HYBRID CONTROL OF SPATIALLY DISTRIBUTED PROCESSES

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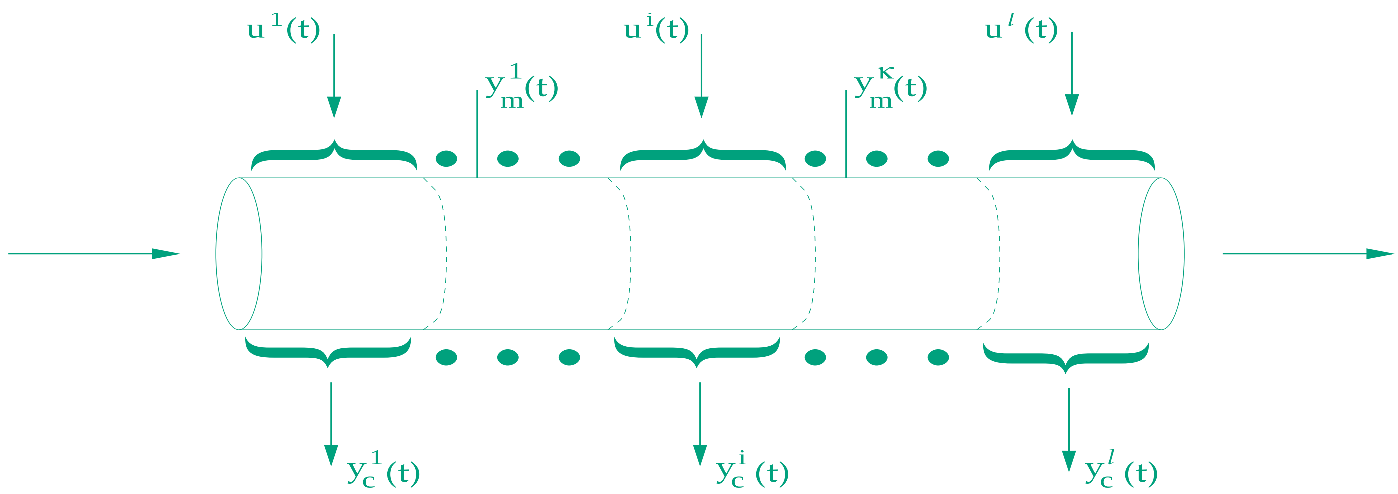
2002 AIChE Annual Meeting
Indianapolis, IN
November 6, 2002

INTRODUCTION

- **Spatially Distributed Processes**
  - ◇ Representative examples:
    - ⋆ Transport-reaction processes
    - ⋆ Fluid flows
  - ◇ Regulation of spatially distributed variables using:
    - ⋆ Spatially distributed control actuators/measurement sensors
  - ⋆ Highly dissipative partial differential equation (PDE) systems:
    - ⋆ Infinite-dimensional systems
    - ⋆ Two time-scale separation of eigenspectrum

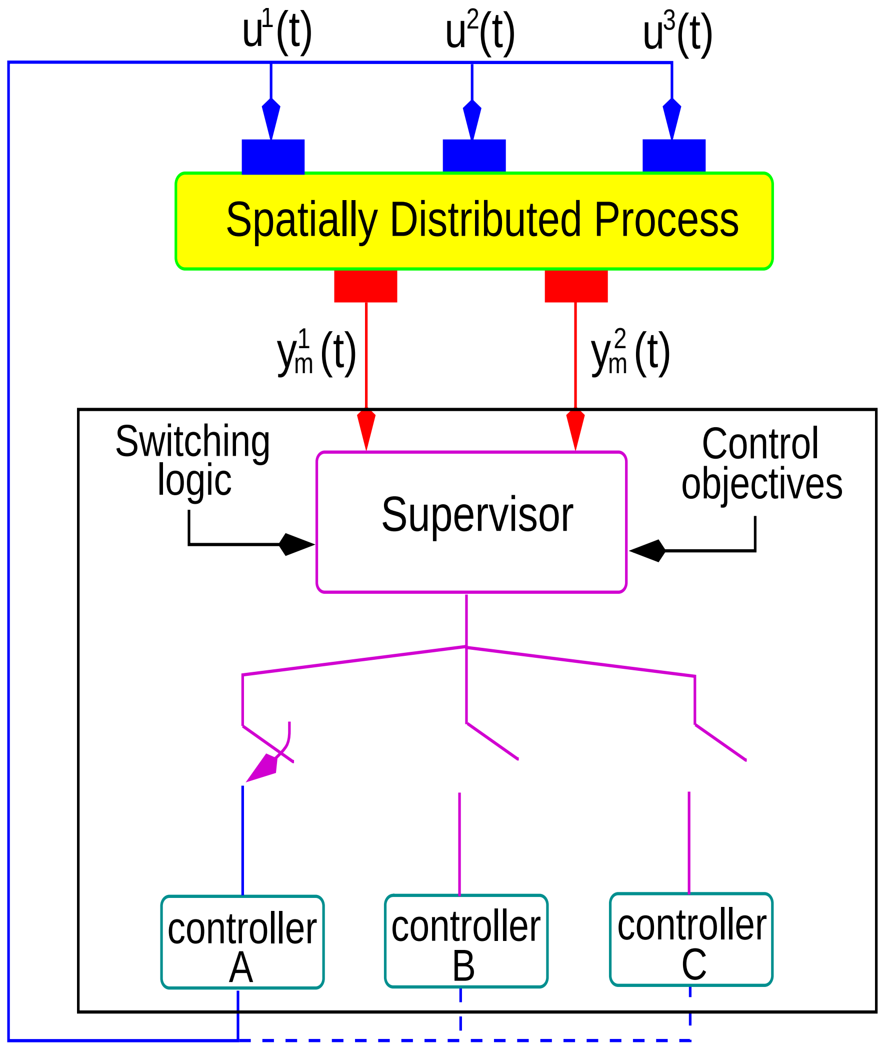


BACKGROUND ON CONTROL OF DISSIPATIVE PDEs

- **Standard approach:**
  (e.g., Balas, IJC, 1979; Ray, 1981, Curtain, 1982).
  - ◇ Derivation of ODE models using eigenfunction expansions
  - ◇ Controller design using methods for ODEs
  - ◇ High-dimensionality of the controller?
- **Synthesis of nonlinear low-order controllers:**
  (Christofides, Birkhäuser, 2001)
  - ◇ Derivation of low-order ODE models using Galerkin's method and approximate inertial manifolds
    - ▷ Nonlinear and robust control
    - ▷ Control of parabolic PDEs with moving domains.
- **Lyapunov-based control:**
  - ◇ Stabilization via boundary feedback (e.g. Liu and Krstic, NA, 2001)

Fixed control actuator/measurement sensor configuration

HYBRID CONTROL STRUCTURE



⋆ Controller combines discrete & continuous elements

- **Finite family of control configurations:**
  - ⋆ Feedback controllers
  - ⋆ Actuator/sensor spatial arrangements
- **Motivation for switching:**
  - ⋆ Fault-tolerant control
  - ⋆ Optimizing performance
  - ⋆ Enforcing state & control constraints
  - ⋆ Flexibility in reconciling multiple conflicting control objectives

PRESENT WORK

(El-Farra & Christofides, Comp. & Chem. Eng., *submitted*, 2002)

- **Scope:**
  - ◇ Highly dissipative PDE systems with input constraints
- **Objectives:**
  - ◇ Development of an integrated approach for hybrid control
    - ▷ Feedback controller design (control algorithm)
    - ▷ Switching between multiple actuator configurations
      - ⋆ Ensure fault-tolerance
      - ⋆ Ensure constraint satisfaction
      - ⋆ Guarantee closed-loop stability
  - ◇ Application to a diffusion-reaction process

DISSIPATIVE PDE SYSTEMS

- **Infinite-dimensional system description:**

$$\begin{aligned} \dot{x} &= \mathcal{A}x + \mathcal{B}u(t) + f(x), \quad x(0) = x_0 \\ y_c &= \mathcal{C}x, \qquad y_m = \mathcal{S}x \end{aligned}$$

⋆ $x(t) \in \mathcal{H}$: state, $u(t) \in \mathbb{R}^m$: control input

⋆ $\mathcal{A}$: differential operator

⋆ $\mathcal{B}$: input operator: actuators location & type

⋆ $\mathcal{S}$: measurement operator: sensors location & type

- **Eigenvalue problem for $\mathcal{A}$:**

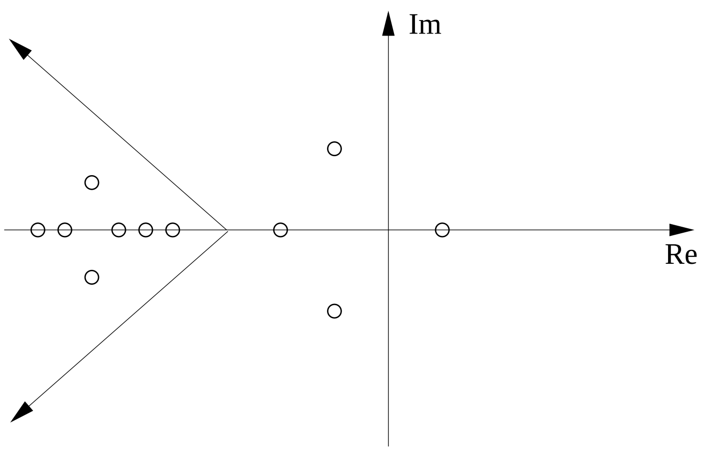
$$\mathcal{A}\phi_j = \lambda_j \phi_j, \quad j = 1, \ldots, \infty$$

λ_j : eigenvalue; ϕ_j eigenfunction

⋆ $Re\{\lambda_1\} \geq Re\{\lambda_2\} \geq \cdots$

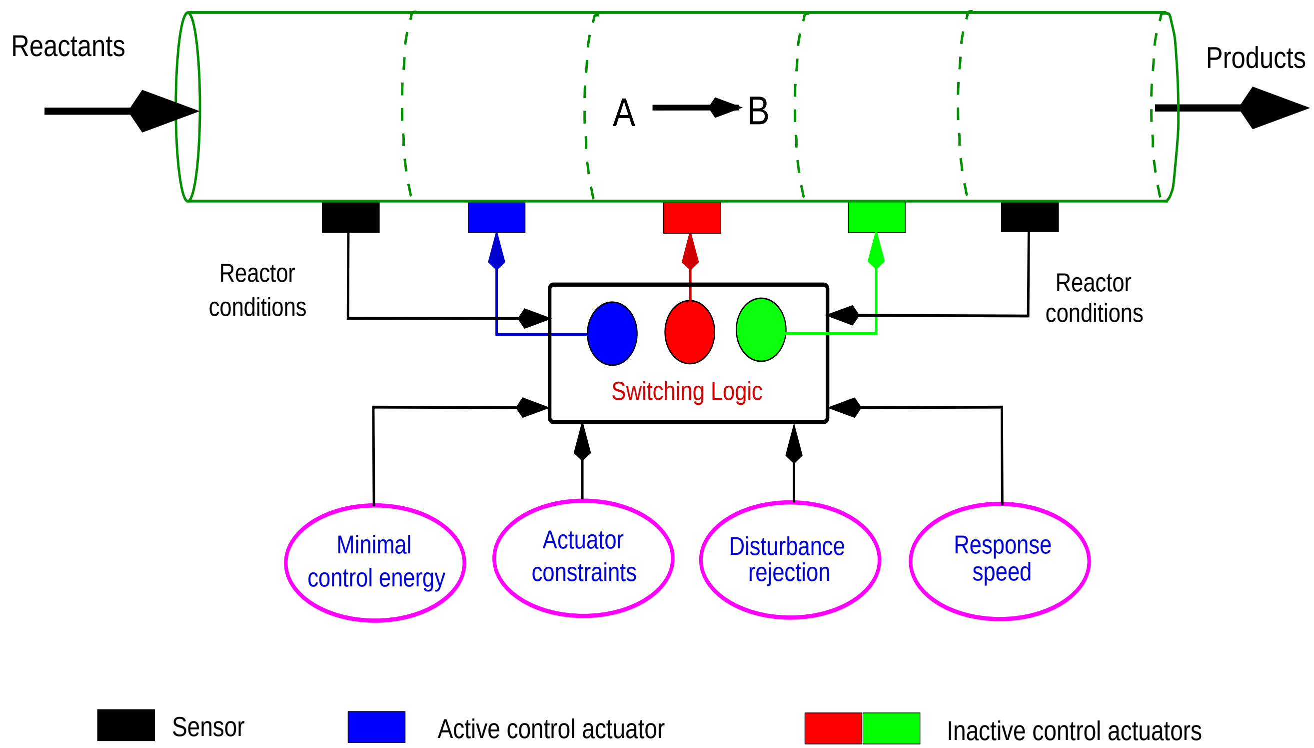
⋆ $\{\phi_1, \phi_2, \ldots,\}$ complete orthonormal set in $\mathcal{H}$

- **Typical structure of eigenspectrum:**



⋆ **A finite number of dominant modes** practically determines dynamics

INTEGRATING FEEDBACK AND SWITCHING



- **Problem specifications:**
  - ◇ N spatially-distinct actuator configurations available
    - ⋆ $\bar{z}_k(t)$: location vector for actuators in k-th configuration
    - ⋆ $k(t) \in \{1, 2, \cdots, N\}$ indexes the actuator configuration
  - ◇ Constraints on each actuator configuration $u_i^k \in [-u_{max}^k, u_{max}^k]$
  - ◇ Only one configuration can be active at a given time
    - ⋆ Finite switches over finite time

INTEGRATING FEEDBACK AND SWITCHING

- **Methodology:**
  - ◇ Model reduction
    - ⋆ Derivation of low-dimensional ODE models:
  - ◇ Feedback controller synthesis
    - ⋆ Bounded Lyapunov-based nonlinear controller design
  - ◇ Characterizing stability regions for N actuator configurations
    - ⋆ Set of feasible initial states for each configuration
  - ◇ Derivation of switching rules
    - ⋆ When can each configuration can be activated
  - ◇ Analysis of switched closed-loop system
    - ⋆ Singular perturbation theory

MODAL DECOMPOSITION/GALERKIN'S METHOD

- $\mathcal{H}_s = span\{\phi_1, \phi_2, \ldots, \phi_m\}$, $\mathcal{H}_f = span\{\phi_{m+1}, \phi_{m+2}, \ldots,\}$
  P_s, P_f: orthogonal projection operators

  $x_s(t) = P_s x(t)$: state vector corresponding to slow eigenmodes

  $x_f(t) = P_f x(t)$: state vector corresponding to fast eigenmodes

- Set of infinite ODEs:

$$\begin{aligned} \frac{dx_s}{dt} &= A_s x_s + \mathcal{B}_s u + f_s(x_s, x_f) \\ \frac{\partial x_f}{\partial t} &= A_f x_f + \mathcal{B}_f u + f_f(x_s, x_f) \end{aligned}$$

- Neglecting the fast dynamics:

$$\frac{d\bar{x}_s}{dt} = A_s \bar{x}_s + \mathcal{B}_s u + f_s(\bar{x}_s, 0), \quad \bar{x}_f = 0$$

- When $u(t) \equiv 0$, $\|x - \bar{x}_s\|_2 = O(\epsilon), \;\; \forall\, t \geq 0,\; \epsilon = \dfrac{|Re\lambda_1|}{|Re\lambda_{m+1}|}$

FORMULATION OF CONTROL PROBLEM

(El-Farra, Armaou & Christofides, Automatica, *to appear*, 2003)

- **Constrained low-dimensional ODE system**

$$\begin{aligned} \frac{d\bar{x}_s}{dt} &= \tilde{f}(\bar{x}_s) + \tilde{G}(\bar{x}_s, \bar{z}_k)u \\ y_{cs} &= \mathcal{C}\bar{x}_s \end{aligned}$$

$$u_i \in U = [-u_{max}, u_{max}]$$

- **Control Objectives:**
  - ◇ Desired closed-loop properties:
    - ⋆ Exponential stability
    - ⋆ Reference input tracking
  - ◇ Explicit characterization of the region of closed-loop stability

- **Approach:**
  - ◇ Bounded Lyapunov-based control techniques

FEEDBACK CONTROLLER DESIGN

- **Bounded controller synthesis:**

$$u \quad = \quad -k(x_s, u_{max}, \bar{z}_k)(L_G V)^T(\bar{z}_k)$$

  - ◇ V: control Lyapunov function
  - ◇ $k(\cdot)$: scalar nonlinear "gain" shaped so that:
    - ⋆ $|u(x)| \leq u_{max}$ and $\dot{V} < 0$
    - ⋆ Example gain: (Sontag's bounded law)

$$k(x_s, u_{max}, \bar{z}_k) = \left(\frac{L_f V + \sqrt{(L_f V)^2 + (u_{max}|(L_G V)^T(\bar{z}_k)|)^4}}{|(L_G V)^T(\bar{z}_k)|^2 \left[1 + \sqrt{1 + (u_{max}|(L_G V)^T(\bar{z}_k)|)^2}\right]} \right)$$

- **Closed-loop properties:**
  - ◇ Asymptotic stability
  - ◇ Reference input tracking

$$\lim_{t \to \infty} |\bar{y}_{cs_i} - v_i| \quad = \quad 0$$

CHARACTERIZATION OF STABILITY PROPERTIES

$$D(u_{max}, \bar{z}_k) \quad = \quad \{x_s \in \mathcal{H}_s : L_f V \leq u_{max} |(L_G V)^T (\bar{z}_k)|\}$$

- **Properties of inequality:**
  - ◇ Describes an open unbounded region where:
    - ⋆ $|u(x)| \leq u_{max} \;\; \forall \, x \in D$
    - ⋆ $\dot{V}(x) < 0 \;\; \forall \;\; 0 \neq x \in D$
  - ◇ Parameterized by control actuator location $\bar{z}_k$:
    - ⋆ Identify feasible initial conditions, for a fixed $\bar{z}_k$
    - ⋆ Identify feasible actuator locations, for a fixed initial condition
  - ◇ Explicit guidelines for switching between actuator configurations
- **Some design implications:**
  - ◇ Given the desired stability region, determine u_{max}
  - ◇ u_{max} determines capacity & size of control actuators
    - ⋆ Valves, pumps, heaters, etc.

STABILIZING SWITCHING LAWS

- **Switched closed-loop ODE system:**

$$\begin{aligned} \frac{d\bar{x}_s}{dt} &= \tilde{f}(\bar{x}_s) + \tilde{G}(\bar{x}_s, \bar{z}_k)u(\bar{x}_s, \bar{z}_k) \\ k(t) &\in \mathcal{I} = \{1, 2, \cdots, N\} \end{aligned}$$

◇ Multiple configurations represent multiple modes:

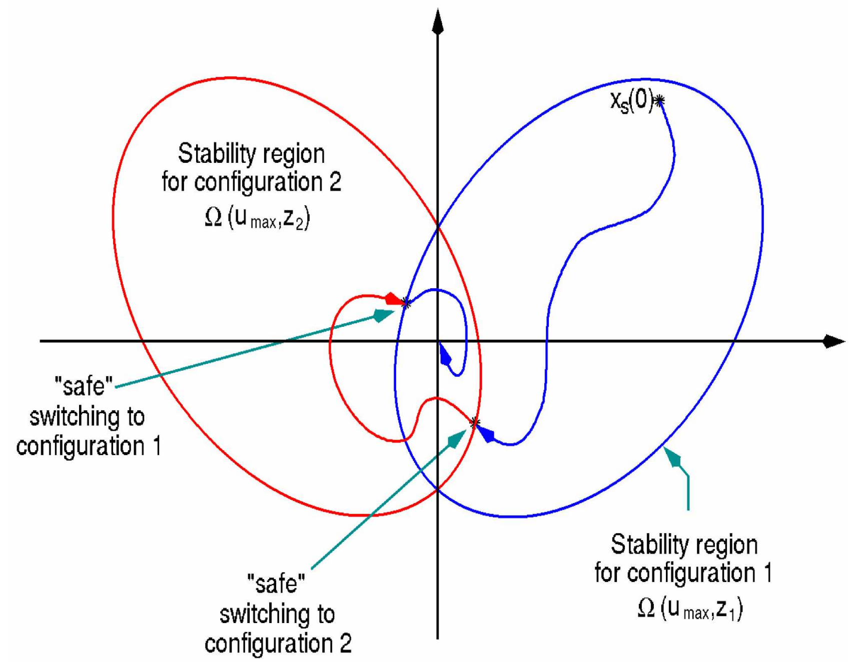
⋆ Stability of constituent modes & transitions

- **Switching rule:**

$$k(T^+) = j \text{ if } x_s(T) \in \Omega(u_{max}, \bar{z}_j)$$

◇ Tracks evolution of slow state (ϵ sufficiently small)

◇ Implicitly determines the switching times



Switching between stability regions

COMPARING LINEAR & NONLINEAR SYSTEMS

- **For linear systems:** modal decomposition yields a cascade

$$\begin{aligned} \dot{x}_s &= A_s x_s + \mathcal{B}_s u(x_s) \\ \dot{x}_f &= A_f x_f + \mathcal{B}_f u(x_s) \end{aligned}$$

  - ◇ Evolution of slow states independent of the fast states
  - ◇ Fast subsystem exponentially stable with bounded, decaying input

- **For nonlinear systems:** modal decomposition yields an interconnection

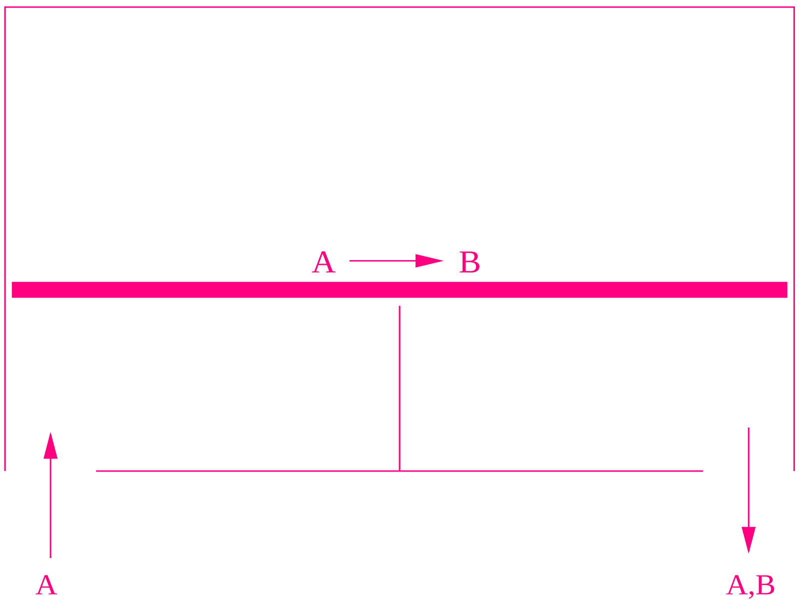
$$\begin{aligned} \dot{x}_s &= A_s x_s + \mathcal{B}_s u(x_s) + f_s(x_s, x_f) \\ \dot{x}_f &= A_f x_f + \mathcal{B}_f u(x_s) + f_s(x_s, x_f) \end{aligned}$$

  - ◇ Evolution of slow states depends on the fast subsystem

- **Implications for the stability region:**
  - ◇ Linear case: stability region of slow system is exactly preserved
  - ◇ Nonlinear case: stability region is recovered asymptotically (as $\epsilon \longrightarrow 0$)

APPLICATION TO A DIFFUSION-REACTION PROCESS

CATALYTIC ROD



- **Process dynamic model:**

$$\frac{\partial x}{\partial t} = \frac{\partial^2 x}{\partial z^2} + \beta_T e^{-\frac{\gamma}{1+x}} + \beta_U (u(z,t) - x) - \beta_T e^{-\gamma}$$

- **Dirichlet boundary conditions:**

$$x(0,t) = 0, \quad x(\pi,t) = 0$$

EIGENSPECTRUM/OPEN-LOOP DYNAMICS

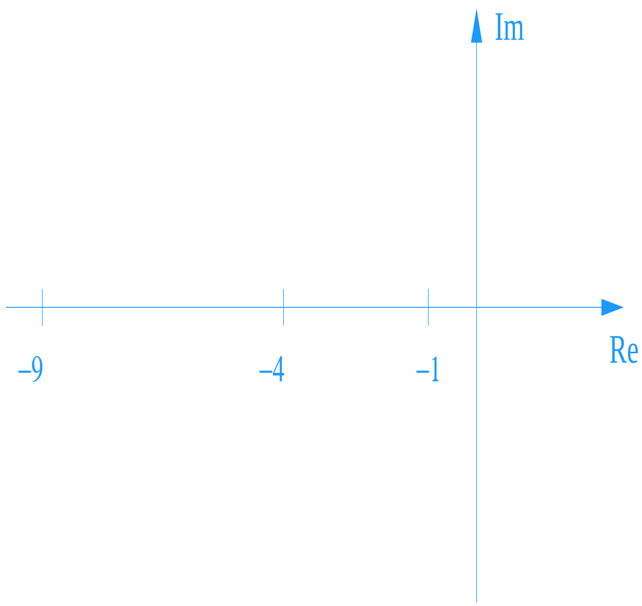
- **Eigenvalue problem:**

$$\mathcal{A}\phi_j = \frac{\partial^2 \phi_j}{\partial z^2} = \lambda_j \phi_j$$
$$\phi_j(0) = 0, \quad \phi_j(\pi) = 0$$

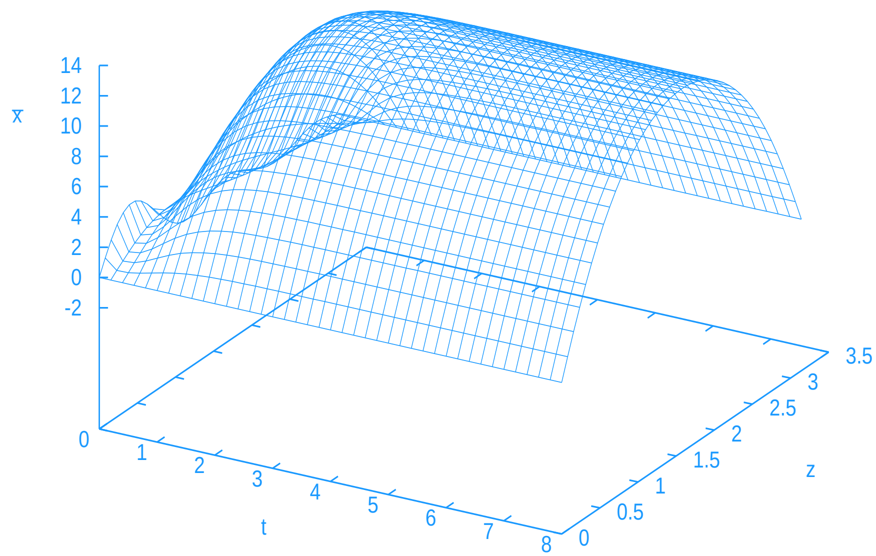
- **Eigenvalues:** $\lambda_j = -j^2$, $j = 1, \ldots, \infty$

- **Eigenfunctions:** $\phi_j = \sqrt{\frac{2}{\pi}} sin(jz)$, $j = 1, \ldots, \infty$

Structure of Eigenspectrum



Open-loop temperature profile



steady-state $x(z,t) = 0$ unstable

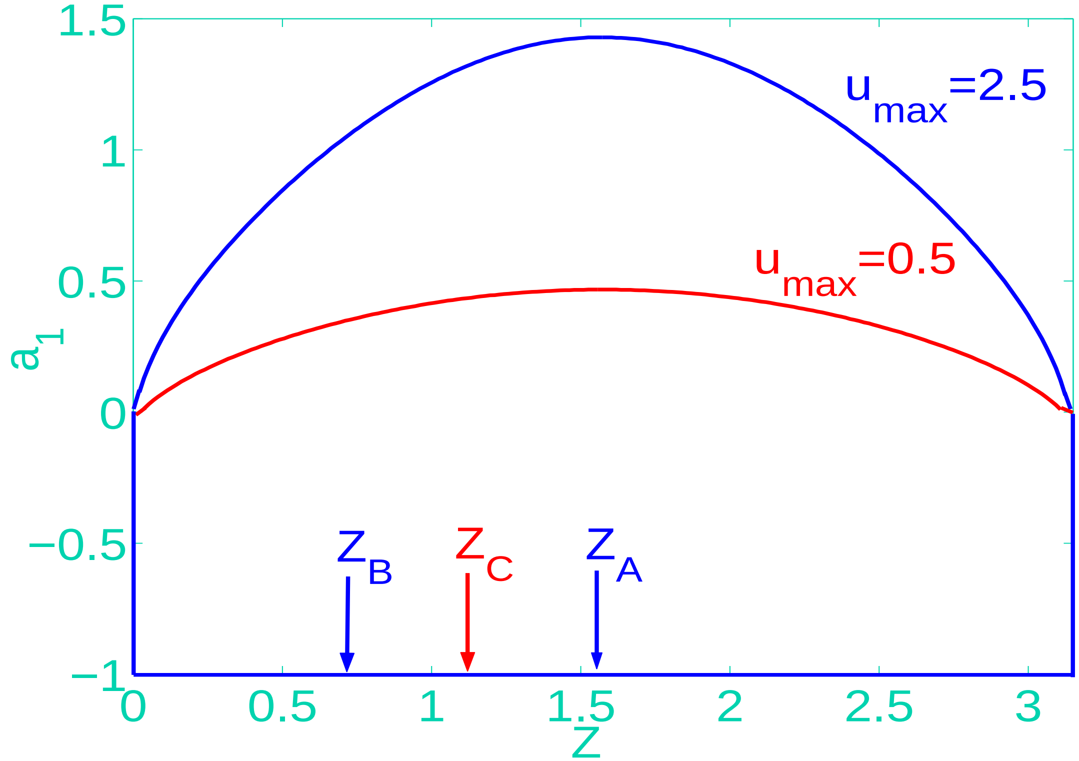
PROBLEM FORMULATION
Diffusion-reaction process

- **Control problem:**
  - ◇ First eigenmode is dominant
  - ◇ Controlled output: $y_c(t) = \int_0^{\pi} \sqrt{\frac{2}{\pi}} sin(z) x(z,t) dz$
  - ◇ Manipulated input, $u(t)$:
    - ▷ One point control actuator with $b(z) = \delta(z - z_c)$

- **Actuator switching problem:**
  - ◇ Three point actuators available:
    - ⋆ Configuration A $(z_c = 0.5\pi,\ u_{max} = 2.5)$
    - ⋆ Configuration B $(z_c = 0.23\pi,\ u_{max} = 2.5)$
    - ⋆ Configuration C $(z_c = 0.36\pi,\ u_{max} = 0.5)$
  - ◇ Actuator failure: which actuator to activate when the operating actuator fails?

CONTROLLER SYNTHESIS/STABILITY ANALYSIS

- One-dimensional system used for controller design
  - ◇ First eigenmode is dominant, $m = 1$, $\epsilon = \frac{|Re\lambda_1|}{|Re\lambda_2|} = 0.25$
- Dependence of stability region on actuator location & constraints
  - ⋆ region size peaks at $z = 0.5\pi$ & vanishes at $z = 0$, $z = \pi$

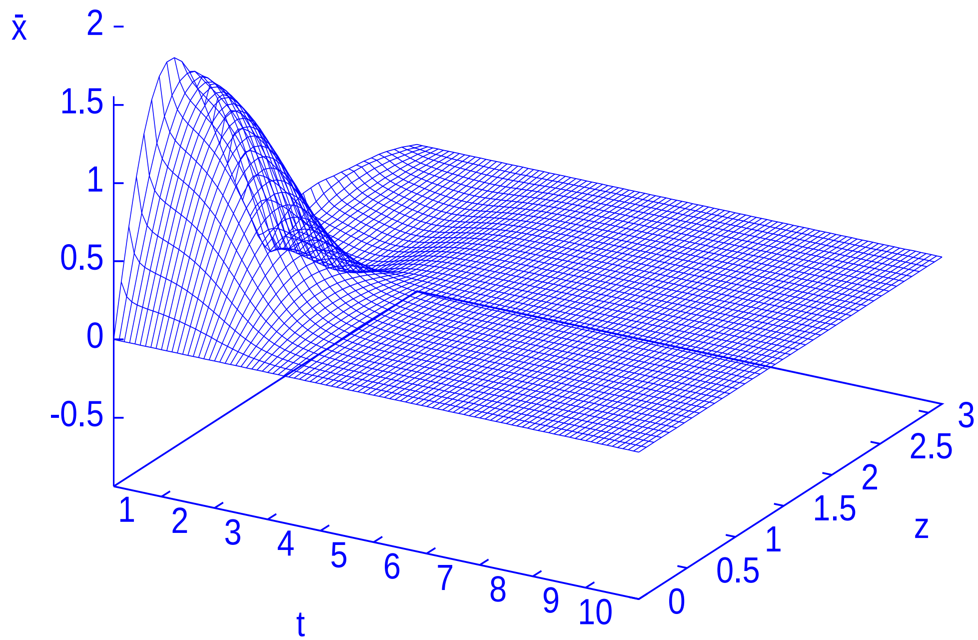


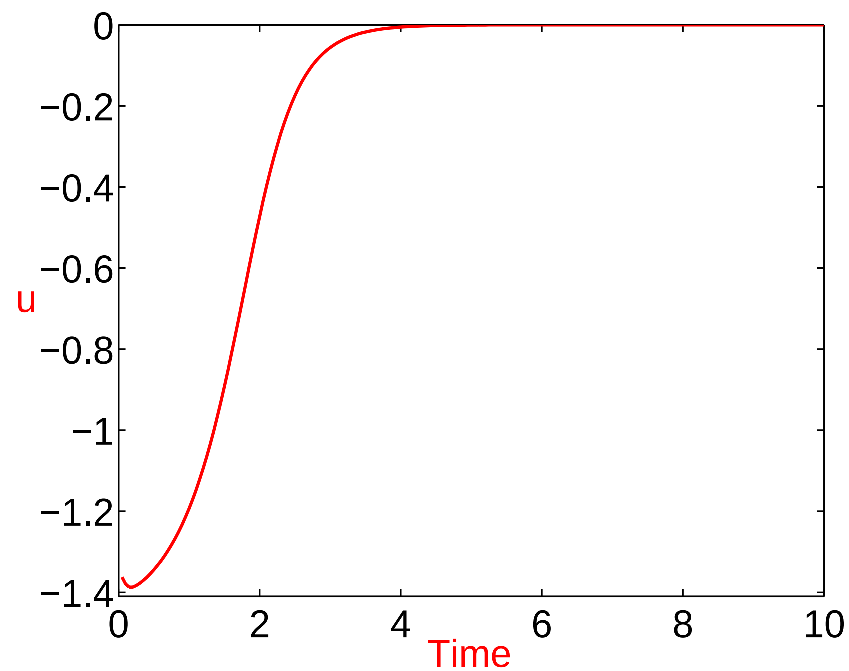
$a_1(t)$: amplitude of first eigenmode

CLOSED-LOOP SIMULATION RESULTS

- Temperature & manipulated input profiles under no actuator failure

  ($a_1(0) = 1.3$, only configuration A is active, $\bar{z}_A = 0.5\pi$, $u_{max} = 2.5$)





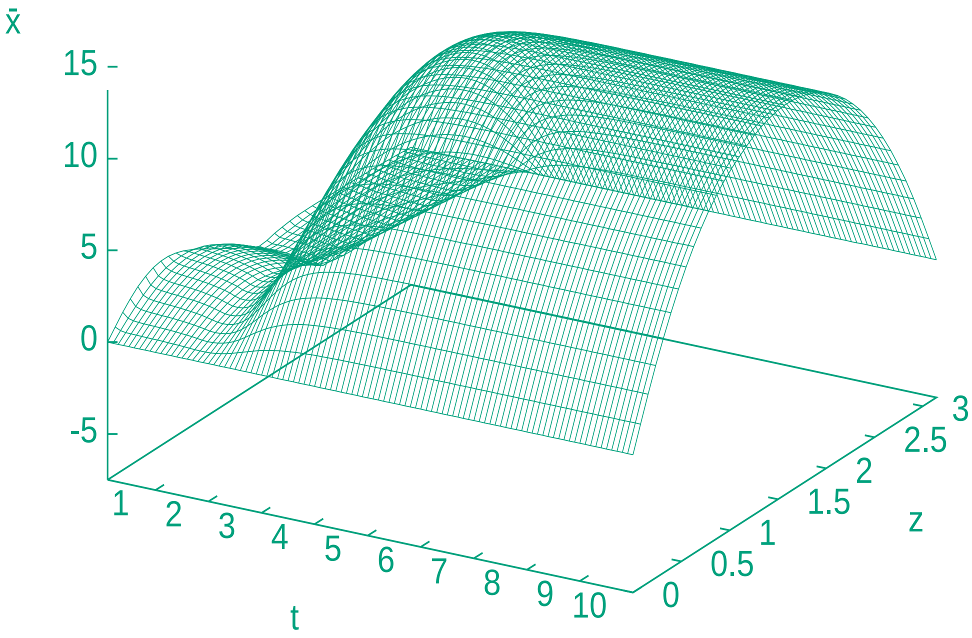
- Closed-loop system stable

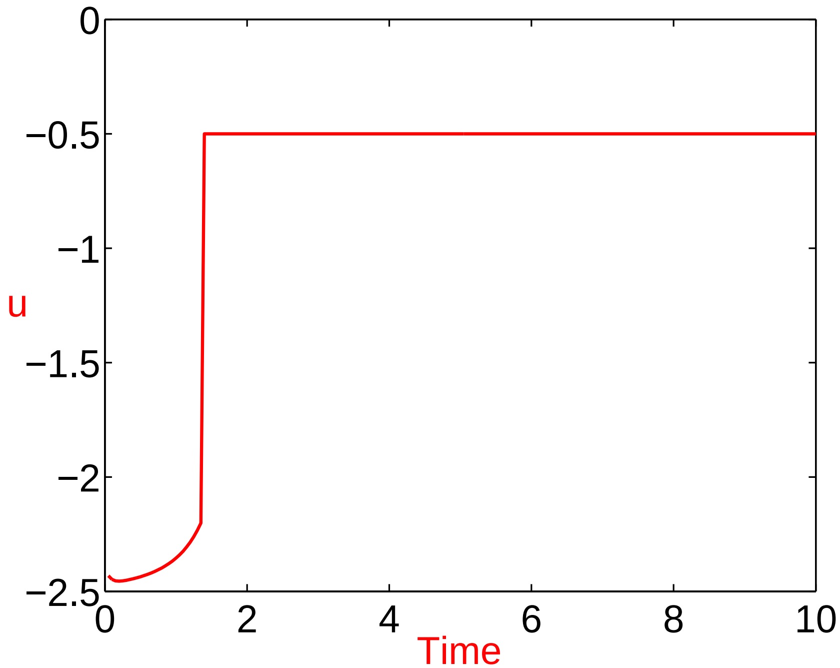
CLOSED-LOOP SIMULATION RESULTS

- Temperature & manipulated input profiles under actuator failure

(Configuration A fails at $t = 1.4$ & configuration C activated)

$$a_1(0) = 1.3,\ \bar{z}_C = 0.36\pi,\ u_{max} = 0.5$$





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CLOSED-LOOP SIMULATION RESULTS

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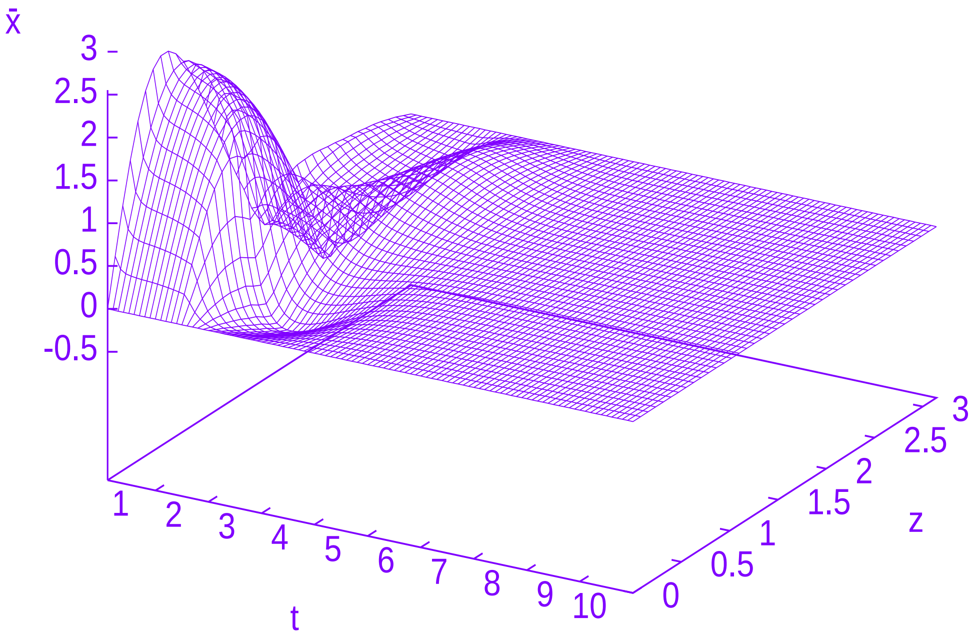
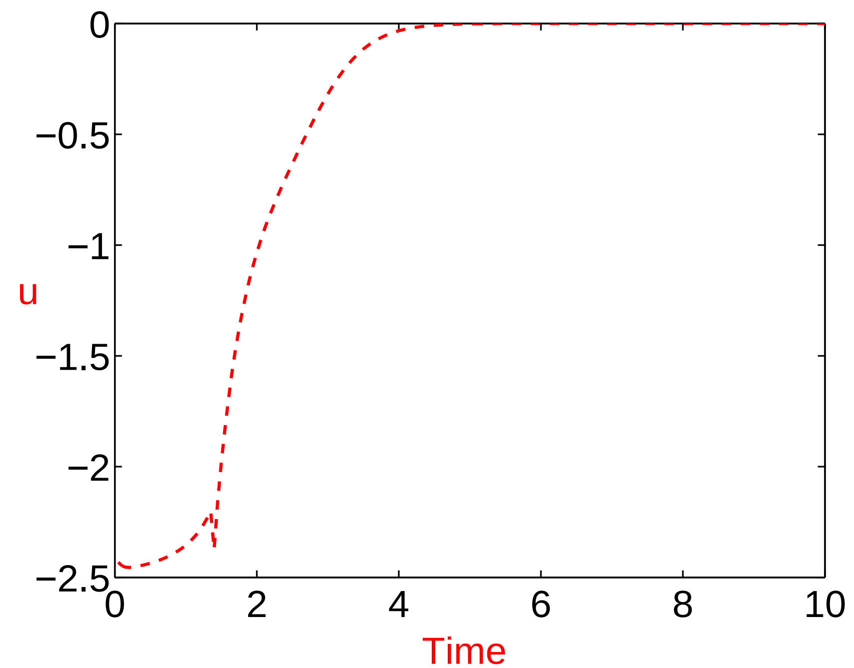
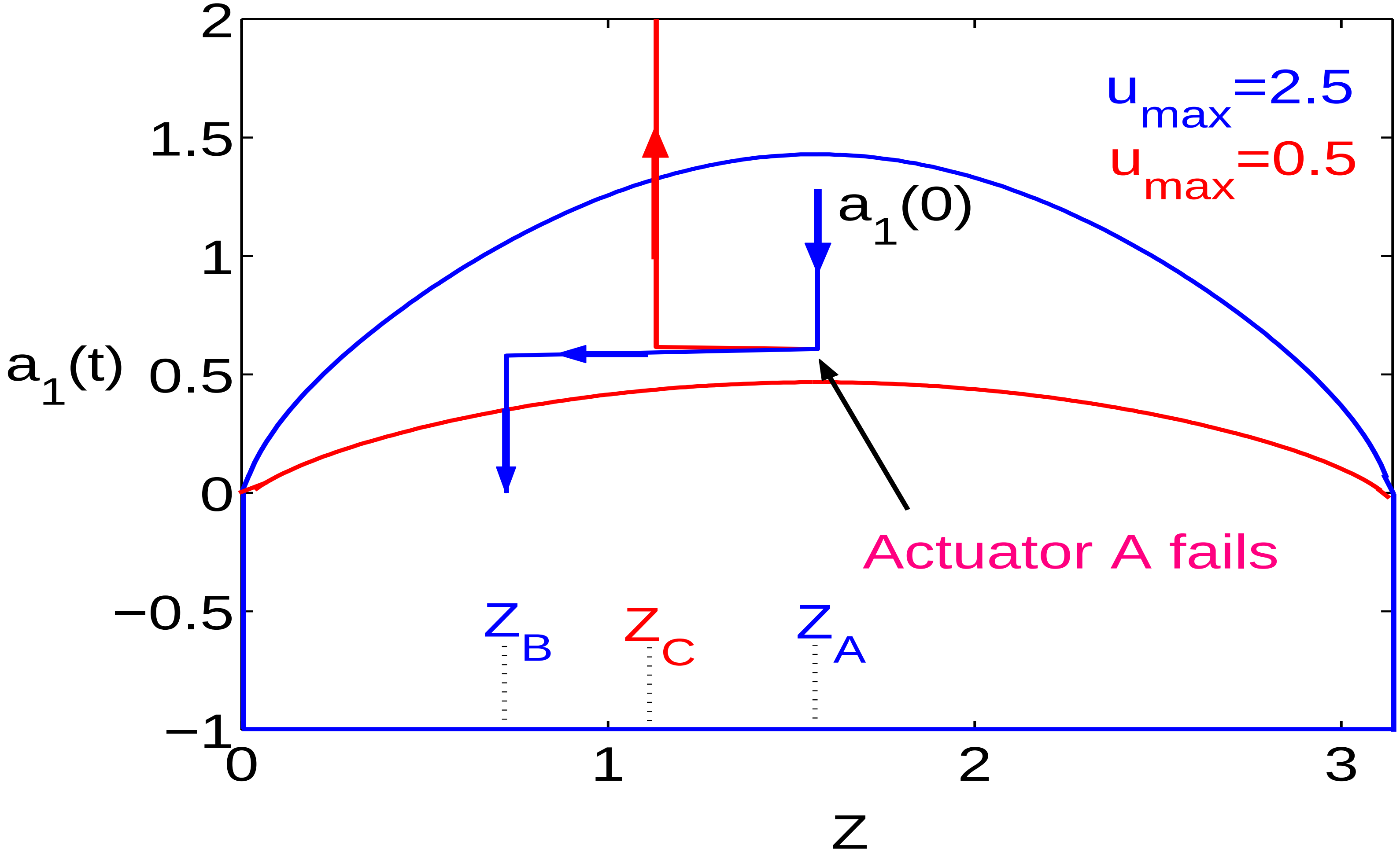




- Closed-loop system stable

CLOSED-LOOP SIMULATION RESULTS

- Implementing the switching logic



CONCLUSIONS

- Highly dissipative PDE systems with input constraints
- Methodology for integrating feedback and switching
  - ◇ Feedback controller design
  - ◇ Switching between multiple actuator configurations
    - ▷ Ensure constraint satisfaction
    - ▷ Guaranteed closed-loop stability
    - ▷ Reconcile conflicting control objectives
  - ◇ Application to fault-tolerant control of a diffusion-reaction process

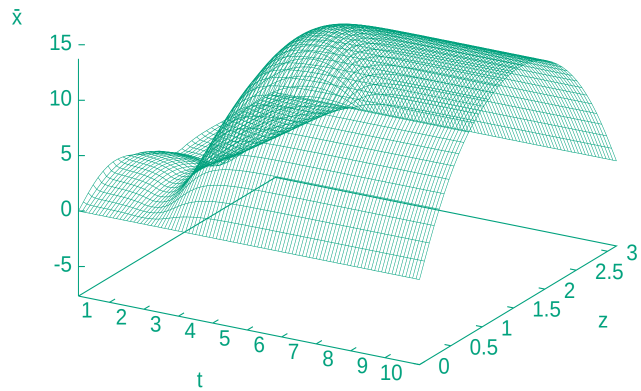
ACKNOWLEDGMENT

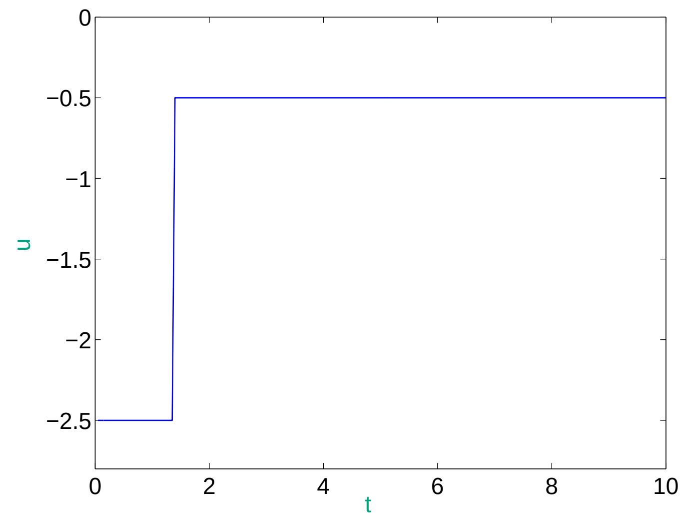
- Financial support from UCLA Chancellor's Fellowship (Nael H. El-Farra) is gratefully acknowledged

SIMULATION RESULTS: OUTPUT FEEDBACK

$$y_{m_1}(t) = \bar{x}(0.33\pi, t)$$

- Closed-loop state/manipulated input (A fails at $t = 1.4$, C activated)





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