

# TOWARDS BRIDGING THE GAP BETWEEN NONLINEAR CONTROL THEORY AND PROCESS CONTROL PRACTICE

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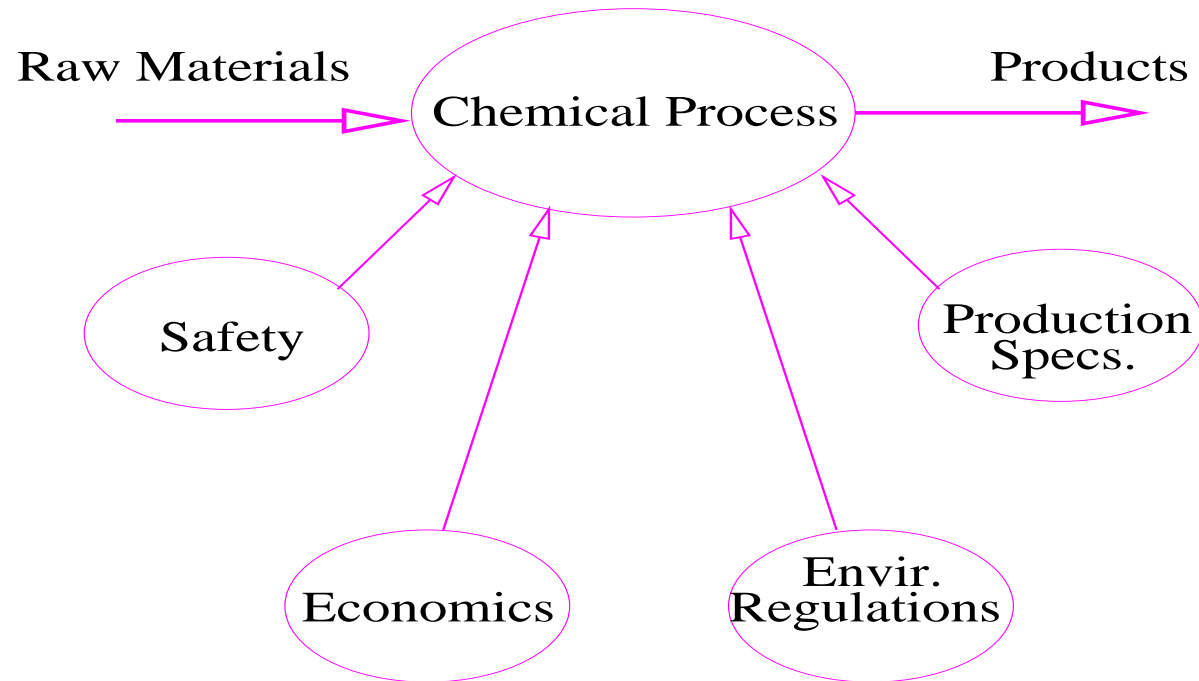


Meeting of Texas-Wisconsin  
Modeling and Control Consortium  
September 22, 2003



# INTRODUCTION

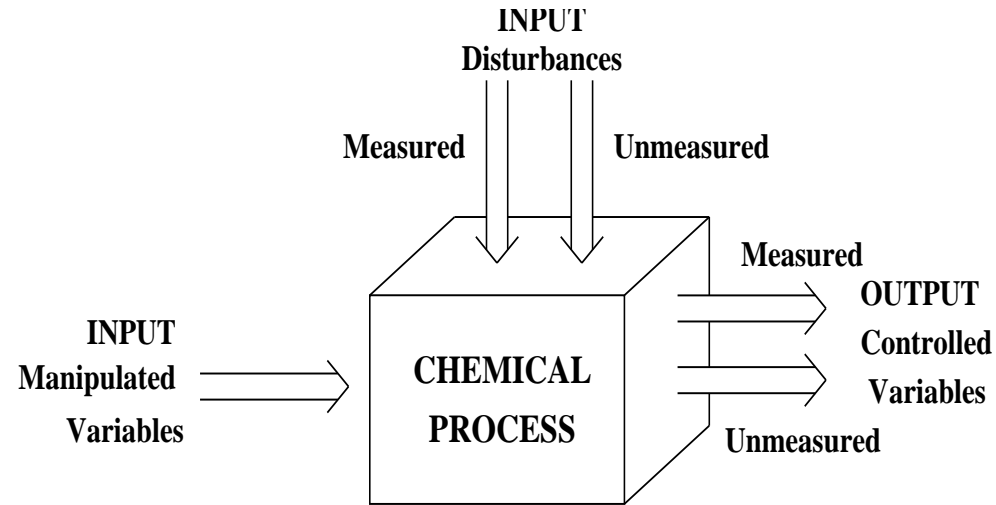
- Incentives for chemical process control.



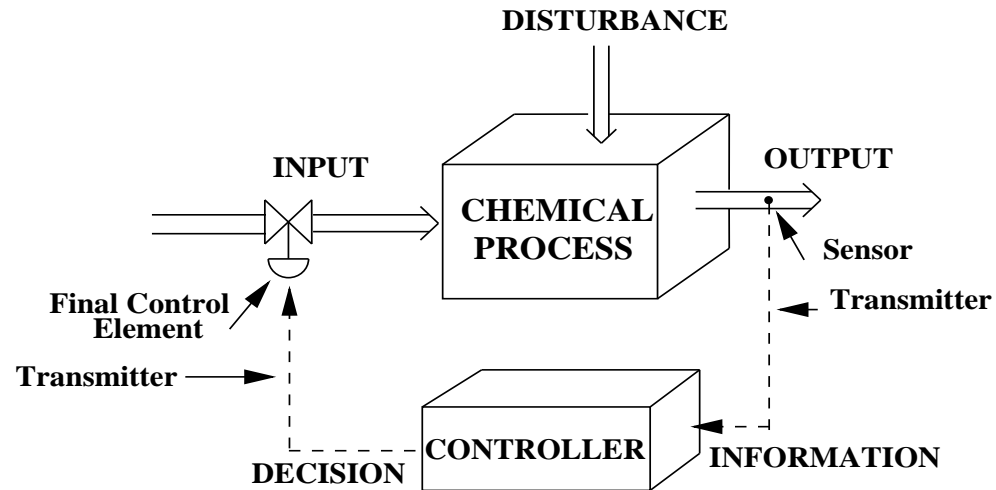
- ◇ Need for continuous monitoring and external intervention (control).
- Objectives of a process control system.
  - ◇ Ensuring stability of the process.
  - ◇ Suppressing the influence of external disturbances.
  - ◇ Optimizing process performance.

# BASIC CONCEPTS IN PROCESS CONTROL

- Process variables.

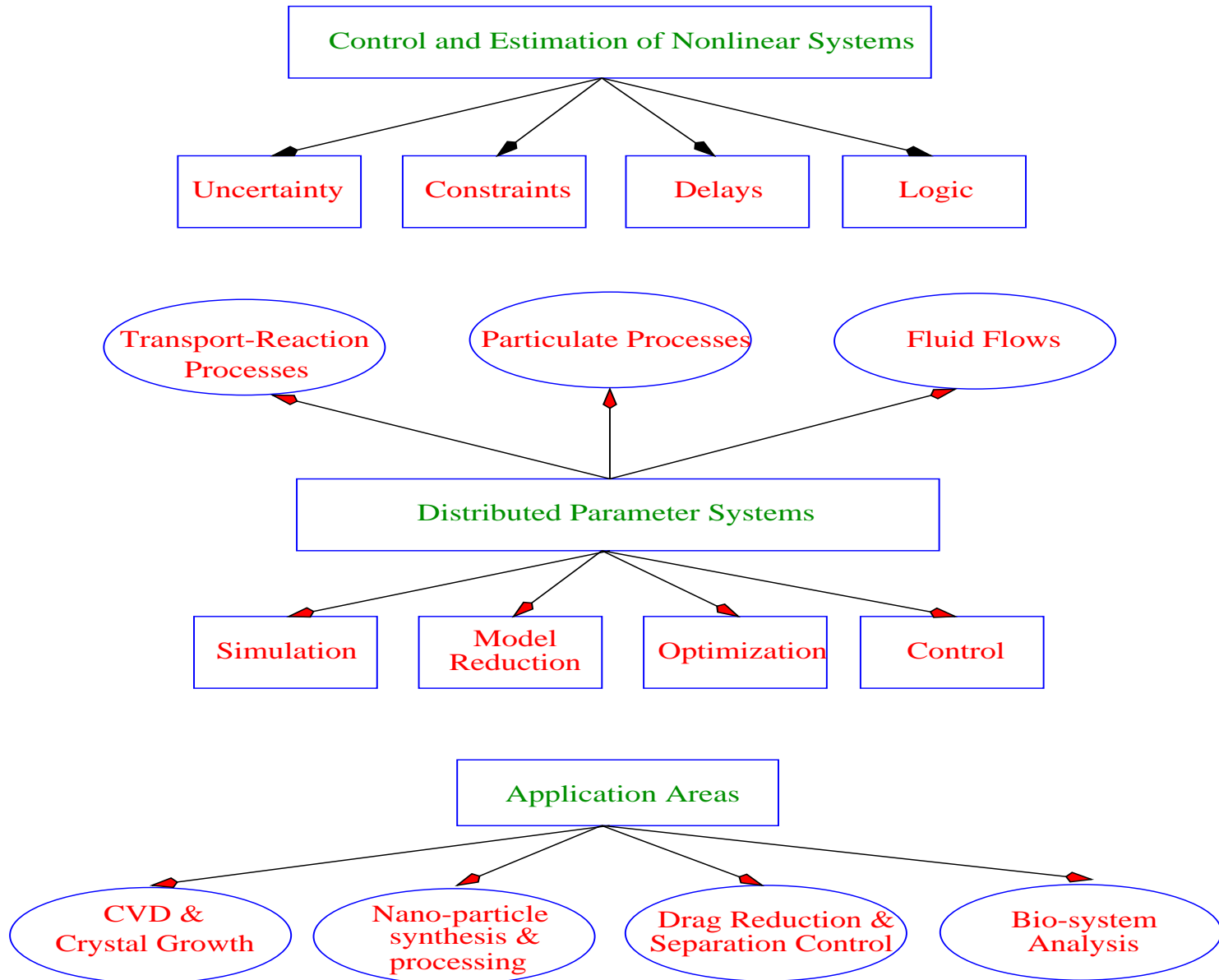


- Feedback control loop.



- ◇ Controller synthesis based on a fundamental process model.

# PROCESS CONTROL RESEARCH IN OUR GROUP



# ISSUES IN CHEMICAL PROCESS CONTROL

## ◇ Nonlinear behavior:

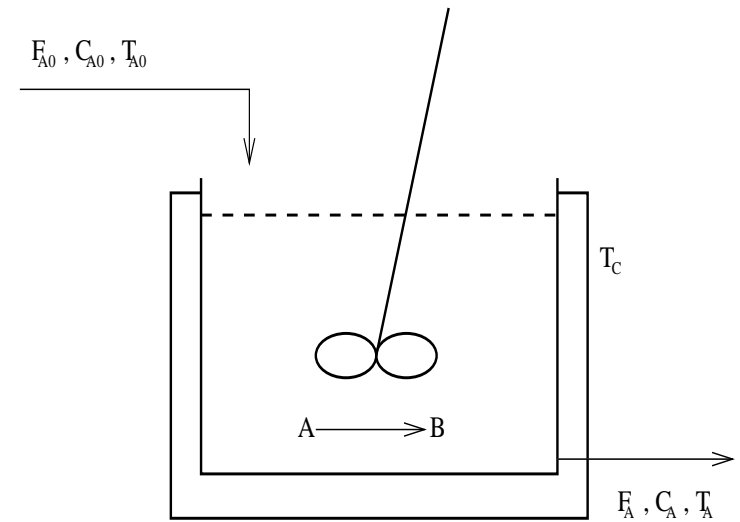
- ▷ Arrhenius dependence of reaction rates on temperature.
- ▷ Complex reaction mechanisms.

## ◇ Model uncertainties:

- ▷ Unknown process parameters.
- ▷ Exogenous disturbances.

## ◇ Input and state constraints:

- ▷ Limited capacity of control actuators.
- ▷ Operating ranges for process variables (environmental, safety, quality reasons).



Continuous-stirred tank reactor

# NONLINEAR CONTROL THEORY VS. PROCESS CONTROL PRACTICE

- Nonlinear control theory and tools:
  - ◇ Requires a nonlinear process model.
  - ◇ Geometric control, Lyapunov-based control, etc.
  - ◇ Allow rigorous analysis of closed-loop stability and performance properties.
- Process control practice:
  - ◇ Proportional-Integral-Derivative (PID) control.  
Linear Model Predictive Control (MPC).
  - ◇ Do not account for the complex dynamics of the process.
- Nonlinear control implementation requires **redesign** of control hardware:

Use nonlinear control theory to aid process control practice

## FOCUS OF THE PRESENT TALK

- Integrating Lyapunov-based control & predictive control algorithms:
  - ◇ Design of a hybrid predictive control structure.
  - ◇ Applications to linear systems under state and output feedback.
  - ◇ Application to nonlinear systems.
- Development of “nonlinear” tuning guidelines for PID controllers:
  - ◇ Optimization-based approach to make the PID controller response emulate that of a nonlinear controller.

## LINEAR SYSTEMS WITH CONSTRAINTS

- State-space description:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) \\ u(t) &\in \mathcal{U}\end{aligned}$$

- ◇  $x(t) \in \mathbb{R}^n$ , vector of process variables: concentrations, temperatures, etc.
- ◇  $u(t) \in \mathcal{U} \subset \mathbb{R}^m$ , control input: flow rates, heat input etc.
- ◇  $u = 0 \in$  interior of  $\mathcal{U}$ , nominal operating point.
- ◇  $\mathcal{U} \subset \mathbb{R}^m$ , compact & convex: limited flow rates, heat duties etc.
  - ◇  $(A, B)$  : controllable pair
  - ◇  $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$

- Stabilization of origin under constraints.



# MODEL PREDICTIVE CONTROL

(Rawlings, IEEE CSM, 2000)

- Control problem formulation

- ★ Finite-horizon optimal control:

$$P(x, t) \quad : \quad \min\{J(x, t, u(\cdot)) \mid u(\cdot) \in U_{\Delta}\}$$

- ★ Performance index:

$$J(x, t, u(\cdot)) = F(x(t+T)) + \int_t^{t+T} [\|x^u(s; x, t)\|_Q^2 + \|u(s)\|_R^2] ds$$

- ◇  $\|\cdot\|_Q$  : weighted norm

- ◇  $T$  : horizon length

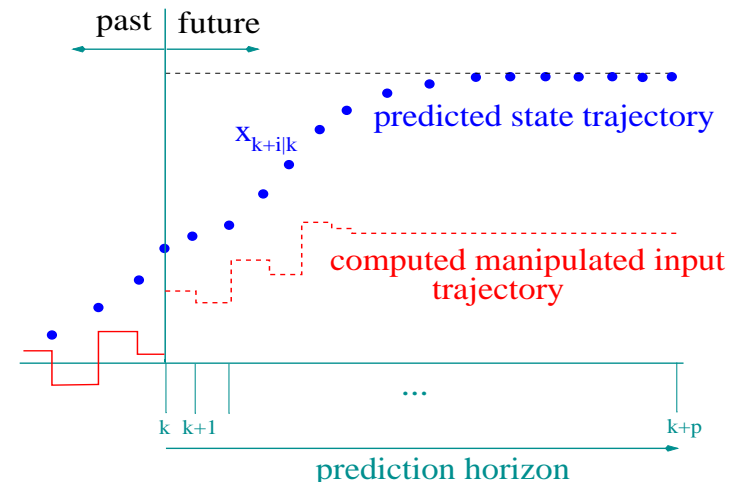
- ◇  $Q, R > 0$  : penalty weights

- ◇  $F(\cdot)$  : terminal penalty

- ★ Implicit feedback law

$$M(x) = u^0(t; x, t)$$

“repeated on-line optimization”



# MODEL PREDICTIVE CONTROL

- **Formulations for closed-loop stability:**

(Mayne et al, Automatica, 2000)

- ◇ Adjusting horizon length, terminal penalty, weights, etc.

- ◇ Imposing stability constraints on optimization:

- ▷ Terminal equality constraints:  $x(t + T) = 0$

- ▷ Terminal inequality constraints:  $x(t + T) \in W$

- ▷ Control Lyapunov functions:  $V(x(t + T)) < V(x(t))$

- **Issues of practical implementation:**

- ◇ Lack of explicit characterization of stability region:

- ▷ Extensive closed-loop simulations.

- ▷ Restrict implementation to small neighborhoods.

## BOUNDED LYAPUNOV-BASED CONTROL

- **Explicit bounded nonlinear control law:**

$$u = -k(x, u_{max})(L_G V)^T$$

◇ An example gain: (Lin & Sontag, 1991)

$$k(x, u_{max}) = \left( \frac{L_f V + \sqrt{(L_f V)^2 + (u_{max} \|(L_G V)^T\|)^4}}{\|(L_G V)^T\|^2 \left[ 1 + \sqrt{1 + (u_{max} \|(L_G V)^T\|)^2} \right]} \right)$$

$$\begin{aligned} V &= x^T P x, \quad A^T P + P A - P B B^T P < 0 \\ L_f V &= x^T (A^T P + P A) x, \quad L_G V = 2x^T P B \end{aligned}$$

◇ Nonlinear gain-shaping procedure:

★ **Accounts explicitly for constraints & closed-loop stability.**

- **Constrained closed-loop properties:**

◇ **Asymptotic stability.**

◇ **Inverse optimality.**

# CHARACTERIZATION OF STABILITY PROPERTIES

(El-Farra & Christofides, Chem. Eng. Sci., 2001, 2003)

$$D(u_{max}) = \{x \in \mathbb{R}^n : L_f V < u_{max} | (L_G V)^T |\}$$

- **Properties of inequality:**

- ◇ Describes open unbounded region where:

- ▷  $|u| \leq u_{max} \quad \forall x \in D$

- ▷  $\dot{V} < 0 \quad \forall 0 \neq x \in D$

- ◇ Captures constraint-dependence of stability region.

- ◇  $D$  not necessarily invariant.

- **Region of guaranteed closed-loop stability:**

$$\Omega(u_{max}) = \{x \in \mathbb{R}^n : V(x) \leq c_{max}\}$$

- ◇ Region of invariance:  $x(0) \in \Omega \implies x(t) \in \Omega \subset D \quad \forall t \geq 0$ .

- ◇ Provides larger estimate than saturated linear/nonlinear controllers.

# UNITING BOUNDED CONTROL AND MPC

(El-Farra, Mhaskar & Christofides, Automatica, to appear)

- Objectives:

- ◇ Development of a framework for merging the two approaches:
  - ▷ Reconcile tradeoffs in stability and optimality properties
    - ★ Explicit characterization of constrained stability region.
    - ★ Efficient implementation of MPC.
    - ★ Possibility of improved performance.

- Central idea:

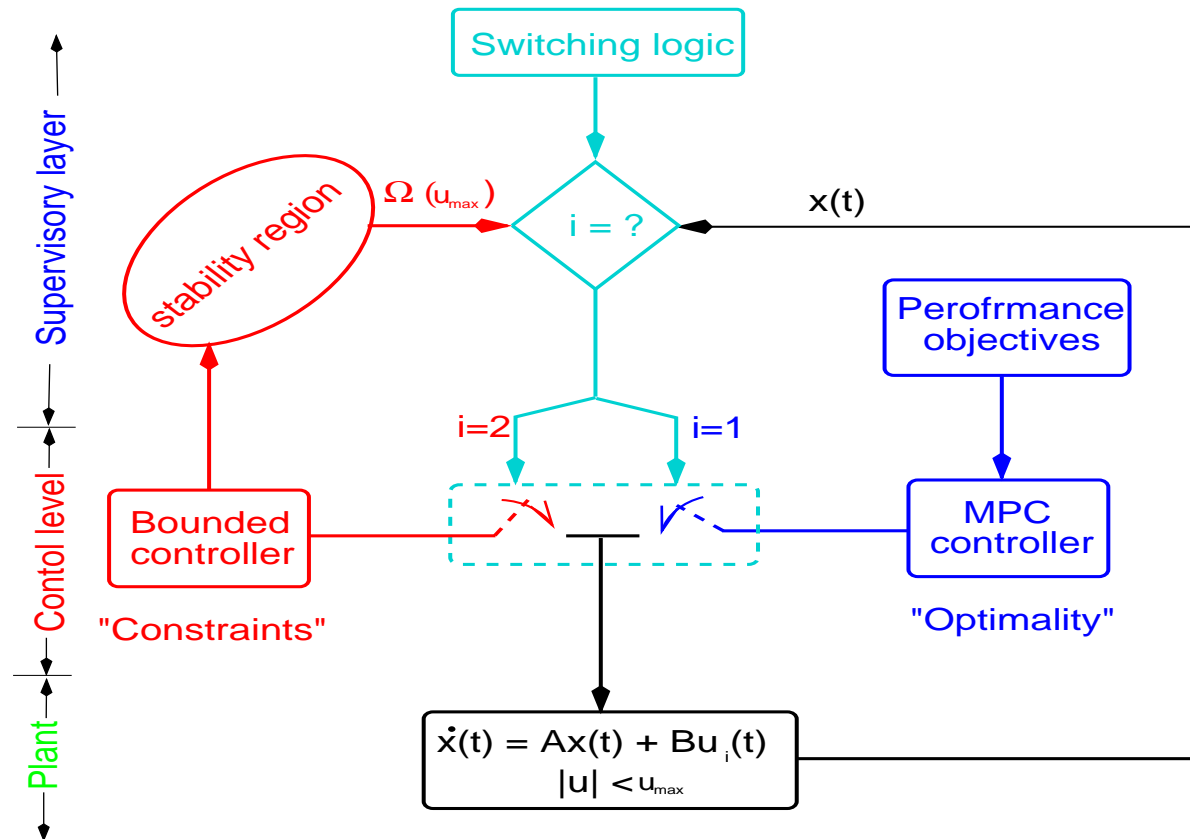
Decoupling “optimality” & “constrained stabilizability”

- ◇ Stability region provided by bounded controller.
- ◇ Optimal performance provided by MPC.

- Approach:

- ◇ Switching between MPC & bounded controller.

# HYBRID PREDICTIVE CONTROL STRUCTURE



- Hierarchical control structure
  - ◇ Plant level
  - ◇ Control level
  - ◇ Supervisory level
- Overall structure **independent** of specific MPC algorithm used:
  - ◇ Switching rules may vary.
- Numerous variants of switching scheme possible.

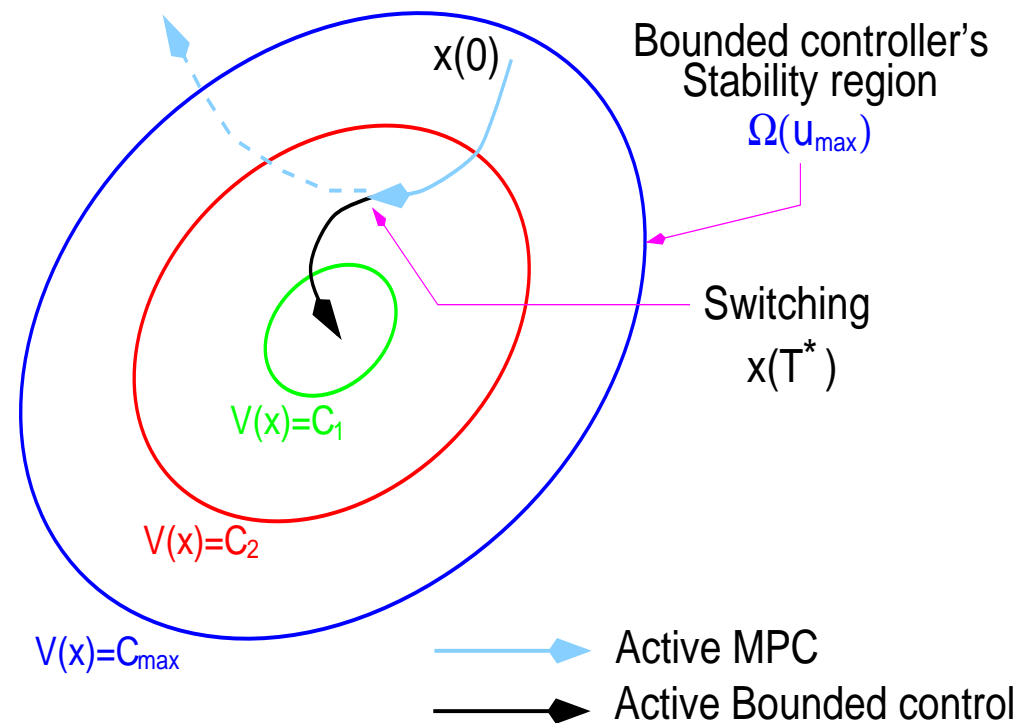
# STABILITY-BASED CONTROLLER SWITCHING

- Switching logic:

$$u_{\sigma}(x(t)) = \begin{cases} M(x(t)), & 0 \leq t < T^* \\ b(x(t)), & t \geq T^* \end{cases}$$

$$T^* = \inf \{ T^* \geq 0 : -\|x^M(T^*)\|_{\bar{Q}}^2 + \|B^T P x^M(T^*)\|^2 + 2x^{M^T}(T^*) P B u(T^*) \geq 0 \}$$

- ◇ Initially implement MPC,  $x(0) \in \Omega(u_{max})$ .
- ◇ Monitor temporal evolution of  $V(x^M(t)) = x^T(t) P x(t)$ .
- ◇ Switch to bounded controller only if  $V(x^M(t))$  starts to increase.



# STABILITY-BASED CONTROLLER SWITCHING

- Implications of switching scheme:

- ◇ Switched closed-loop inherits bounded controller's stability region:

- ▷ A priori guarantees for all  $x(0) \in \Omega(u_{max})$ .

- ◇ Lyapunov stability condition checked & enforced by “supervisor”:

- ▷ Reduce computational complexity of optimization.

- ▷ Scheme does not require stability of MPC within  $\Omega(u_{max})$ .

- ▷ Provides a safe mechanism for implementing MPC.

- ▷ Stability independent of horizon length.

- Conceptual differences from other schemes:

- ◇ Switching does not occur locally.

- ◇ Provides stability region explicitly.

- ◇ No switching occurs if  $V(x^M(t))$  decays continuously:

- ▷ Only MPC is implemented  $\implies$  optimal performance recovered.



# ENHANCING CLOSED-LOOP PERFORMANCE

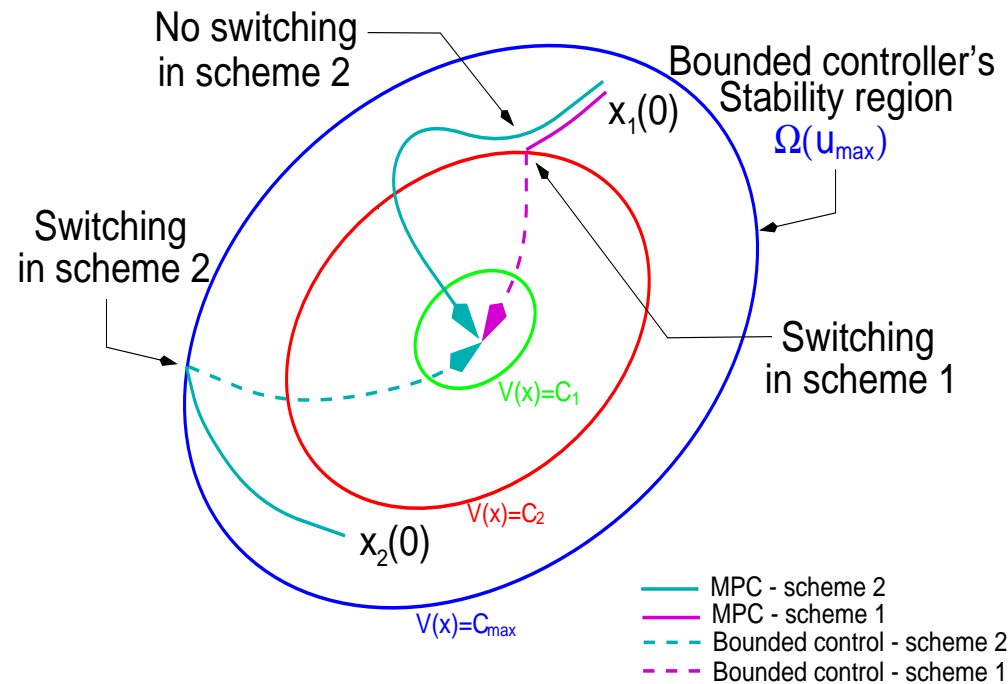
- Relaxing switching rules:

$$u_{\sigma}(x(t)) = \begin{cases} M(x(t)), & 0 \leq t < \min\{T^*, T_N\} \\ b(x(t)), & t \geq \min\{T^*, T_N\} \end{cases}$$

$$T^* = \inf\{T^* \geq 0 : V(x^M(T^*)) = c_{max}\}$$

$$T_N = \sup\{T_i \geq 0 : \dot{V}(x^M(T_i)) = 0, i = 1, \dots, N < \infty\}$$

- ◇ Allow increases in  $V(x^M(t))$ .
- ◇ Stability safeguards: switch if
  - ▷ Trajectory hits  $\partial\Omega(u_{max})$ , or
  - ▷  $\dot{V}(x(t))$  changes sign  $N$  times.
- ◇ Greater flexibility for MPC implementation.



# SWITCHING WITH “STABLE” MPC FORMULATIONS

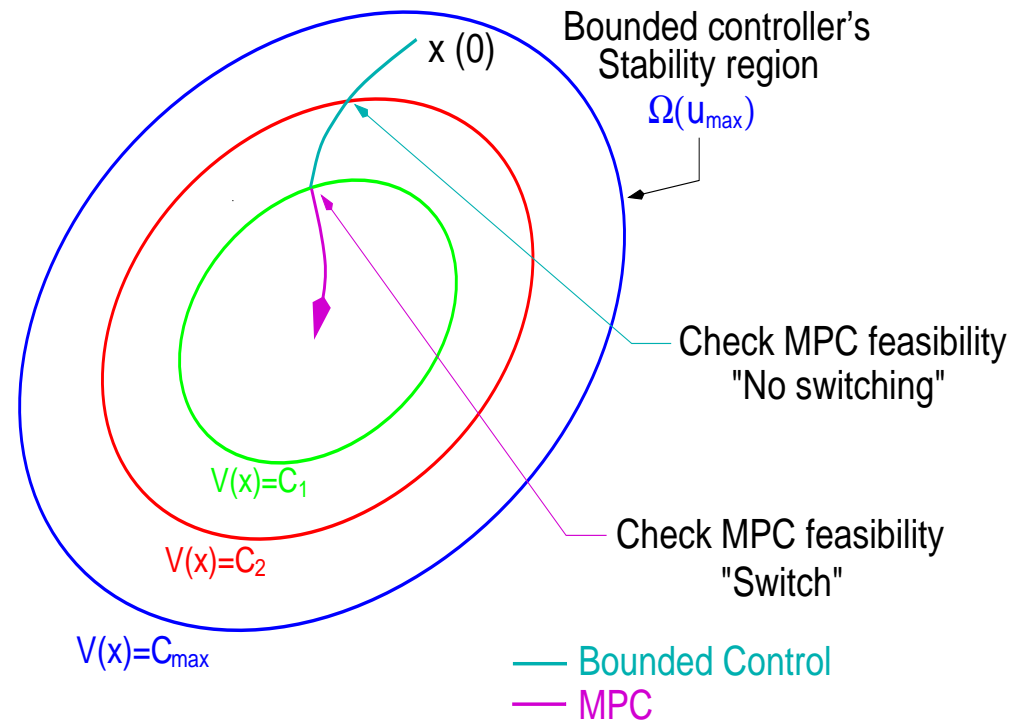
- MPC with stability constraints:

$$\begin{aligned}
 & \min_{u(\cdot)} \int_t^{t+T} (x^T(s)Qx(s) + u^T(s)Ru(s))ds \\
 \text{s.t. } & \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \\
 & u(\cdot) \in U_{\Delta} \\
 & x(T) \in \Omega_{Terminal}
 \end{aligned}$$

- Feasibility-based switching:

$$u_{\sigma}(x(t)) = \left\{ \begin{array}{l} b(x(t)), \quad 0 \leq t < T^* \\ M(x(t)), \quad t \geq T^* \end{array} \right\}$$

- ◇  $T^*$  : earliest time for which MPC yields feasible solution.
- ◇ Frequent off-line supervisory “checks” of MPC feasibility.



## NUMERICAL EXAMPLE

- State space description:

$$\dot{x} = \begin{bmatrix} 0.5 & 0.25 \\ 0.5 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

- ◇ Origin unstable (two real positive eigenvalues).
- ◇ Input constraints:  $u_i \in [-5, 5]$ ,  $i = 1, 2$ .

- Bounded controller:

- ◇ Lyapunov function:  $V = x^T P x$ ,  $P = \begin{bmatrix} 1.03 & 0.306 \\ 0.306 & 1.18 \end{bmatrix}$
- ◇ Stability region:  $x^T (A^T P + P A) x < 10 \|B^T P x\|$

- Model predictive controller:

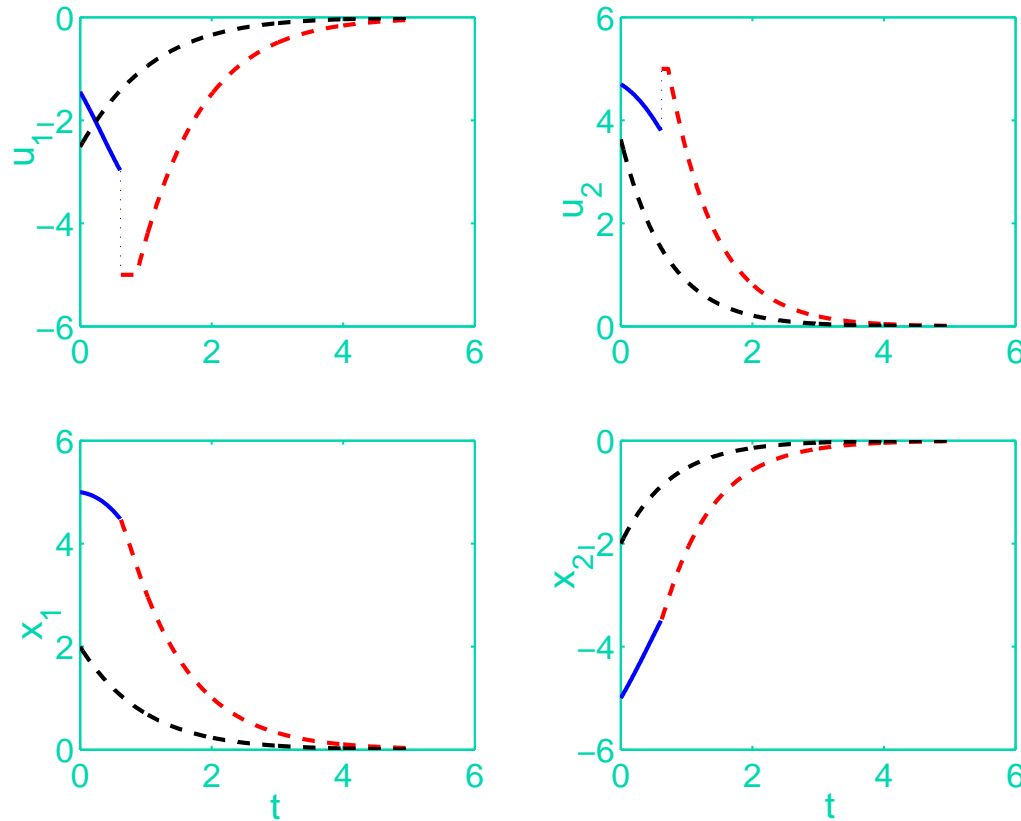
- ◇ Performance index:

$$J = \int_t^{t+T} [\|x(\tau)\|_Q^2 + \|u(\tau)\|_R^2 + \|\dot{u}(\tau)\|_S^2] d\tau$$

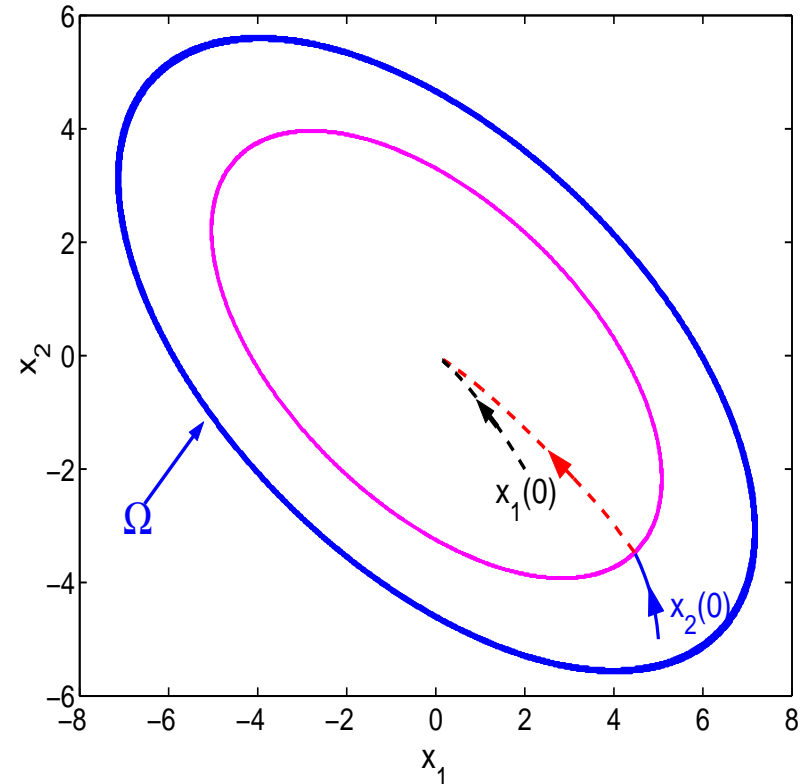
$$Q = qI > 0, R = rI > 0, S = sI > 0$$

# CLOSED-LOOP SIMULATION RESULTS

“Feasibility-based switching”



★ Input & state profiles



★ Closed-loop trajectories

◇  $x_1(0) = [2 \ -2]^T \in \Omega \implies$  MPC ( $q = 1; r = 4; T = 1$ ) feasible

◇  $x_2(0) = [5 \ -5]^T \in \Omega \implies$  MPC ( $q = 1; r = 4; T = 1$ ) NOT feasible

★ Use bounded controller

★ Switch to MPC at  $t = 0.6$

# STATE ESTIMATION & OUTPUT FEEDBACK CONTROL

- **Lack of full state measurements:**
  - ◇ Inaccessibility of some process variables for measurement.
  - ◇ Estimation of states from measured outputs necessary.
- **Main objectives for output feedback controller design:**
  - ◇ To establish guaranteed stability from an explicitly characterized set of initial conditions:
    - ▷ Design technique for the state estimator.
    - ▷ Devise switching rules, based on available state measurements.
  - ◇ Controlled rate of convergence of state estimation error.
- **Approach for output feedback controller design:**
  - ◇ Relies on separation principle: combination of
    - ▷ State feedback controllers.
    - ▷ State observers.

# DESIGN OF STATE OBSERVER

- State space description of estimator:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y - C\hat{x})$$

- ◇  $\hat{x}$ : state estimate.

- ◇  $L$ : Observer gain matrix.

- Structural features:

- ◇ Observer pole placement:

- ▷ Enforce desired decay of estimation error

- ◇ Effect of observer peaking eliminated through:

- ▷ Input saturation (indirect).

- ▷ Estimate-saturation (direct) (e.g., El-Farra & Christofides, IJC, 2001).

- Closed-loop analysis:

- ◇ Fall back (bounded controller) robust to a certain allowable error.

- ◇ For a given choice of initial conditions:

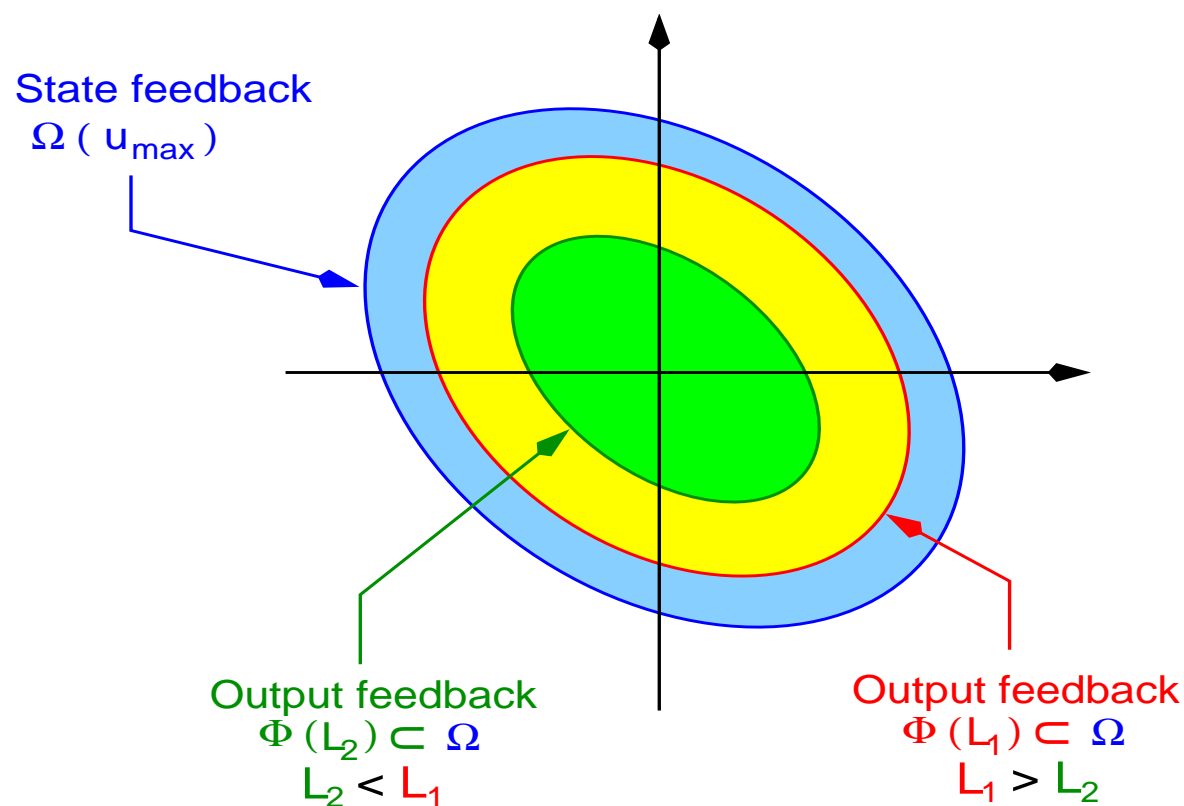
- ▷ State estimator designed to force error under the allowable error.

- ▷ Switching laws, based on the state estimates, for “safe” MPC implementation.

# STATE ESTIMATION & OUTPUT FEEDBACK CONTROL

- Practical implications:

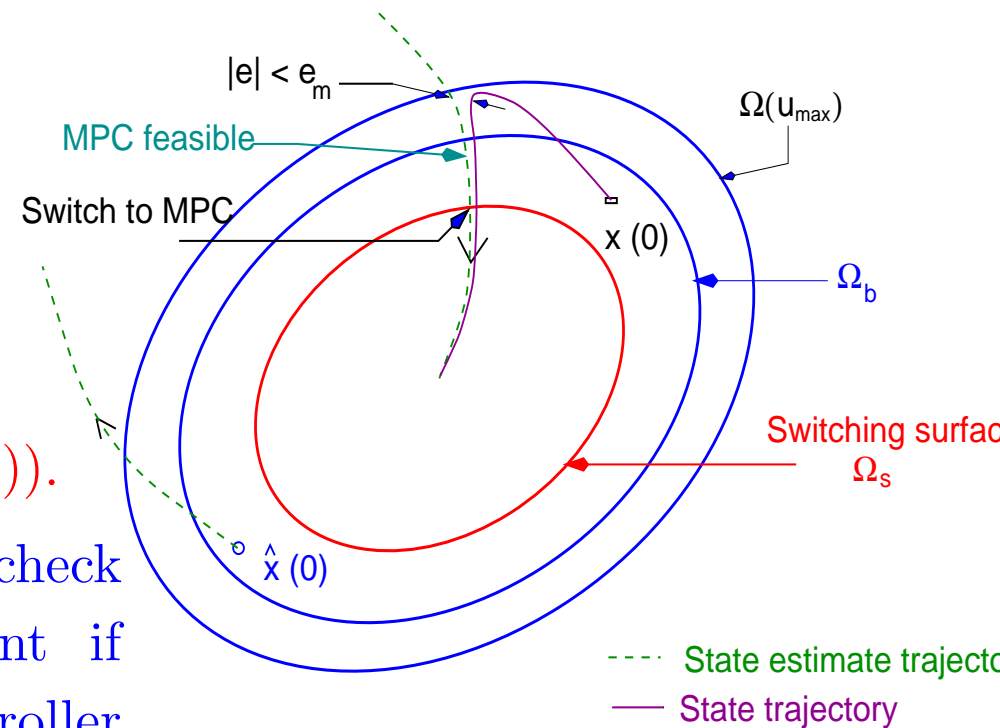
- ◇ Any other estimation scheme, such as Moving Horizon Estimation (MHE), can be used.
- ◇ Requires a transparent relationship between error decay and estimator parameters.
- ◇ MPC implemented in a region where the fall back controller can step in any time to rescue stability.



# OUTPUT FEEDBACK IMPLEMENTATION OF SWITCHING

(Mhaskar, El-Farra & Christofides, AIChE J., to appear)

- Bounded controller design  
( $u = -k(x)L_g V, \Omega(u_{max})$ ).
- State observer design  
(given  $\Omega_b \subset \Omega$ , compute  $L$ ).
- Estimate 'safe' region  
(given  $\hat{x} \in \Omega_s \Rightarrow x \in \Omega$ ).
- Initialize:  $\hat{x}(0) \in \Omega_b, u(0) = b(\hat{x}(0))$ .
- After  $\hat{x}$  enters 'safe' region,  $\Omega_s$ , check feasibility of MPC & implement if  $\dot{V}(\hat{x}) < 0$  else keep bounded controller active.





# NONLINEAR SYSTEMS WITH INPUT CONSTRAINTS

- State–space description:

$$\begin{aligned}\dot{x}(t) &= f(x(t)) + g(x)u(t) \\ u(t) &\in \mathcal{U}\end{aligned}$$

- ◇  $x(t) \in \mathbb{R}^n$  : state vector
- ◇  $u(t) \in \mathcal{U} \subset \mathbb{R}^m$  : control input
- ◇  $\mathcal{U} \subset \mathbb{R}^m$ : compact & convex
- ◇  $u = 0 \in$  interior of  $\mathcal{U}$
- ◇  $(0, 0)$  an equilibrium point

- Stabilization of origin under constraints

## NONLINEAR MPC: IMPLEMENTATION ISSUES

- Optimization problem non-convex:
  - ◇ Does not reduce to a Quadratic Program.
  - ◇ Possibility of multiple, local optima.
  - ◇ Optimization problem hard to solve (e.g., algorithm failure).
  - ◇ Stability requirements harder to implement.
  - ◇ Difficult to obtain solution within “reasonable” time.
- Lack of explicit characterization of stability region:
  - ◇ Extensive closed-loop simulations.
  - ◇ Restrict implementation to small neighborhoods.

## BOUNDED LYAPUNOV-BASED CONTROL

- **Explicit bounded nonlinear control law:**

$$u = -k(x, u_{max})(L_G V)^T$$

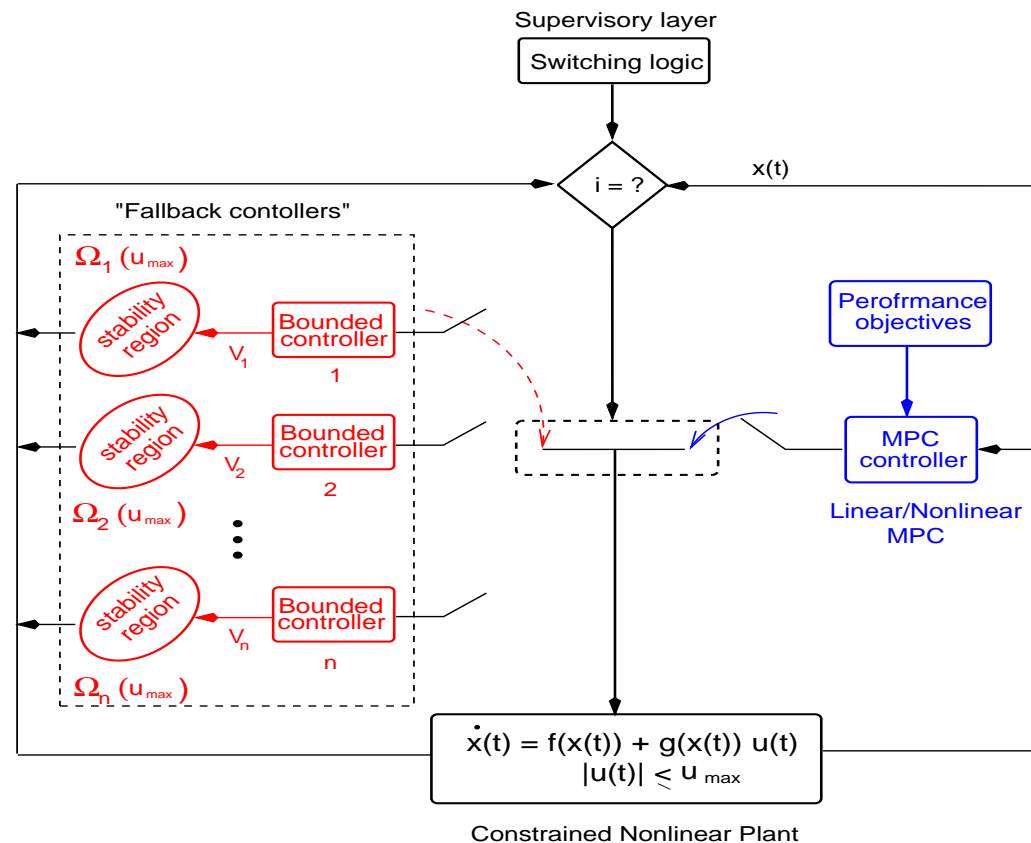
- **Region of guaranteed closed-loop stability:**

$$\Omega(u_{max}) = \{x \in \mathbb{R}^n : V(x) \leq c_{max}\}$$

- ◇ Region of invariance:  $x(0) \in \Omega \implies x(t) \in \Omega \subset D \forall t \geq 0$ .
- ◇ Larger estimates using a combination of several Lyapunov functions.
- ◇ Other Lyapunov-based bounded control designs can be used.

# HYBRID PREDICTIVE CONTROL STRUCTURE

(El-Farra, Mhaskar & Christofides, IJRNC, to appear)



- **Hierarchical control structure**

- ◇ Plant level
- ◇ **Control level**
- ◇ Supervisory level

- Overall structure **independent** of specific MPC algorithm used:

- ◇ Could use linear/nonlinear MPC with or without stability constraints.

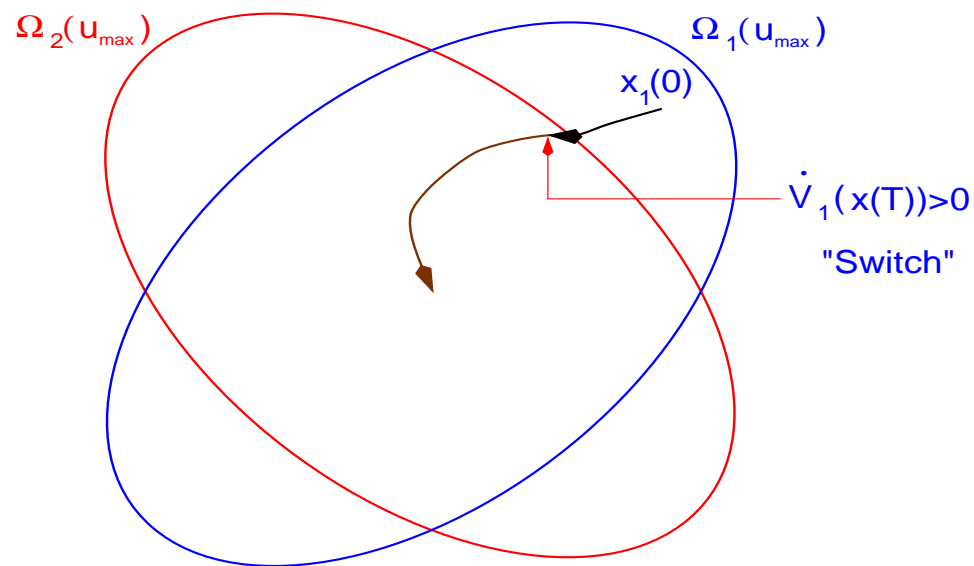
# STABILITY-BASED CONTROLLER SWITCHING

- Switching logic:

$$u_{\sigma}(x(t)) = \begin{cases} M(x(t)), & 0 \leq t < T^* \\ b(x(t)), & t \geq T^* \end{cases}$$

$$L_f V_k(x) + L_g V_k(x) M(x(T^*)) \geq 0$$

- ◇ Initially implement MPC,  $x(0) \in \Omega_k(u_{max})$ .
- ◇ Monitor temporal evolution of  $V_k(x^M(t))$ .
- ◇ Switch to bounded controller only if  $V_k(x^M(t))$  starts to increase.



 MPC  
 Bounded control

# PRACTICAL IMPLEMENTATION

- A “typical” predictive control design:

- ◇ Nonlinear process model:

$$\dot{x} = f(x) + g(x)u$$
$$u_{min}^i \leq u_i \leq u_{max}^i$$

- ◇ Linear representation:

$$\dot{x} = Ax + Bu$$
$$u_{min}^i \leq u_i \leq u_{max}^i$$

- ▷ Linearization

- (around desired steady-state)

- ▷ Model identification

- (e.g., through step tests)

- ◇ Use of computationally efficient linear MPC (QP) algorithms.

- ◇ No closed-loop stability guarantees for nonlinear system.

- Practical value of the hybrid control structure:

- ◇ Provides stability guarantees through fall-back controllers.

- ◇ Entails no modifications in existing predictive controller design.

## APPLICATION TO A CHEMICAL REACTOR

- Process dynamic model:

$$\begin{aligned}\dot{C}_A &= \frac{F}{V}(C_{A0} - C_A) - k_0 e^{\frac{-E}{RT_R}} C_A \\ \dot{T}_R &= \frac{F}{V}(T_{A0} - T_R) + \frac{(-\Delta H)}{\rho c_p} k_0 e^{\frac{-E}{RT_R}} C_A + \frac{UA}{\rho V c_p} (T_c - T_R)\end{aligned}$$

- ◇ Multiple steady-states.
- ◇ Control objective:
  - ▷ Stabilization at open-loop unstable equilibrium point,  
 $(C_{As}, T_s) = (0.52 \text{ mol/L}, 398 \text{ K})$ .
- ◇ Manipulated input:  $u = T_c \in [275, 370]$ .

# CONTROLLER DESIGN

- **Model predictive controller:**

- ◇ Performance index:

$$J = \int_t^{t+T} [\|x(\tau)\|_Q^2 + \|u(\tau)\|_R^2 + \|\dot{u}(\tau)\|_S^2] d\tau$$

$$Q = qI > 0, R = rI > 0, S = sI > 0$$

- ◇ Prediction model:

$$\dot{x} = Ax + Bu$$

- ▷  $(A, B)$  from linearizing the nonlinear model around  $(C_{As}, T_s)$ .

- ◇ Terminal equality constraint:  $x(t + T) = 0$ .

- **Family of bounded controllers:**

- ◇ Designed using a normal form representation.

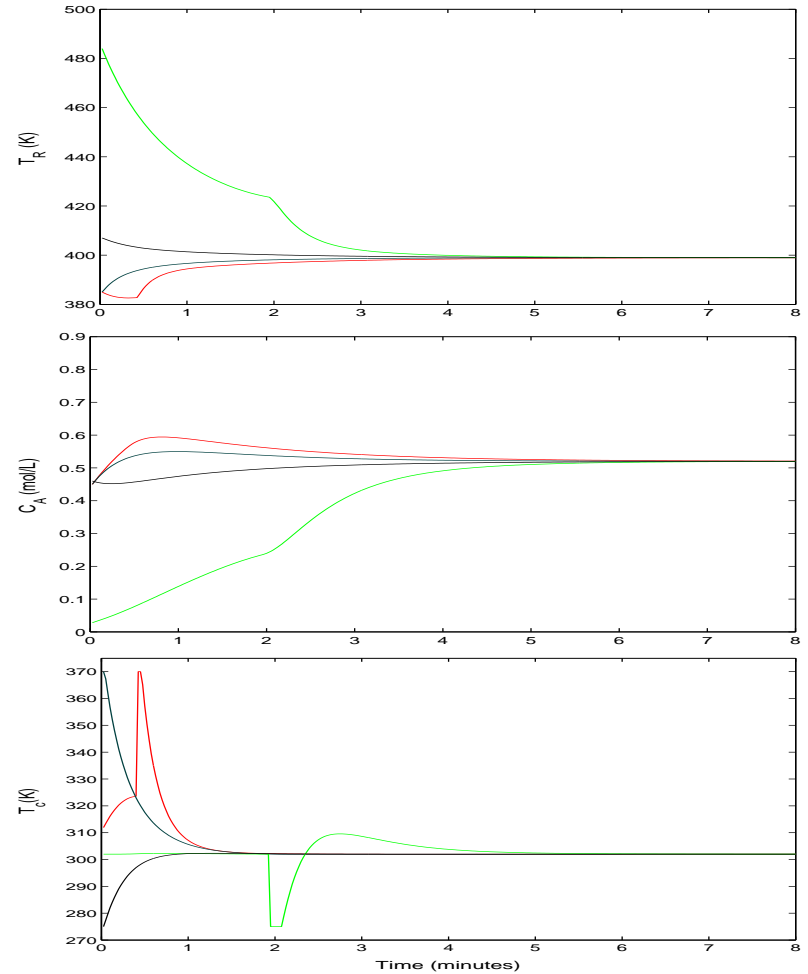
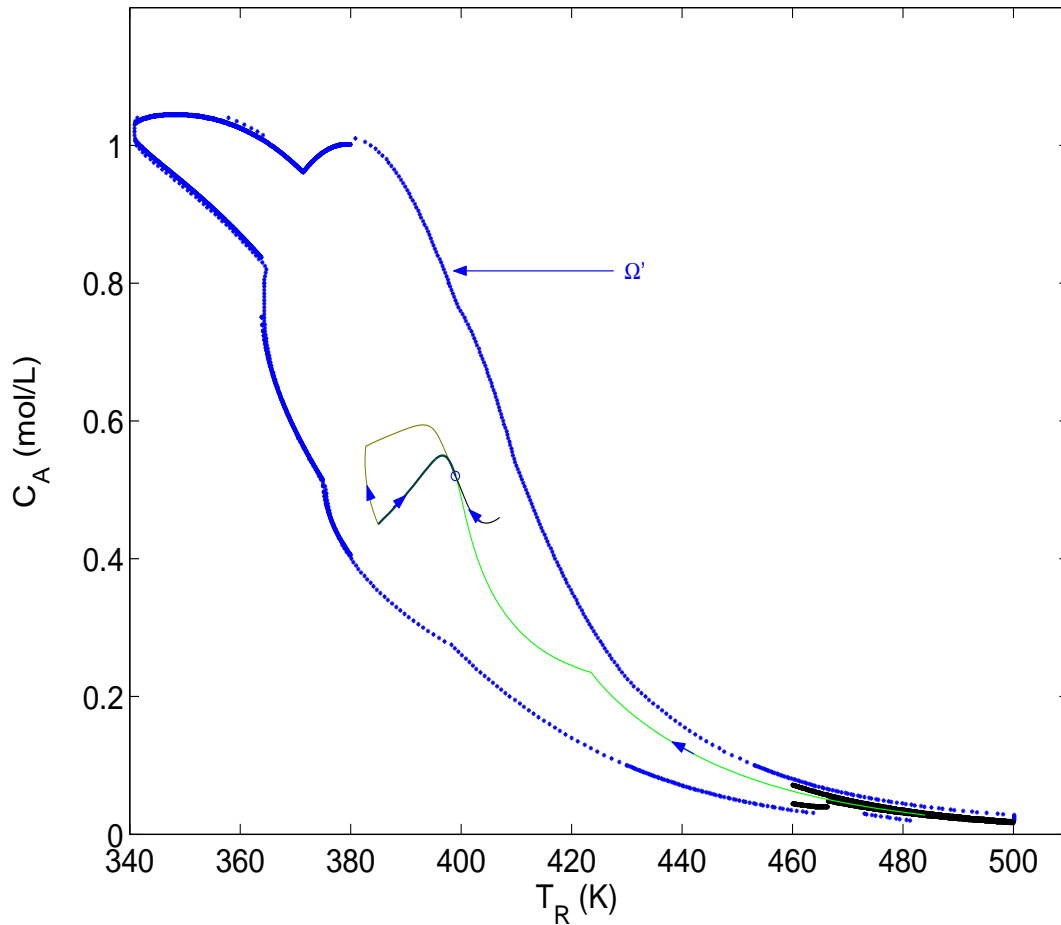
- ◇ Several Lyapunov functions:  $V_k = \xi^T P_k \xi, k = 1, 2, 3, 4$ .

- ◇ Stability region:  $\Omega = \bigcup_{k=1}^4 \Omega_k$



# CLOSED-LOOP SIMULATION RESULTS

“Stability-based switching”



◇ MPC with  $T = 0.25$ ; MPC/BC(4) switching ( $t = 0.4$ ).

◇ MPC with  $T = 0.5$ ; MPC/BC(3) switching ( $t = 1.9$ ).

## APPLICATION TO A CONTINUOUS CRYSTALLIZER

- Crystallizer moments model:

$$\begin{aligned} \dot{x}_0 &= -x_0 + (1 - x_3)Da \exp\left(\frac{-F}{y^2}\right) \\ \dot{x}_1 &= -x_1 + yx_0 \\ \dot{x}_2 &= -x_2 + yx_1 \\ \dot{x}_3 &= -x_3 + yx_2 \\ \dot{y} &= \frac{1 - y - (\alpha - y)yx_2}{1 - x_3} + \frac{u}{1 - x_3} \end{aligned}$$

- ◇ Unstable equilibrium point surrounded by stable limit cycle.

- ◇ Control objective:

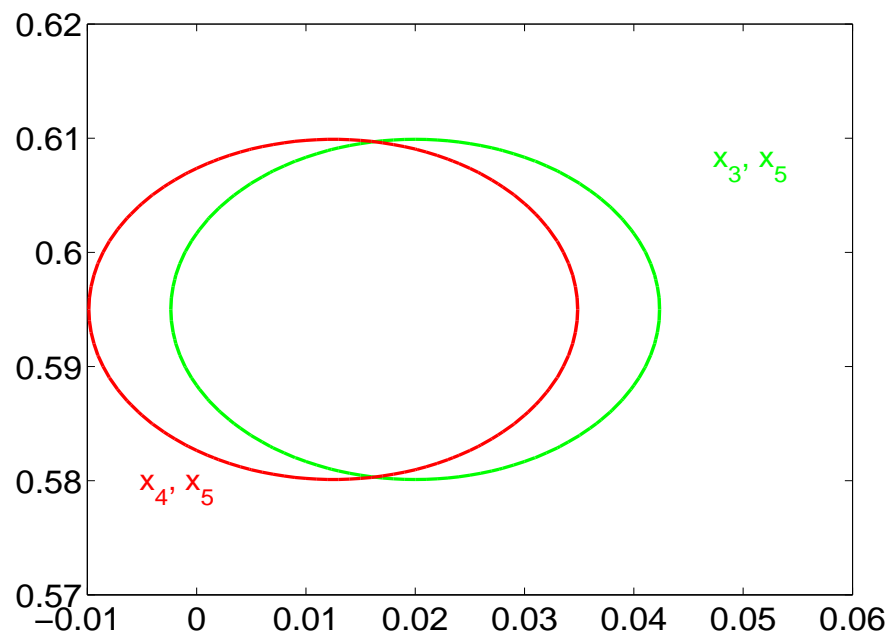
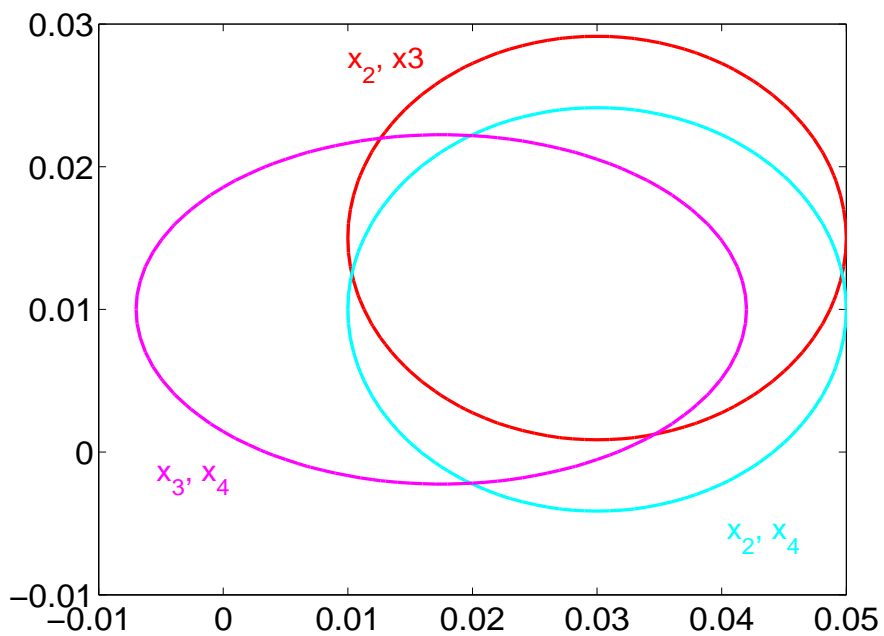
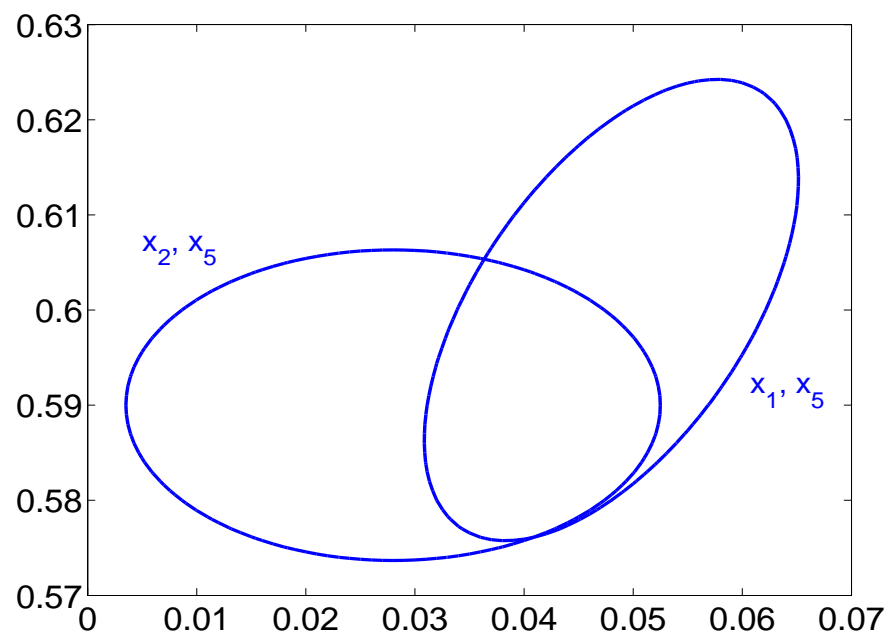
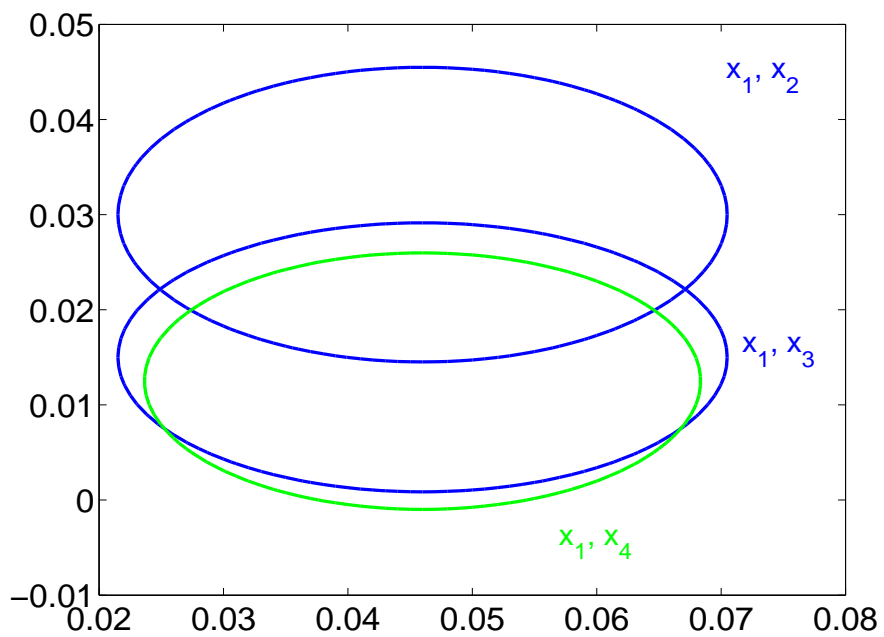
- ▷ Stabilization at unstable equilibrium point.

- ▷ Input constraints:  $u \in [-1, 1]$ .

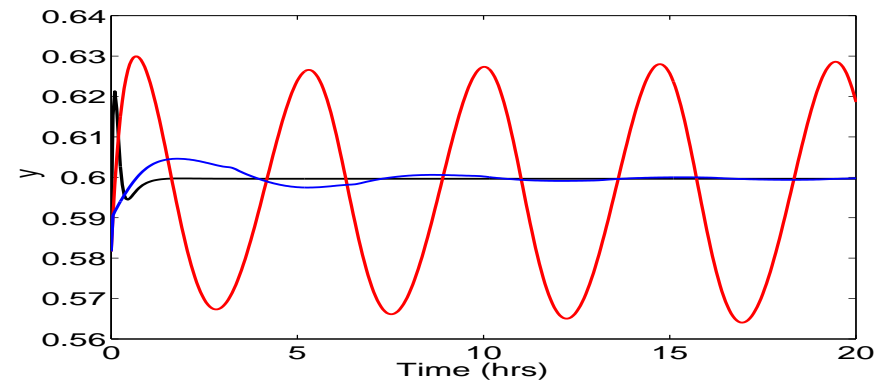
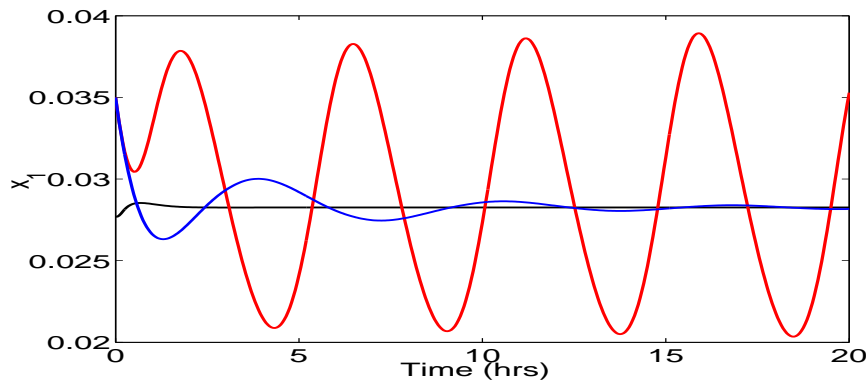
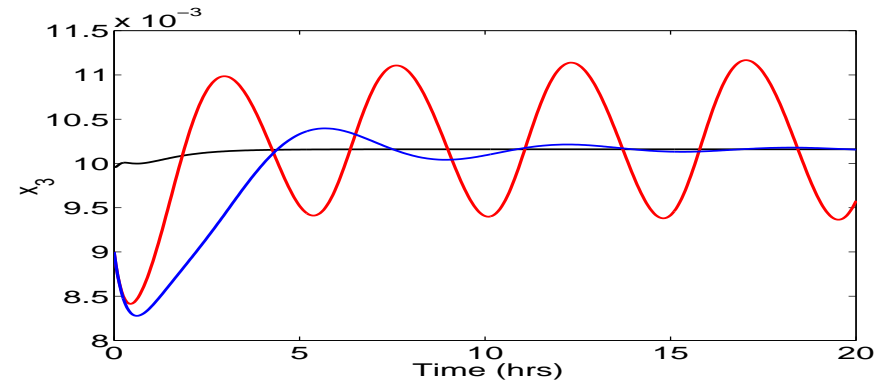
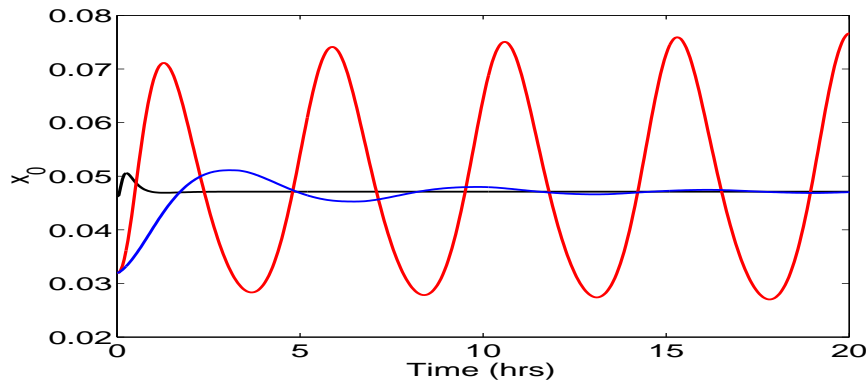
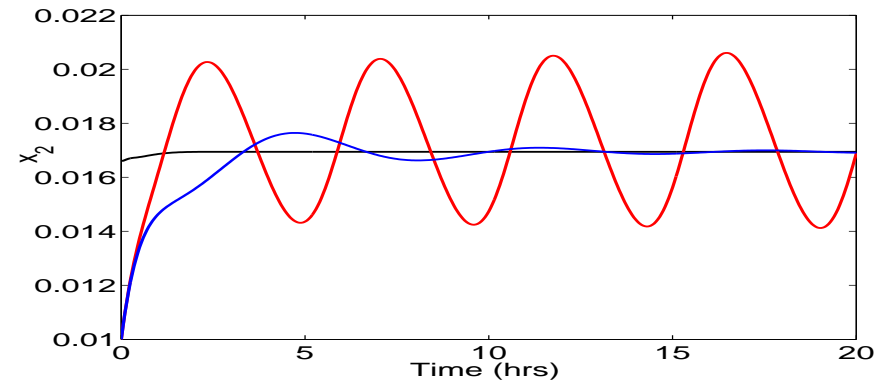
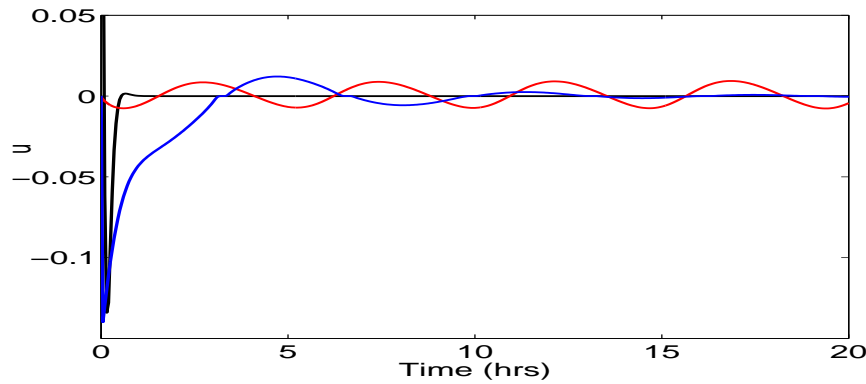
- Bounded controller: designed using normal form.

- Predictive controller: linear prediction model with stability constraints.

# PROJECTIONS OF STABILITY REGION



# CLOSED-LOOP SIMULATION RESULTS



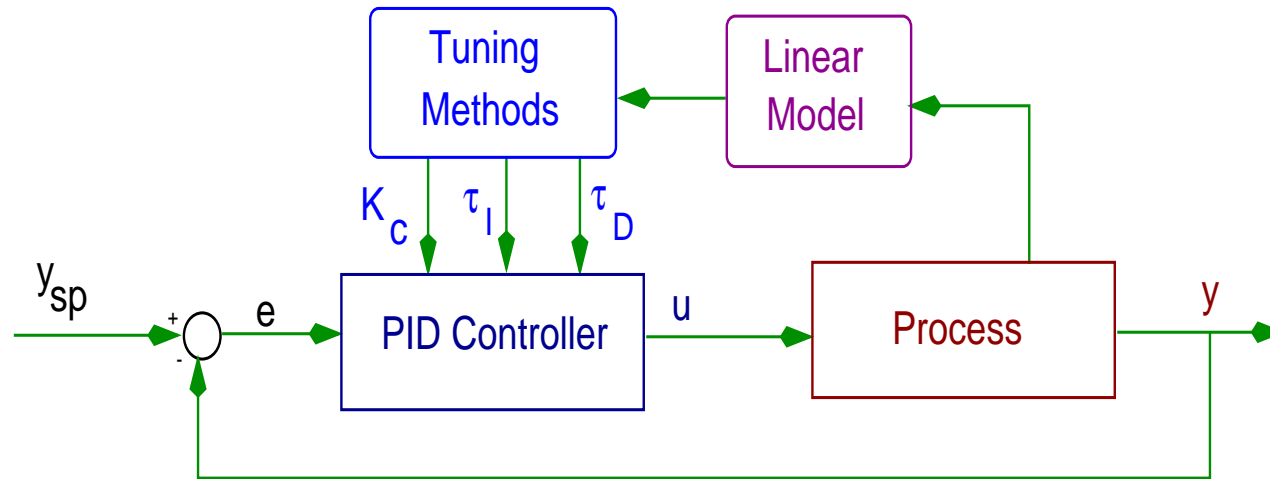
◇  $x_1(0)$ : MPC with  $T = 0.25$  feasible

◇  $x_2(0)$ : MPC with  $T = 0.25$  (no terminal constraints)

◇  $x_2(0)$ : switching to bounded controller

# TUNING CLASSICAL CONTROLLERS USING NONLINEAR CONTROL THEORY

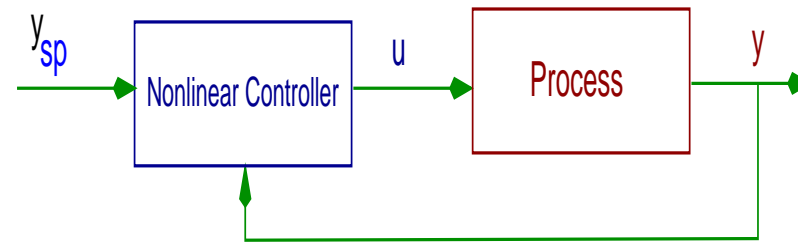
- Proportional-Integral-Derivative (PID) control:



- “Easy” to use and implement:
  - ◇ Tuning rules based on linear process models.
- Do not account for
  - ◇ Process nonlinearities, uncertainties, constraints etc.
- Extensive re-tuning / poor performance.

# TUNING CLASSICAL CONTROLLERS USING NONLINEAR CONTROL THEORY

- Nonlinear controllers:



- ◇ Handle process uncertainties/time delays/state estimation/...
- ◇ Provide rigorous results and analysis.
- ◇ Require better understanding of the process (nonlinear models).
- ◇ Implementation requires **redesign** of existing control hardware.

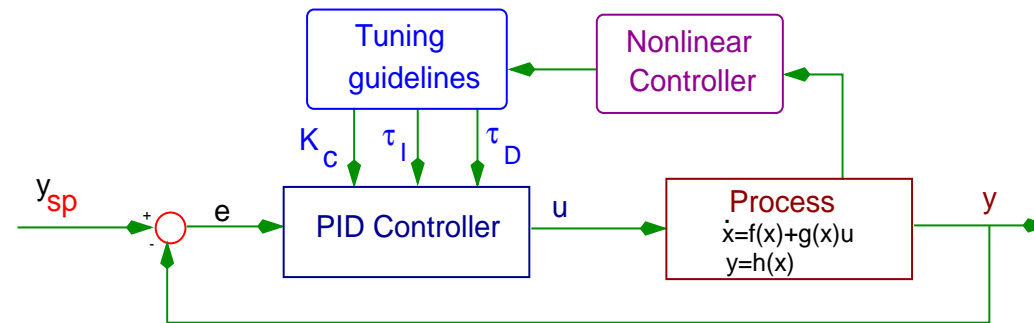
- Gap between

- ▷ Nonlinear control theory tools.
- ▷ Process control practice ( $K_c, \tau_i, \tau_d$ ).

Use/develop nonlinear control tools for PID controller tuning

# TUNING METHOD

- Tuning method:



- ◇ Design a nonlinear controller that accounts for the complex process dynamics.
- ◇ Compute, **but not implement**, the control action as prescribed by the nonlinear controller.
- ◇ Set up and solve an optimization problem:
  - ▷ The objective function ‘measures’ the difference between the control action of the PID and the nonlinear controller.
  - ▷ The decision variables are the PID controller parameters.

## APPLICATION TO A NONLINEAR PROCESS MODEL

- Process description:

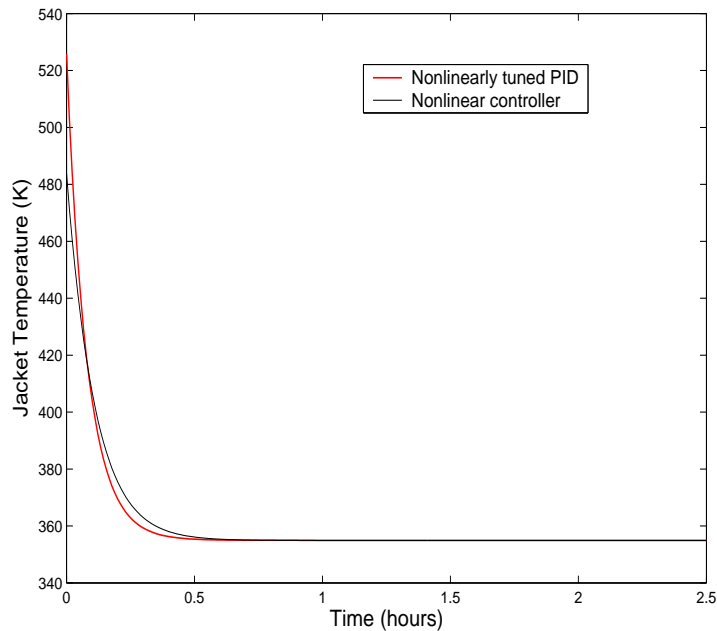
$$\begin{aligned}\frac{dT}{dt} &= \frac{F}{V}(T_{A0} - T) + \sum_{i=1}^3 \frac{(-\Delta H_i)}{\rho c_p} k_{i0} e^{\frac{-E_i}{RT}} C_A + \frac{Q}{\rho c_p V} \\ \frac{dC_A}{dt} &= \frac{F}{V}(C_{A0} - C_A) - \sum_{i=1}^3 k_{i0} e^{\frac{-E_i}{RT}} C_A\end{aligned}$$

- ◇ Three steady-states (two locally asymptotically stable and one unstable).
- ◇ Stabilize the reactor at the open-loop unstable steady-state using the jacket temperature as the manipulated input.

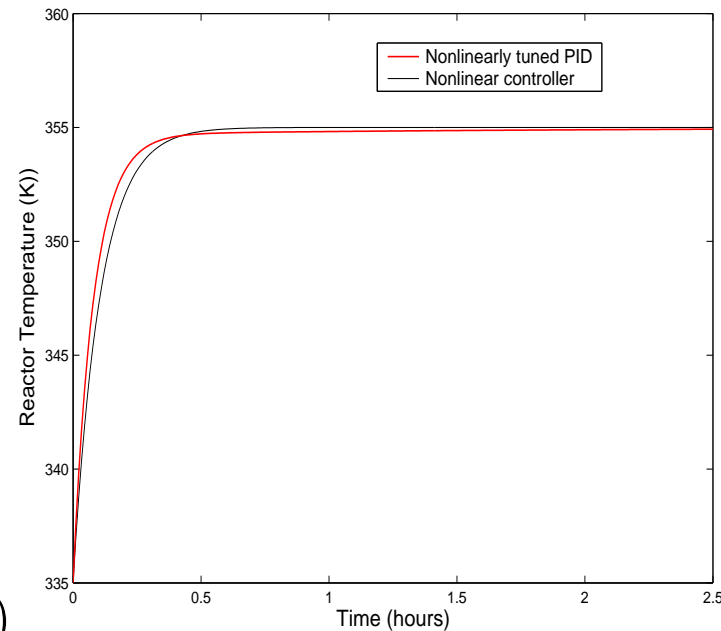


# CLOSED-LOOP SIMULATION RESULTS

“Nonlinear-control based PID tuning”



(a)



(b)

- ◇ (a) Input & (b) state profiles.
- ◇ The nonlinear controller is only designed, **not** implemented.
- ◇ PID controller designed to ‘emulate’ the action of the nonlinear controller.

## CONCLUSIONS

- Issues in Process Control:
  - ◇ Nonlinearities, uncertainties, constraints and state estimation.
- Integrate tools from nonlinear control theory with existing process control practice.
  - ◇ Safety-net for predictive control implementation.
  - ◇ Improved tuning of classical controllers.
- Practical implementation:
  - ◇ Software: Direct incorporation into existing MPC packages.
  - ◇ Hardware: Does not require redesigning existing classical control hardware.

## ACKNOWLEDGEMENT

- Financial support from NSF, CTS-0129571, is gratefully acknowledged