PREDICTIVE CONTROL OF SWITCHED NONLINEAR SYSTEMS

Prashant Mhaskar, Nael H. El-Farra
& Panagiotis D. Christofides

Department of Chemical Engineering
University of California, Los Angeles

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INTRODUCTION

- **Switched systems:**
  - ◇ Discrete transitions in continuous dynamics.
  - ◇ Frequently arise in operation (demand changes, phase changes, etc.).
  - ◇ Nonlinear continuous dynamics (nonlinear expressions of reaction rates).
- **Input constraints:**
  - ◇ Finite capacity of control actuators.
  - ◇ Influence stabilizability of an initial condition.
- **Desired characteristics of an effective control policy:**
  - ◇ Account for the switched dynamics.
  - ◇ Respect input constraints.
  - ◇ Explicit characterization of set of admissible initial conditions.

BACKGROUND

- **Combined discrete-continuous processes:**
  - ◇ Modeling (e.g., Yamalidou et al, C&CE, 1990).
  - ◇ Simulation (e.g., Barton and Pantelides, AIChE J., 1994).
  - ◇ MINLP - Optimization (e.g., Grossman et al., CPC-6, 2001).
- **Stability of switched and hybrid systems:**
  - ◇ Multiple Lyapunov functions (e.g., Branicky, IEEE TAC,1998).
  - ◇ Dwell-time approach (e.g., Hespanha and Morse, CDC,1999).
- **Control of switched and hybrid linear systems:**
  - ◇ Mixed Logical Dynamical systems (Morari and co-workers)
    - ▷ Model predictive control.
    - ▷ Moving horizon estimation and fault detection.
    - ▷ Optimization-based verification.
  - ◇ Optimal control of switched linear systems (e.g., Xu and Anstaklis, IJRNC, 2002).

SWITCHED NONLINEAR SYSTEMS WITH INPUT CONSTRAINTS

- **State–space description:**

$$\begin{aligned} \dot{x}(t) &= f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t))u_{\sigma(t)} \\ u(t) &\in \mathcal{U},\ \sigma(t) \in \mathcal{I} : \{1, \ldots, N < \infty\} \end{aligned}$$

◇ $x(t) \in \mathbb{R}^n$: state vector.

◇ $u(t) \in \mathcal{U} \subset \mathbb{R}^m$: control input.

◇ $\mathcal{U} \subset \mathbb{R}^m$: compact & convex.

◇ $u = 0 \ \in$ interior of $\mathcal{U}$.

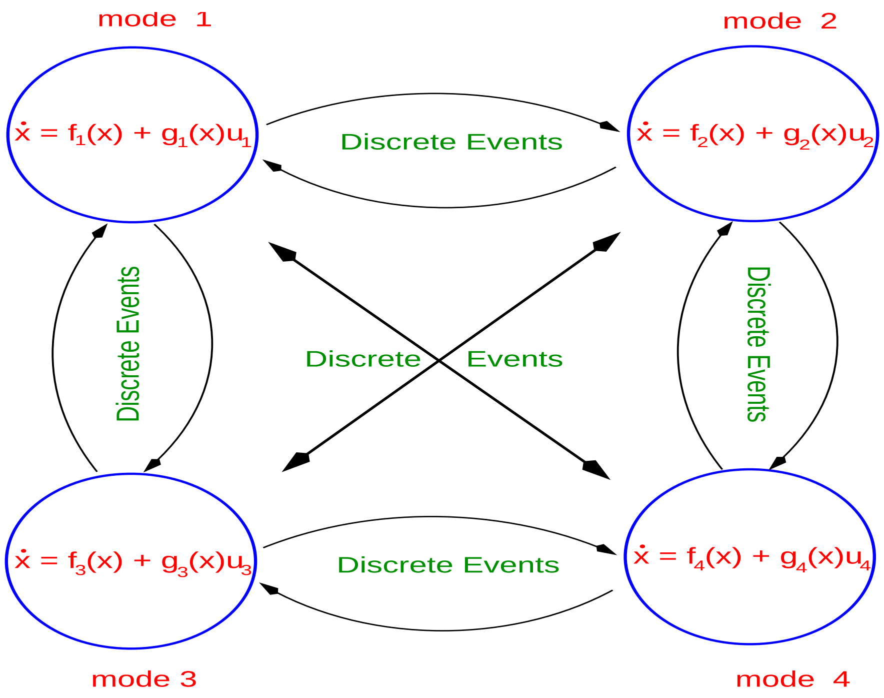
◇ $(0,\ 0)$: an equilibrium point for all σ.

◇ $\sigma : [0, \infty) \rightarrow \mathcal{I}$: the switching signal.

◇ $f_\sigma,\ g_\sigma$: sufficiently smooth functions for all σ.

◇ $\sigma(t)$: piecewise continuous from the right.

SWITCHED NONLINEAR SYSTEMS



- σ is a function of time, or state, or both.
- Autonomous switching:
  - ◇ σ depends **only** on the states.
- Controlled switching:
  - ◇ σ can be chosen by a supervisor.

CONSTRAINED CONTROL OF SWITCHED NONLINEAR SYSTEMS

(El-Farra and Christofides, AIChE J., 2003)

- **Controller design requirements:**
  - ◇ Available model for each mode of the nonlinear plant.

$$\boxed{\dot{x} \quad = \quad f_\sigma(x) + g_\sigma(x)u_\sigma}$$

  - ◇ Input constraints.

$$\boxed{|u_\sigma| \quad \leq \quad u_{max}}$$

  - ◇ Multiple Lyapunov functions.

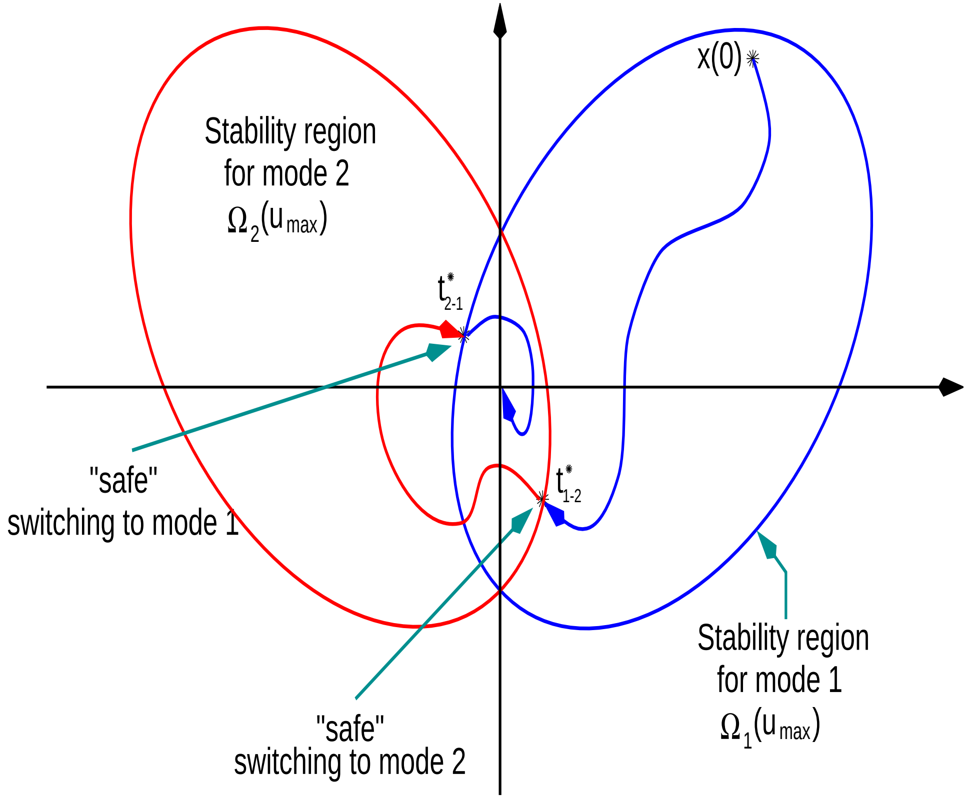
$$\boxed{V_\sigma, \quad \sigma = 1, \cdots, N}$$

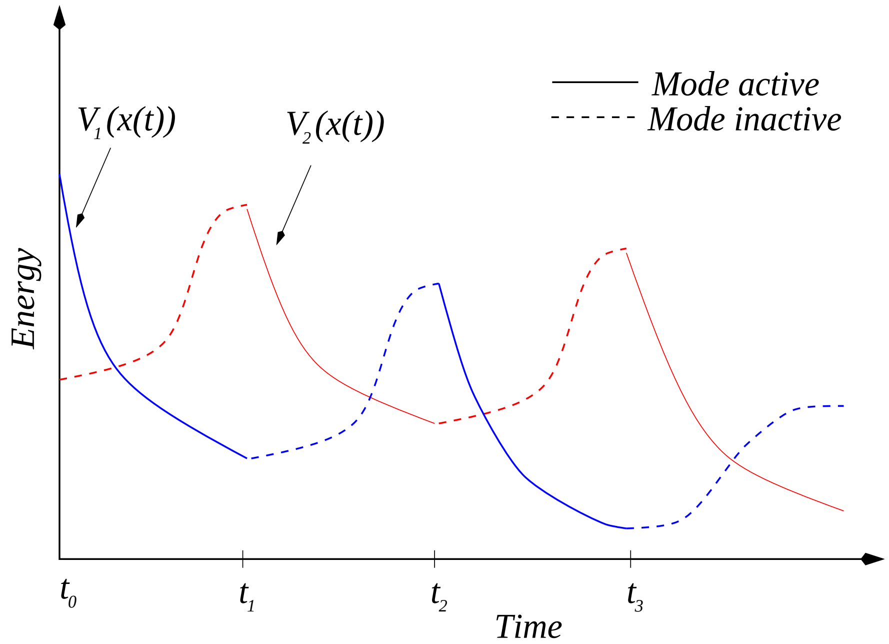
- **Feedback controller design:**
  - ◇ Bounded Lyapunov-based nonlinear control.

- **Switching laws:**
  - ◇ Track evolution of states w.r.t. stability regions of the individual modes.
  - ◇ Multiple Lyapunov Function stability criteria.

AN EXAMPLE

SAFE SWITCHING FOR 2-MODE SYSTEM





◇ Safe transition times determined, not enforced.

OPTIMIZATION-BASED CONTROL OF SWITCHED SYSTEMS

- **Control problem formulation**

  ◇ Compute $u(t)$, $\sigma(t)$, that minimizes the following objective function:

$$
\begin{array}{rcl}
P(x,t) & : & \min\{J(x,t,u(\cdot))|\, u(\cdot) \in U_{\Delta}\} \\
x(T) & \in & \Pi \\
\dot{x}(t) & = & f_{\sigma(t)}(x(t)) + g_{\sigma(t)}(x(t))u_{\sigma(t)} \\
u(t) & \in & \mathcal{U},\ \sigma(t) \in \mathcal{I}
\end{array}
$$

  ◇ Performance index:

$$
J(x,t,u(\cdot),\sigma) = \int_{t}^{t+T} \left[l(x(s)) + r(u(s)) + m(\delta(\sigma)) \right] ds
$$

▷ $l(\cdot)$, $r(\cdot)$, $m(\cdot)$: positive definite.

▷ $m(\cdot)$: penalty on switching.

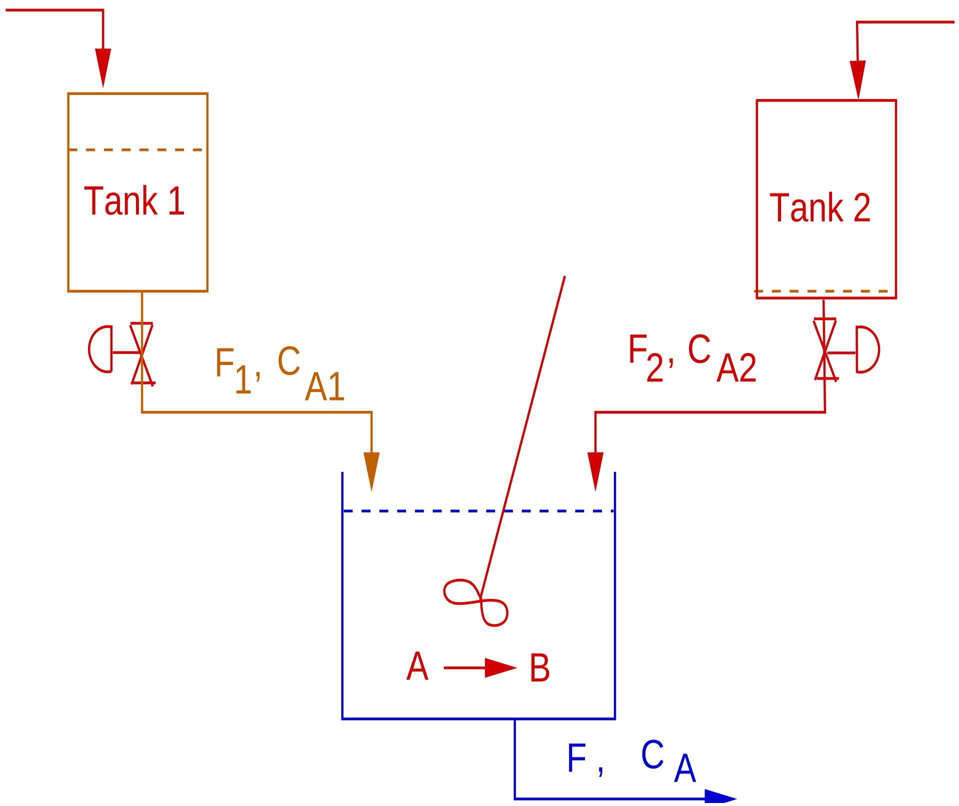
▷ $l(\cdot)$, $r(\cdot)$: penalty on states, control.

▷ T : horizon length.

PROPERTIES AND IMPLEMENTATION ISSUES

- The optimization problem is a Mixed Integer optimization problem and yields:
  - Optimal control, $u(t)$.
  - Optimal switching sequence, $\sigma(t)$.
- "Optimal" control accounting for switched dynamics:
  - Feasibility/stability depends on appropriate choice of horizon length.
- For linear systems/quadratic costs: Mixed Integer Quadratic Program (MIQP).
  - Computational techniques available.
  - Allows for online/receding horizon implementation.
- Nonlinear systems: Mixed Integer Nonlinear Program (MINLP).
  - Computationally intractable.
  - Ill-suited for the purpose of online implementation.

DYNAMIC SCHEDULING IN CHEMICAL PROCESSES: AN EXAMPLE



- ⋄ Storage tanks 1 & 2 empty every 1 hour.
- ⋄ Schedule involves switching between F_1 and F_2 every hour.

PRESENT WORK

- **Scope:**
  - ◇ Switched nonlinear systems with input constraints.
- **Objectives:**
  - ◇ Implement a prescribed switching sequence.
  - ◇ Asymptotic stabilization of the switched system.
- **Approach:**
  - ◇ An MLF-based predictive control framework that brings together:
    - ▷ Lyapunov-based control (stability region).
    - ▷ Model Predictive Controller (enforce transition constraints).

Switching between predictive controllers & Lyapunov-based controllers

LYAPUNOV-BASED CONTROL

- **Explicit nonlinear control law:**

$$u_\sigma \quad = \quad -k(x, u_{max})(L_{g_\sigma} V)^T$$

  ◇ Example: bounded controller (Lin & Sontag, 1991)

    ▷ Controller design accounts for constraints.

- **Explicit characterization of stability region:**

$$\Omega_\sigma(u_{max}) \quad = \quad \{x \in \mathbb{R}^n : V_\sigma(x) \leq c_\sigma^{max} \ \& \ \dot{V}_\sigma(x) < 0\}$$

  ◇ Larger estimates using a combination of several Lyapunov functions

MODEL PREDICTIVE CONTROL

- **Control problem formulation**

  ◇ Finite-horizon optimal control:

$$P(x,t) \quad : \quad \min\{J(x,t,u(\cdot)) |\, u(\cdot) \in U_\Delta, V_\sigma(x(t+\Delta)) < V_\sigma(x(t))\}$$

  ◇ Performance index:

$$J(x,t,u(\cdot)) \quad = \quad F(x(t+T)) + \int_t^{t+T} \left[\|x^u(s;x,t)\|_Q^2 + \|u(s)\|_R^2 \right] ds$$

▷ $\|\cdot\|_Q$: weighted norm.

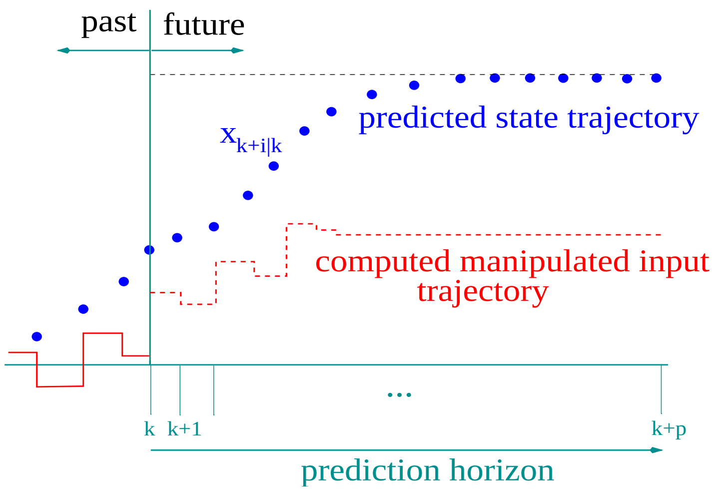
▷ T : horizon length.

▷ $Q,\ R\ > 0$: penalty weights.

▷ $F(\cdot)$: terminal penalty.

◇ Same V_σ as that for bounded controller design.

◇ Bounded controller may provide "good" initial guess.



HYBRID PREDICTIVE CONTROL

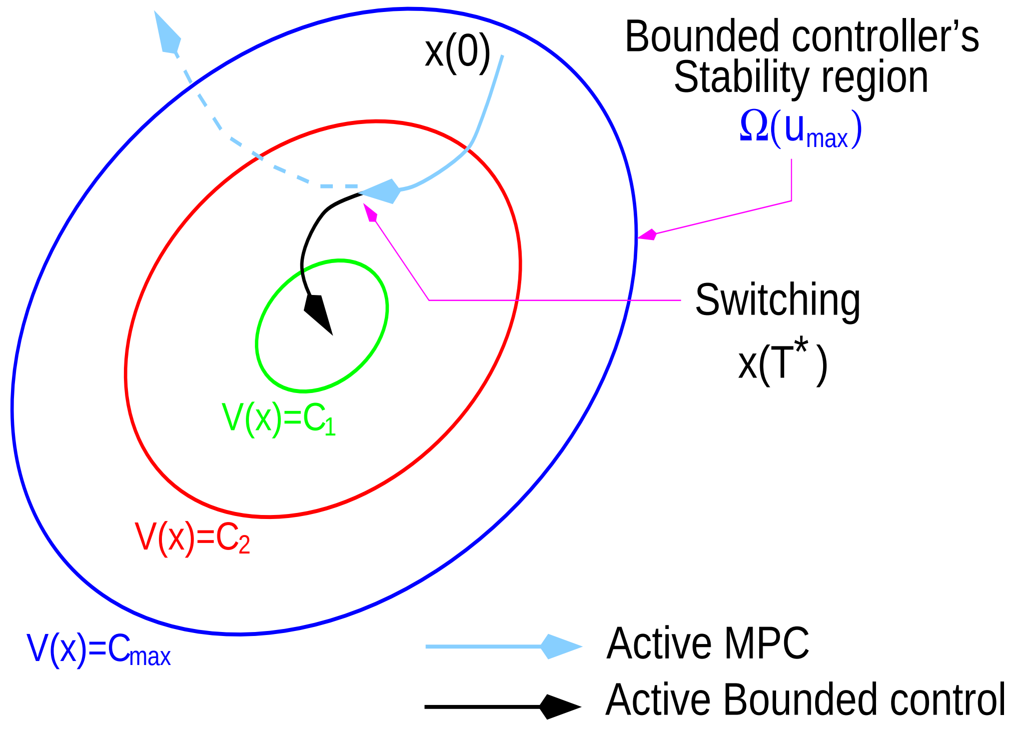
(El-Farra et. al., Automatica, 2004; IJRNC, 2004; AIChE J., 2004)

- **Switching logic:**

$$u_\sigma(x(t)) = \left\{ \begin{array}{ll} M_\sigma(x(t)), & 0 \leq t < T^* \\ b_\sigma(x(t)), & t \geq T^* \end{array} \right\}$$

$$T^* = \inf\{T^* \geq 0 : L_{f_\sigma} V_\sigma(x) + L_{g_\sigma} V_\sigma(x) M_\sigma(x(T^*)) \geq 0\}$$

- ◇ Initially implement MPC, $x(0) \in \Omega_\sigma(u_{max})$
- ◇ Monitor temporal evolution of $V_\sigma(x^M(t))$
- ◇ Switch to bounded controller only if $V_\sigma(x^M(t))$ starts to increase



PREDICTIVE CONTROL OF SWITCHED NONLINEAR SYSTEMS

- **Prescribed switching sequence:** $\sigma(t) = \begin{cases} \sigma_0, & t_0 \leq t < t_1 \\ \sigma_1, & t_1 \leq t < t_2 \\ \vdots & \vdots \end{cases}$

- **Objectives:**
  - ◇ Satisfy switching schedule.
  - ◇ Achieve asymptotic stabilization of the origin.

- **Approach**
  - ◇ Family of hybrid predictive controllers with well characterized stability regions.
  - ◇ To ensure safe transitions, incorporate as constraints in MPC:
    - ▷ Stability region constraint.
    - ▷ MLF stability criteria.

PREDICTIVE CONTROL WITH MLF & STABILITY REGION CONSTRAINTS

- **Specifications**
  - $\sigma(0) = 1$: System in mode 1
  - V_2^{old}: Value of V_2 at last switch to mode 2.
  - $\sigma(t_{switch}) = 2$: Switch to mode 2.
  - Ω_2: Stability region of closed–loop system in mode 2.
- **Optimization problem**
  - Minimize:

$$J(x, t, u(\cdot)) = F(x(t+T)) + \int_t^{t+T} \left[\|x^u(s; x, t)\|_Q^2 + \|u(s)\|_R^2 \right] ds$$

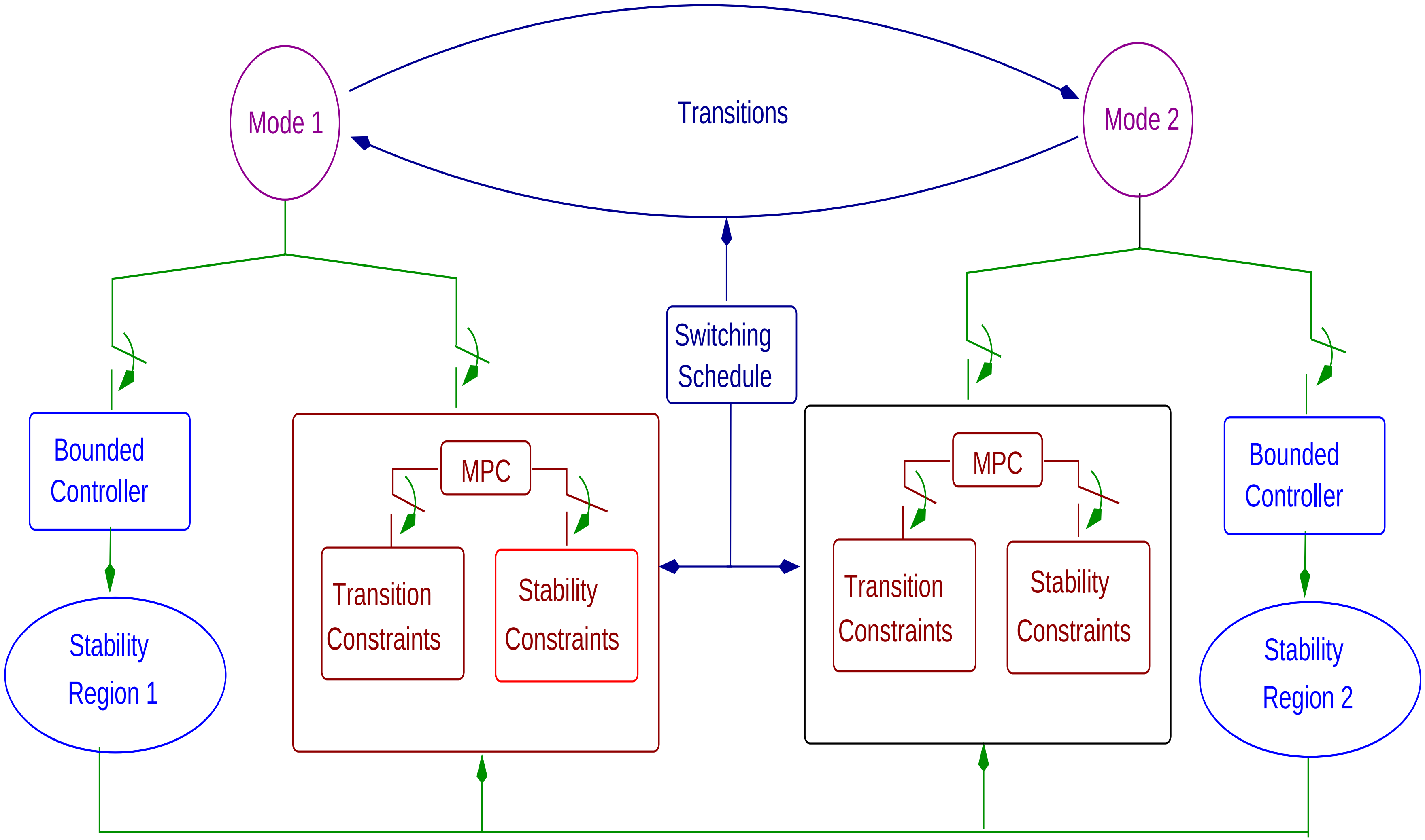
  - Subject to:

$$\begin{aligned} V_1(x(t+\Delta)) &< V_1(x(t)) \\ V_2(x(t_{switch})) &< V_2^{old} \\ x(t_{switch}) &\in \Omega_2 \end{aligned}$$

▷ $T \geq t_{switch}$: Horizon length.

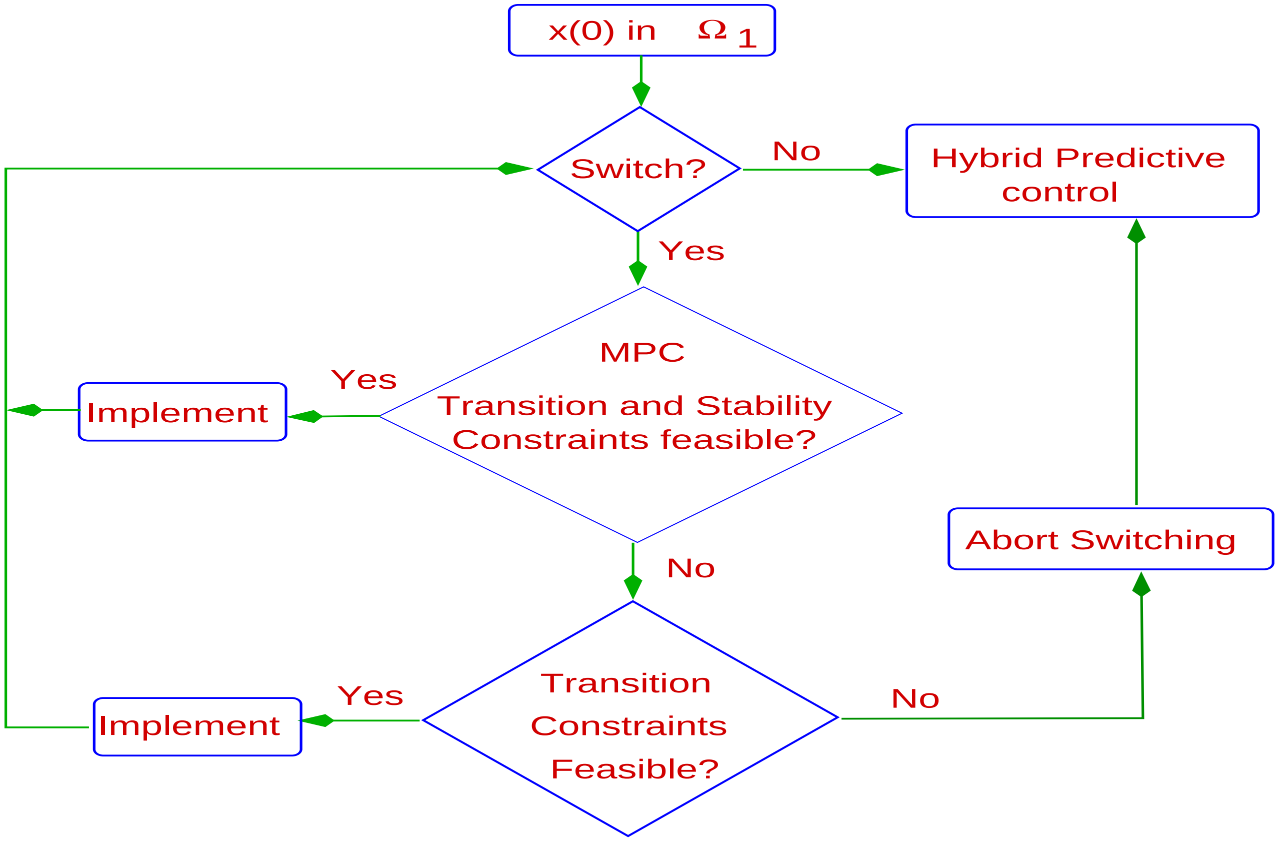
▷ t_{switch} : Remains fixed during receding horizon implementation.

MLF-BASED PREDICTIVE CONTROL STRATEGY



DESIGN AND IMPLEMENTATION

- **Design, for each mode:**
  - ◇ A hybrid predictive controller comprising of:
    - ▷ A bounded controller, and characterize its stability region $\Omega_\sigma(u_{max})$.
    - ▷ A model predictive controller.
  - ◇ MPC with MLF and stability region constraints.
- **Implementation:**



ILLUSTRATIVE EXAMPLE

- **State–space description:**

$$\dot{x} = f_\sigma(x) + g_\sigma(x)u_\sigma$$

$$f_1(x) = \begin{bmatrix} 2x_1^2 + x_1 + 2x_2^3 + 5x_2 \\ -2x_1^2 + 0.3x_1 + x_2^2 + 8x_2 \end{bmatrix}, \; g_1(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f_2(x) = \begin{bmatrix} x_1^2 + 7x_1 + 2x_2^3 + 2x_2 \\ x_1^2 + x_1 + x_2^2 + 0.9x_2 \end{bmatrix}, \; g_2(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- ◇ Origin unstable equilibrium of both modes.
- ◇ Objectives:
  - ▷ Switch to mode 2 at $t = 0.2$.
  - ▷ Asymptotic stabilization of the origin.
- ◇ Manipulated input: $u \in [-1, \; 1]$

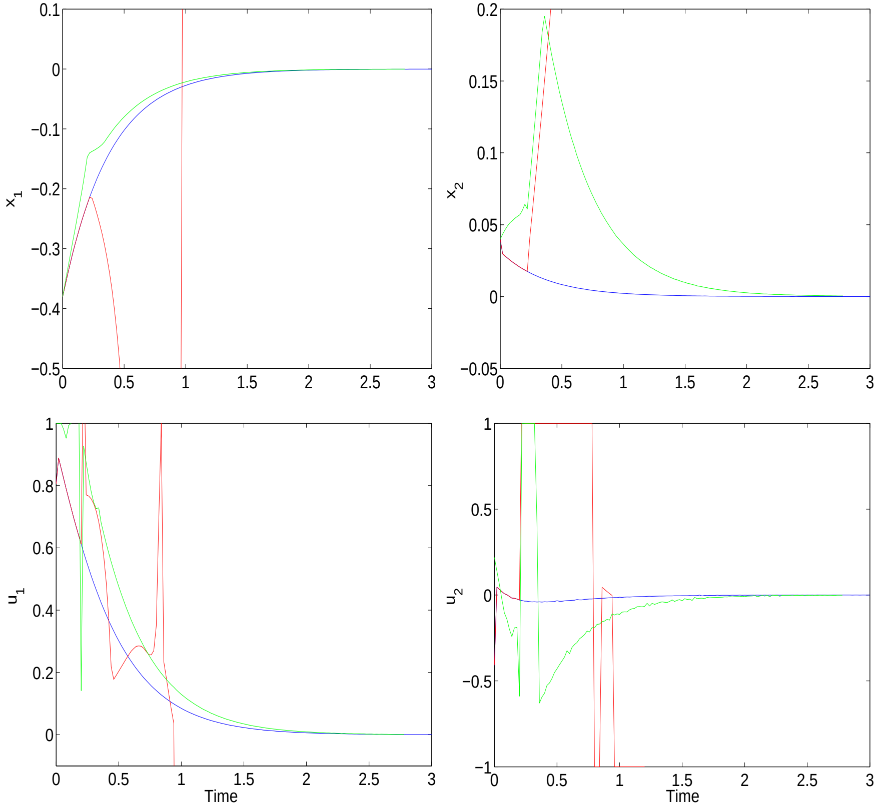
CONTROLLER DESIGN

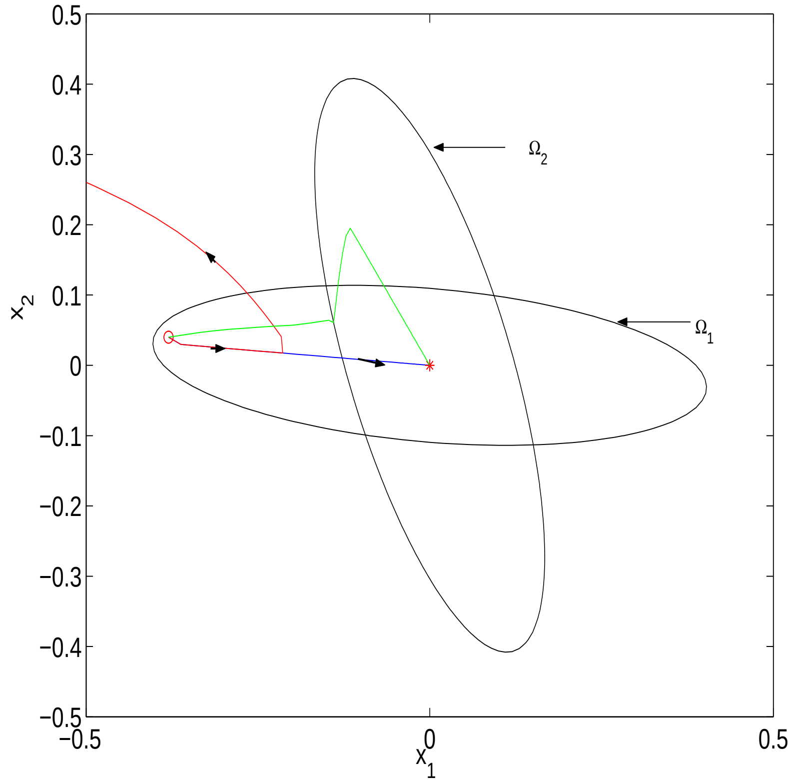
- **Bounded controller:**
  - ◇ Bounded controller designed using a quadratic lyapunov function.
  - ◇ Use $V_k = x^T P_k x,\ k = 1,\ 2.$
- **Lyapunov-based Model predictive controller:**

$$J = \int_t^{t+T} \left[\|x(\tau)\|_Q^2 + \|u(\tau)\|_R^2 \right] d\tau$$

  - ◇ $Q = qI > 0,\ q = 1,\ R = rI > 0,\ r = 1$
  - ◇ Lyapunov constraint: $V_1(x(t+\Delta)) < V_1(x(t))$
- **Transition constraint:**
  - ◇ $x(t = 0.2) \in \Omega_2$

CLOSED-LOOP SIMULATION RESULTS





◇ Input & state profiles

◇ Closed-loop trajectories

◇ $x(0) = [-0.38 \;\; 0.04]^T \in \Omega_1(u_{max})$; $q = 1.0$; $r = 1.0$.

◇ MPC with $T = 0.2$, no switch; MPC with $T = 0.2$, switch at $t = 0.2$; MLF-based MPC, followed by MPC, switch at $t = 0.2$

CONCLUSIONS

- Stabilization of switched nonlinear systems with input constraints.
  - ◇ Implement a prescribed switching sequence.
- An MLF-based predictive control framework that brings together:
  - ◇ Lyapunov-based control (stability region).
  - ◇ Design & implementation of Hybrid Predictive Control structure:
    - ▷ Guaranteed stability region.
    - ▷ Enforce MLF & stability region constraints.

ACKNOWLEDGEMENT

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