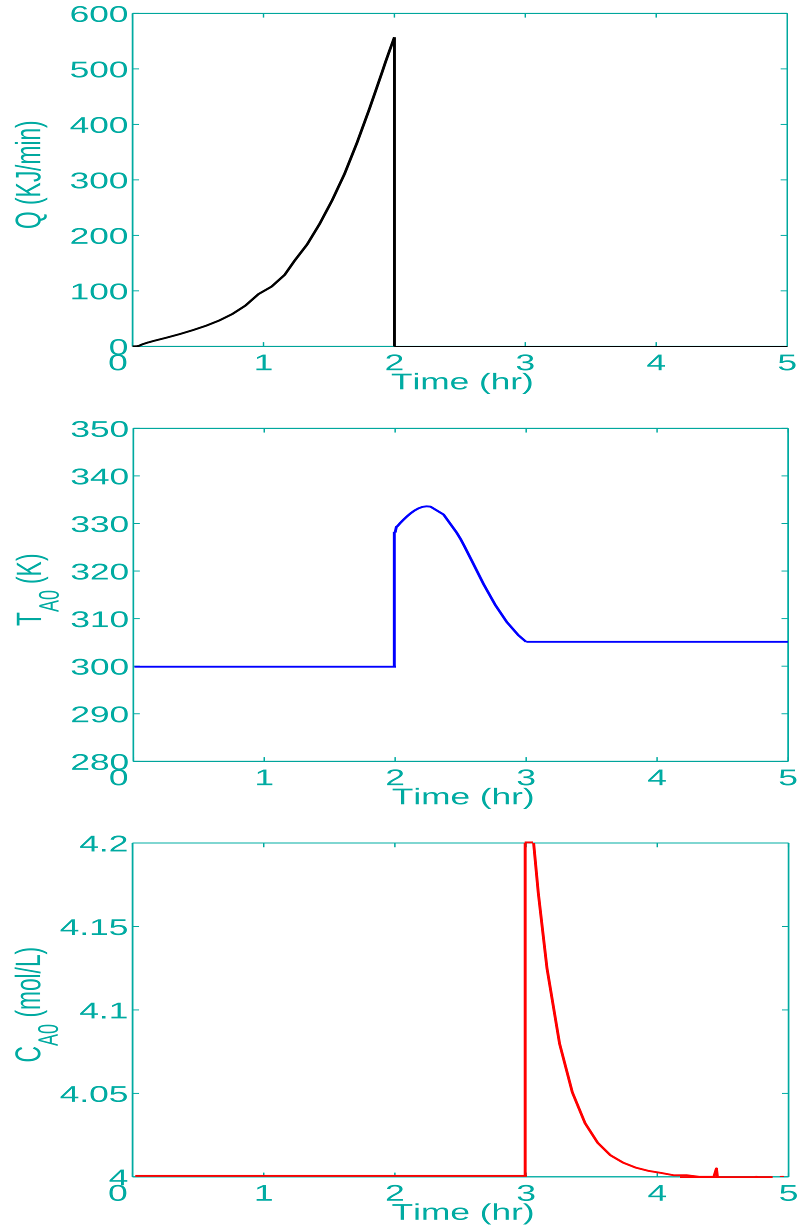


CLOSED-LOOP SIMULATION RESULTS

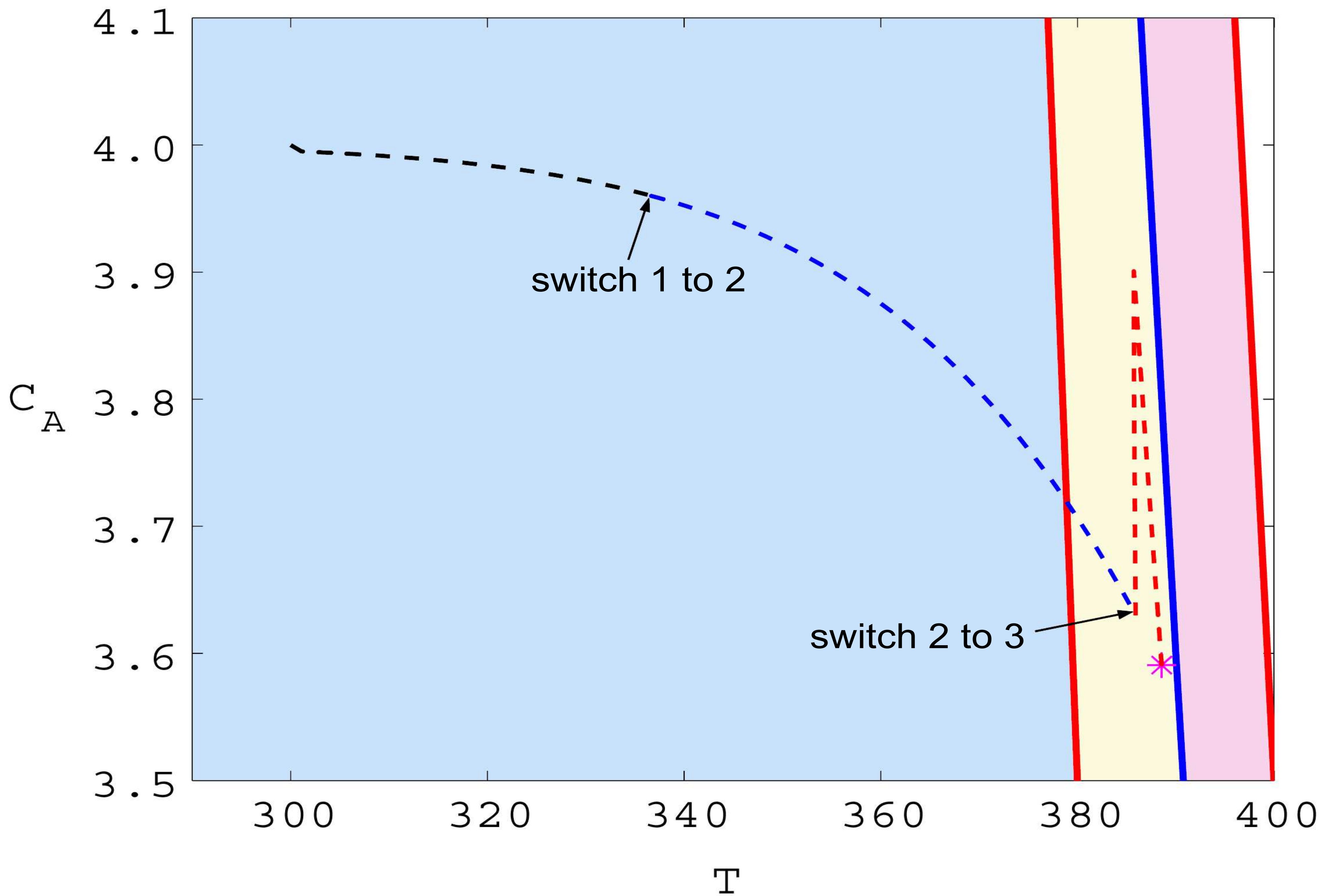
⋆ State profiles



⋆ Input profiles



STABILITY REGION-BASED SWITCHING LOGIC



CONCLUSIONS

- Hybrid nonlinear processes with
  - ⋆ Model uncertainty
  - ⋆ Input constraints
- MLF-based approach for coordinating feedback & supervisory control
  - ◇ Family of bounded robust nonlinear controllers:
    - ⋆ Nonlinear behavior, input constraints & model uncertainty
  - ◇ Design of stabilizing switching laws
    - ⋆ Switching between regions of stability of constituent modes
- Applications to:
  - ◇ Switched chemical reactor with uncertainty & constraints
  - ◇ Fault-tolerant control of an exothermic CSTR

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